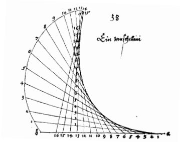

Current and Voltage Excitations for the Eddy Current Model

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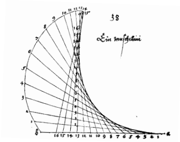

Eddy Current Approximation

material laws

$$
\begin{aligned}
\mathbf{curl}\,\mathbf{H} &= \mathbf{J} \\
\mathbf{curl}\,\mathbf{E} &= -\partial_t \mathbf{B} \\
\operatorname{div}\mathbf{B} &= 0 \\
\operatorname{div}\mathbf{D} &= \rho
\end{aligned}
\qquad\qquad
\begin{aligned}
\mathbf{D} &= \epsilon \mathbf{E} \\
\mathbf{B} &= \mu \mathbf{H} \\
\mathbf{J} &= \sigma \mathbf{E} + \mathbf{J}_g
\end{aligned}
$$

$\mathbf{E}$: electric field strength

$\mathbf{H}$: magnetic field strength

$\mathbf{D}$: dielectric displacement

$\mathbf{B}$: magnetic induction

$\mathbf{J}$: current density

ρ: charge density

ϵ: permittivity

μ: permeability

σ: conductivity

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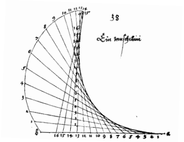
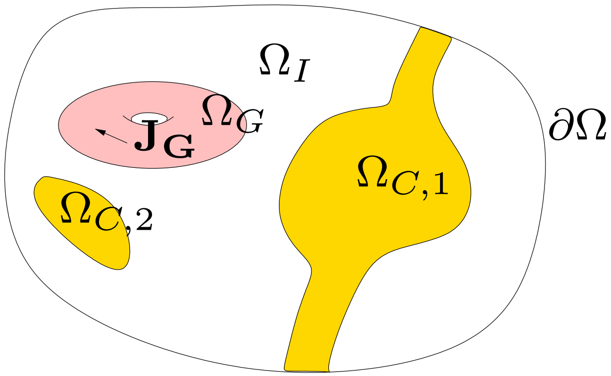


Eddy Current Setting

- Eddy Current Model
- Variational Formulations
- Coupling Fields and Circuits
- Generator Currents
- Excitation by Contacts
- Nonlocal Excitations
- Summary



a typical eddy current setting

Ω_C:	union of all conductors $\Omega_{C,i}$
Ω_I:	insulator
$\overline{\Omega} =$	$\overline{\Omega}_C \cup \overline{\Omega}_I$
$\Omega_G =$	$\operatorname{supp} J_G$
$\partial\Omega =$	$\partial\Omega_e \cup \partial\Omega_h$,
	$\partial\Omega_e \cap \partial\Omega_h = \emptyset$

boundary conditions:

$\mathbf{n} \times \mathbf{E} = \mathbf{f}$ on $\partial\Omega_e \subset \partial\Omega$ and $\mathbf{n} \times \mathbf{H} = \mathbf{g}$ on $\partial\Omega_h \subset \partial\Omega$

important spaces:

$$\boldsymbol{H}(\mathbf{curl};\Omega) := \{\mathbf{u} \in L^2(\Omega), \mathbf{curl}\,\mathbf{u} \in L^2(\Omega)\}$$

$$\boldsymbol{H}_0(\mathbf{curl};\Omega) := \{\mathbf{u} \in L^2(\Omega), \mathbf{curl}\,\mathbf{u} \in L^2(\Omega), \mathbf{n} \times \mathbf{u}|_{\partial\Omega} = 0\}$$

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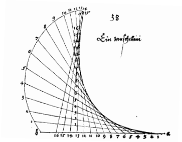

$\mathbb{H}$-based Formulation (magnetic)

$$\boldsymbol{\mathcal{V}}(\mathbf{J}_g, \mathbf{g}) := \{\mathbf{H}' \in \boldsymbol{H}(\mathbf{curl};\Omega), \mathbf{curl}\,\mathbf{H}' = \mathbf{J}_g \text{ in } \Omega_I, \mathbf{n}\times\mathbf{H}' = \mathbf{g} \text{ on } \partial\Omega_h\}$$

$$\boldsymbol{\mathcal{V}}_0 := \boldsymbol{\mathcal{V}}(0,0)$$

Find $\mathbf{H} \in C^1(]0,T[, \boldsymbol{\mathcal{V}}(\mathbf{J}_g, \mathbf{g}))$, such that for all $\mathbf{H}' \in \boldsymbol{\mathcal{V}}_0$

$$\int_{\Omega_C} \frac{1}{\sigma}\,\mathbf{curl}\,\mathbf{H}\cdot\mathbf{curl}\,\mathbf{H}'\,d\mathbf{x} + \int_{\Omega} \partial_t(\mu\mathbf{H})\cdot\mathbf{H}'\,d\mathbf{x}$$

$$= \int_{\Omega_C} \frac{1}{\sigma}\,\mathbf{J}_G\cdot\mathbf{curl}\,\mathbf{H}'\,d\mathbf{x} + \int_{\partial\Omega_e} \underbrace{(\mathbf{n}\times\mathbf{E})}_{=\mathbf{f}}\cdot\mathbf{H}'\,dS\,.$$

(initial value skipped here)

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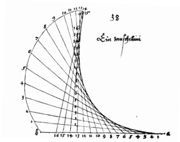

A-based Formulation (electric)

- $\operatorname{div}\mathbf{B} = 0$ in $\mathbb{R}^3 \Longrightarrow \mathbf{B} = \mathbf{curl}\,\mathbf{A}$
- $\Longrightarrow \mathbf{E} = -\partial_t \mathbf{A} - \mathbf{grad}\, v$ (v: scalar potential)
- “temporal gauge” $\Longrightarrow \mathbf{E} = -\partial_t \mathbf{A}$

$$\mathcal{W}(\mathbf{f}) := \{\mathbf{A}' \in \boldsymbol{H}(\mathbf{curl};\Omega), \mathbf{n}\times\mathbf{A}' = -\int \mathbf{f}\,dt \text{ on } \partial\Omega_e\}$$

Find $\mathbf{A} \in C^1(]0,T[,\mathcal{W}(\mathbf{f}))$, such that for all $\mathbf{A}' \in \mathcal{W}(0)$

$$\int_\Omega \frac{1}{\mu}\mathbf{curl}\,\mathbf{A}\cdot\mathbf{curl}\,\mathbf{A}'\,d\mathbf{x} + \int_{\Omega_C} \sigma\partial_t\mathbf{A}\cdot\mathbf{A}'\,d\mathbf{x}$$

$$= \int_\Omega \mathbf{J}_G\cdot\mathbf{A}'\,d\mathbf{x} - \int_{\partial\Omega_h} \underbrace{(\mathbf{n}\times\mathbf{H})}_{=\mathbf{g}}\cdot\mathbf{A}'\,dS\,.$$

(again initial value skipped)

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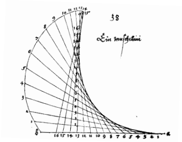

$\mathbf{A}$-based Formulation (electric) II

remark on the uniqueness

- “ungauged” formulation $\Longrightarrow$
  in Ω_I $\mathbf{A}$ and $\mathbf{E} = -\partial_t \mathbf{A}$ are only unique modulo an “electrostatic part”
- $\mathbf{curl}\,\mathbf{E}$ and thus the magnetic field $\mathbf{H}$ is unique
- for uniqueness: fix conductor charges and $\operatorname{div}\mathbf{A}$ in Ω_I
- in most situations the “electrostatic part” is of no interest

▼

don’t care about non-uniqueness

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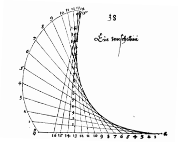
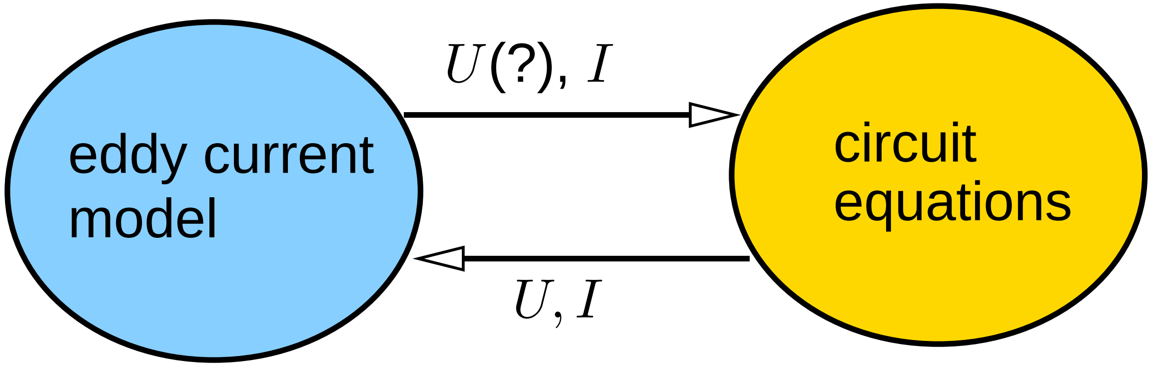


Coupling Quantities

Desirable: coupling by U and I



in eddy current model:

- $I = \int_{\Sigma} \mathbf{J} \cdot \mathbf{n} \, dS$
- however $U_\gamma = \int_{\gamma} \mathbf{E} \cdot d\mathbf{s}$ depends on path γ!

▶ Using U_γ for coupling fields and circuits cannot be accomplished. ▶ *define* voltage through *power*

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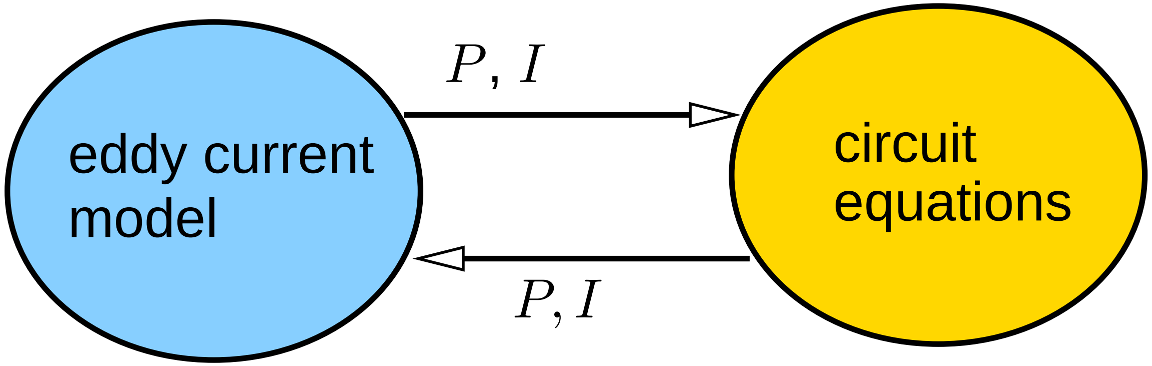
Coupling by I and P —Circuit View

Do coupling by conservation of current I and power P:



circuit view:

- eddy current problem is seen as a one (or multi) port from circuit model
- define voltage drop at (every) port by

$$U = \frac{P}{I}$$

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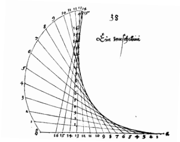
Coupling by I and P —Eddy Current View



eddy current view:

- power balance implied by the eddy current model (magneto-quasistatic Poynting theorem):

$$P_{mag} + P_{Ohm} = P = P_\Omega + P_{\partial\Omega}$$

with

$$P_{mag} := \int_\Omega \partial_t \mathbf{B} \cdot \mathbf{H} \, d\mathbf{x} \, , \qquad P_{Ohm} := \int_{\Omega_C} \sigma \, |\mathbf{E}|^2 \, d\mathbf{x}$$

$$P_\Omega := -\int_\Omega \mathbf{E} \cdot \mathbf{J}_G \, d\mathbf{x} \, , \qquad P_{\partial\Omega} = -\int_{\partial\Omega} (\mathbf{E} \times \mathbf{H}) \cdot \mathbf{n} \, dS \, .$$

- sources are generator current distributions or inhomogeneous boundary conditions

Now look at several different variational formulations for coupling...

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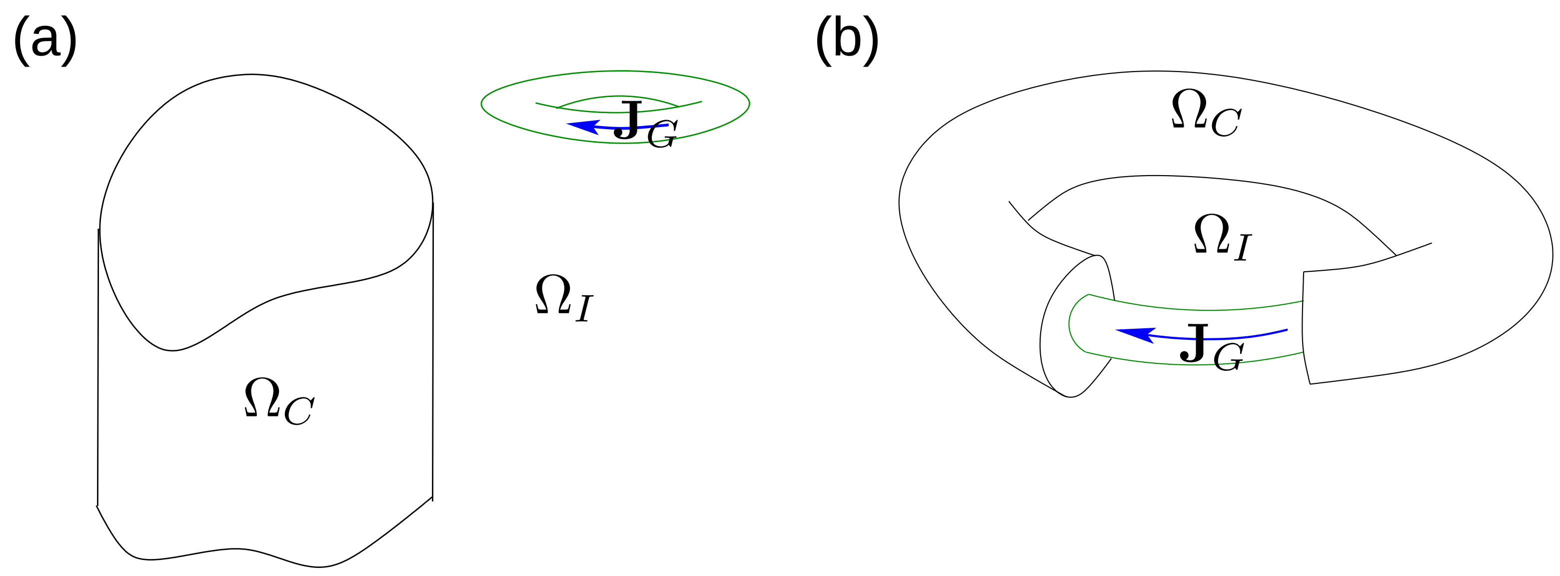
Generator Current Settings



(a) closed current loops in Ω_I, that is $\operatorname{supp}\mathbf{J}_g \subset \Omega_I$, which model coils with known currents

(b) current sources adjacent to conductors, i.e., $\operatorname{supp}\mathbf{J}_g \cap \overline{\Omega_C} \neq \emptyset$

- we only consider $\operatorname{supp}\mathbf{J}_g \subset \overline{\Omega_I}$, $\mathbf{n}\times\mathbf{E} = 0$ on $\partial\Omega$ for simplicity

$\mathbb{H}$-based Current Excitation

- choose a $\mathbf{J}_G$ such that $I = \int_\Sigma \mathbf{J}_G \cdot \mathbf{n}\, dS$
- choose $\mathbf{H}_G \in C^1(]0,T[, \boldsymbol{H}(\mathbf{curl};\Omega))$ such that $\mathbf{curl}\,\mathbf{H}_G = \mathbf{J}_G$ for all times (e.g. by the Biot-Savart law)
- variational formulation
  Seek $\mathbf{H} \in \mathbf{H}_G + C^1(]0,T[, \boldsymbol{\mathcal{V}}_0)$ such that for all $\mathbf{H}' \in \boldsymbol{\mathcal{V}}_0$

$$\int_{\Omega_C} \frac{1}{\sigma} \mathbf{curl}\,\mathbf{H} \cdot \mathbf{curl}\,\mathbf{H}'\, d\mathbf{x} + \int_\Omega \partial_t(\mu \mathbf{H}) \cdot \mathbf{H}'\, d\mathbf{x} = 0\,.$$

- power:
  $P = \int_{\Omega_C} \frac{1}{\sigma} |\mathbf{curl}\,\mathbf{H}|^2\, d\mathbf{x} + \int_\Omega \partial_t(\mu\mathbf{H}) \cdot \mathbf{H}\, d\mathbf{x} = \int_\Omega \partial_t(\mu\mathbf{H}) \cdot \mathbf{H}_G\, d\mathbf{x}\ ,$
- voltage is given by $U \cdot I = \int_\Omega \partial_t(\mu\mathbf{H}) \cdot \mathbf{H}_G\, d\mathbf{x}$

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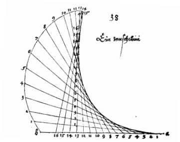

$\mathbf{H}$-based Voltage Excitation

- use *scaled* quantities to represent a unit current source

$$\mathbf{J}_G = I\,\mathbf{J}_0\,, \qquad \int_\Sigma \mathbf{J}_0 \cdot \mathbf{n}\, dS = 1\,, \qquad \mathbf{curl}\,\mathbf{H}_0 = \mathbf{J}_0$$

▼

voltage can be written as

$$U = \int_\Omega \partial_t(\mu\mathbf{H}) \cdot \mathbf{H}_0\, d\mathbf{x}$$

- variational formulation for voltage excitation: Seek $\mathbf{H} \in C^1(]0,T[, \boldsymbol{\mathcal{V}}_0)$ and $I \in C^1(]0,T[)$ such that for all $\mathbf{H}' \in \boldsymbol{\mathcal{V}}_0$

$$\begin{aligned} \int_{\Omega_C} \frac{1}{\sigma}\,\mathbf{curl}\,\mathbf{H} \cdot \mathbf{curl}\,\mathbf{H}'\, d\mathbf{x} + \int_\Omega \partial_t(\mu(\mathbf{H} + I\,\mathbf{H}_0)) \cdot \mathbf{H}'\, d\mathbf{x} &= 0 \\ \int_\Omega \partial_t(\mu\mathbf{H}) \cdot \mathbf{H}_0\, d\mathbf{x} &= U\,. \end{aligned}$$

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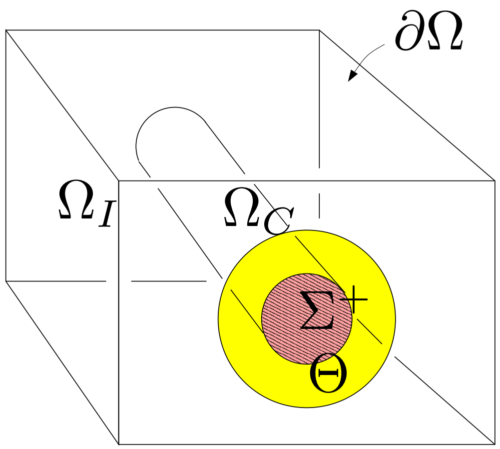
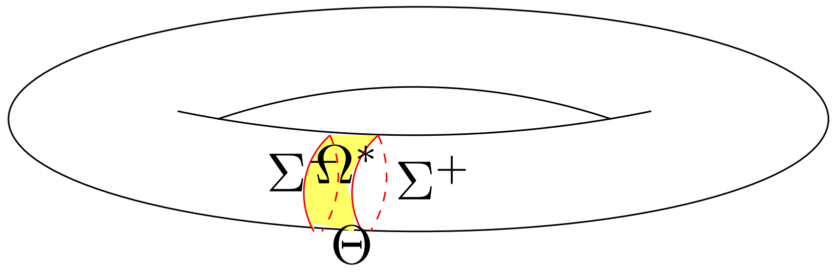
Contact Settings

(a)



(b)

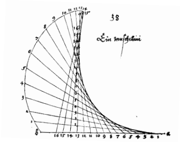


(a) contacts are located where Ω_C meets $\partial\Omega$ ("exterior" boundary conditions)

(b) contacts Σ^+, Σ^- and Θ bound electromotive region Ω^* ("hole in the universe")

- Note: adding Ω^* to Ω_C creates a new loop.

- In both situations there will be an energy flux through Θ or $\Theta \cup \Sigma^+ \cup \Sigma^-$.



H-based Voltage Excitation

- Voltage excitation realized by means of BC for the electric field

$$\mathbf{n} \times \mathbf{E} = 0 \ \text{ on } \partial\Omega \setminus \Theta\,, \qquad \mathbf{n} \times \mathbf{E} = -U(t)\,\mathbf{grad}_\Gamma\, v \ \text{ on } \Theta\,,$$

where $v|_{\Sigma^+} = 1\,, \quad v|_{\partial\Omega\setminus(\Theta\cup\Sigma^+)} = 0\,, \quad v \in H^{\frac{1}{2}}(\partial\Omega)\,.$

- plugging into the boundary term of the variational formulation

$$\int_{\partial\Omega} (\mathbf{n} \times \mathbf{E}) \cdot \mathbf{H}'\, dS = U \int_{\partial\Omega} \mathbf{grad}_\Gamma\, v \cdot (\mathbf{n} \times \mathbf{H}')\, dS = U \int_{\gamma^+} \mathbf{H}' \cdot d\mathbf{s}\,,$$

where $\gamma^+ = \partial\Sigma^+$.

- variational formulation with voltage excitation:

Seek $\mathbf{H} \in C^1(]0,T[,\boldsymbol{\mathcal{V}}_0)$ such that for all $\mathbf{H}' \in \boldsymbol{\mathcal{V}}_0$

$$\int_{\Omega_C} \frac{1}{\sigma}\,\mathbf{curl}\,\mathbf{H} \cdot \mathbf{curl}\,\mathbf{H}'\, d\mathbf{x} + \int_{\Omega} \partial_t(\mu\mathbf{H}) \cdot \mathbf{H}'\, d\mathbf{x} = U \int_{\gamma^+} \mathbf{H}' \cdot d\mathbf{s}\,.$$

$\mathbf{H}$-based Voltage Excitation II

Some Observations

- by Ampere's law we have $I = -\int_{\gamma^+} \mathbf{H} \cdot d\mathbf{s}$
- using the power balance again we get $P = U\ I$
  - introduced U matches the definition based on power
- The RHS in the variational formulation is of the form

$$U \cdot f(\mathbf{H}')\,,$$

  where f is a continuous functional on $\boldsymbol{\mathcal{V}}_0$ measuring the total current through a contact.
- in situation (a) the geometry of Θ does not enter the variational formulation at all

Θ has no impact on $\mathbf{H}$! (But on $\mathbf{E}$!)

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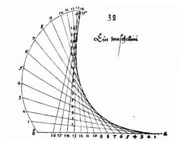

H-based Current Excitation

- one possibility to impose total current $I \in C^1(]0,T[)$ through contacts: prescribe $\mathbf{J}\cdot\mathbf{n} \rightsquigarrow I = -\int_{\Sigma^+} \mathbf{J}\cdot\mathbf{n}\,dS$
- chose $\mathbf{H}_{j_n} \in C^1(]0,T[,\boldsymbol{H}(\mathbf{curl};\Omega))$ such that
  - $\mathrm{div}_\Gamma(\mathbf{H}_{j_n}\times\mathbf{n}) = \mathbf{curl}\,\mathbf{H}_{j_n}\cdot\mathbf{n} = (\mathbf{J}\cdot\mathbf{n})/I$ on $\partial\Omega$,
  - $\mathbf{curl}\,\mathbf{H}_{j_n} = 0$ in Ω_I
- define $\boldsymbol{\mathcal{V}}_0^+ := \{\mathbf{H}' \in \boldsymbol{H}(\mathbf{curl};\Omega);\ \mathbf{curl}\,\mathbf{H}' = 0$ in Ω_I, $\mathrm{div}_\Gamma(\mathbf{H}'\times\mathbf{n}) = 0$ on $\partial\Omega\}$

▼

- variational formulation:

  Seek $\mathbf{H} \in I\,\mathbf{H}_{j_n} + C^1(]0,T[,\boldsymbol{\mathcal{V}}_0^+)$ such that for all $\mathbf{H}' \in \boldsymbol{\mathcal{V}}_0^+$

$$\int_{\Omega_C} \frac{1}{\sigma}\,\mathbf{curl}\,\mathbf{H}\cdot\mathbf{curl}\,\mathbf{H}'\,d\mathbf{x} + \int_{\Omega} \partial_t(\mu\mathbf{H})\cdot\mathbf{H}'\,d\mathbf{x} = 0\,.$$

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H-based Current Excitation II

Some Remarks

- the variational formulation implies the boundary condition

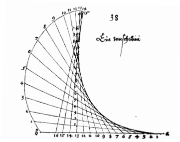
$$\partial_t \mathbf{B} \cdot \mathbf{n} = 0$$

  at the contacts

- for the voltage (defined by power) we have:

$$U = \frac{P}{I} = \int_{\Omega_C} \frac{1}{\sigma} \,\mathbf{curl}\,\mathbf{H} \cdot \mathbf{curl}\,\mathbf{H}_{j_n} \, d\mathbf{x} + \int_{\Omega} \partial_t(\mu \mathbf{H}) \cdot \mathbf{H}_{j_n} \, d\mathbf{x}$$

- Another option for enforcing a particular total current through the contacts is by means of a constraint, together with $\mathbf{n} \times \mathbf{E} = 0$ at the contacts.



$\mathbf{A}$-based Current Excitation by a Constraint

Eddy Current Model
Variational Formulations
Coupling Fields and Circuits
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Nonlocal Excitations
Summary

- contact touching an exterior PEC boundary
- No “temporal gauge” here: $\mathbf{E} = -\partial_t \mathbf{A} - U\,\mathbf{grad}\,\widetilde{v}$,
  $\widetilde{v}$ is $H^1(\Omega)$-extension of $v \in H^{\frac{1}{2}}(\partial\Omega)$,
  $v = 1$ on Σ^+, $v = 0$ on Σ^-, $\mathbf{A} \in \boldsymbol{H}_0(\mathbf{curl};\Omega)$
- variational formulation with constraint enforcing the current:

  Seek $\mathbf{A} \in C^1(]0,T[, \boldsymbol{H}_0(\mathbf{curl};\Omega))$ and $U \in C^1(]0,T[)$ such that for all $\mathbf{A}' \in \boldsymbol{H}_0(\mathbf{curl};\Omega)$

$$
\begin{aligned}
\int_\Omega \frac{1}{\mu}\,\mathbf{curl}\,\mathbf{A}\cdot\mathbf{curl}\,\mathbf{A}'\,d\mathbf{x} + \int_{\Omega_C} \sigma\,\partial_t\mathbf{A}\cdot\mathbf{A}'\,d\mathbf{x} + U\int_{\Omega_C}\sigma\;\mathbf{grad}\,\widetilde{v}\cdot\mathbf{A}'\,d\mathbf{x} &= 0\\
\int_{\Omega_C} \sigma\,\partial_t\mathbf{A}\cdot\mathbf{grad}\,\widetilde{v}\,d\mathbf{x} \qquad + U\int_{\Omega_C}\sigma\,|\,\mathbf{grad}\,\widetilde{v}\,|^2\,d\mathbf{x} &= I\,.
\end{aligned}
$$

- One can show: $\mathbf{B} = \mathbf{curl}\,\mathbf{A}$ *and thus* $\mathbf{E}|_{\Omega_C}$ *independent of the choice of* $\widetilde{v}$.

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Separated Contacts

Eddy Current Model
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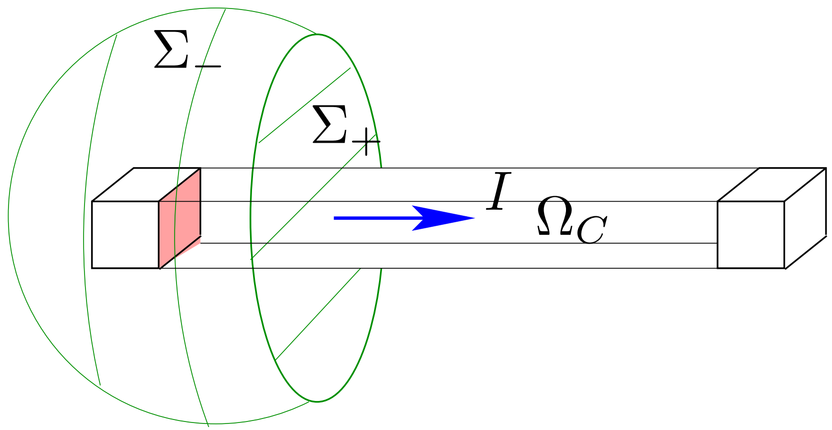
Summary

- The eddy current model cannot accommodate a current flowing out of Ω_C into Ω_I!



A nonzero current through a separated contact violates Ampere's law

- A contradiction:

$$0 \neq I = \int_{\Sigma_+} \mathbf{J} \cdot \mathbf{n}\, dS = \int_{\partial\Sigma^+} \mathbf{H} \cdot d\mathbf{s} = \int_{\partial\Sigma^-} \mathbf{H} \cdot d\mathbf{s} = \int_{\Sigma_-} \mathbf{J} \cdot \mathbf{n}\, dS = 0 \qquad \square$$

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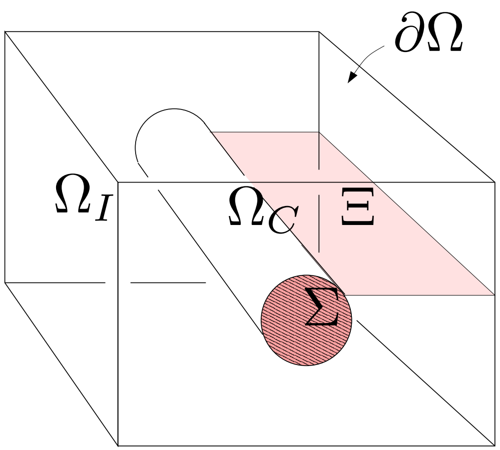
Nonlocal Excitations

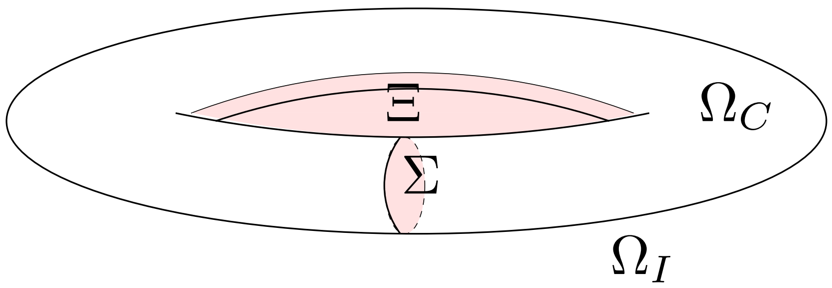
Eddy Current Model
Variational Formulations
Coupling Fields and Circuits
Generator Currents
Excitation by Contacts
Nonlocal Excitations
Summary

- In many situations detailed information about contacts and/or exciting current distributions is not available.
- Are there nevertheless possibilities to impose currents and voltages?
- Idea: Remove Θ and use topological concepts!

(a)



(b)



(a) Conductor touching $\partial\Omega$. Cutting surface Ξ in Ω_I is depicted.

(b) Conducting loop away from $\partial\Omega$, closed by Seifert surface Ξ in Ω_I and cut by surface Σ inside Ω_C.

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H-based Voltage Excitation

Eddy Current Model
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Summary

- variational formulation copied from the excitation by contacts case:

  Seek $\mathbf{H} \in C^1(]0,T[,\boldsymbol{\mathcal{V}}_0)$ such that for all $\mathbf{H}' \in \boldsymbol{\mathcal{V}}_0$

$$\int\limits_{\Omega_C} \frac{1}{\sigma} \,\mathbf{curl}\,\mathbf{H} \cdot \mathbf{curl}\,\mathbf{H}'\, d\mathbf{x} + \int\limits_{\Omega} \partial_t(\mu \mathbf{H}) \cdot \mathbf{H}'\, d\mathbf{x} = U \int\limits_{\gamma^+} \mathbf{H}' \cdot d\mathbf{s}\,.$$

- In situation (a) nothing has changed!
- In situation (b) we incorporated Ω^* into Ω_C.

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H-based Current Excitation

- remember definition

  $$\boldsymbol{\mathcal{V}}_0 := \{\mathbf{H}' \in \boldsymbol{H}(\mathbf{curl};\Omega), \mathbf{curl}\,\mathbf{H}' = 0 \text{ in } \Omega_I\}$$

- $\mathbf{curl}\,\mathbf{H}' = 0$ in $\Omega_I \quad \Rightarrow$

  $$\boldsymbol{\mathcal{V}}_0 \ni \mathbf{H}'|_{\Omega_I} = \mathbf{grad}\,\phi + \mathbf{q}\,, \qquad \phi \in H^1(\Omega_I)$$

  $\mathbf{q}$: representative of first co-homology group $\mathcal{H}^1(\Omega_I, \mathbb{R})$

- construction of $\mathbf{q}$ with help of cut (Seifert surface) Ξ:

  $$\mathbf{q} := \widetilde{\mathbf{grad}\,\theta}\,, \qquad \theta \in H^1(\Omega_I \setminus \Xi)\,, \qquad [\theta]_\Xi = 1$$

- $\tilde{\mathrm{q}}$: $\boldsymbol{H}(\mathbf{curl};\Omega)$ extension of $\mathbf{q}$

- Idea to impose current I: fix contribution from $\tilde{\mathrm{q}}$ to $\boldsymbol{\mathcal{V}}_0$ (and remove it from the test space)

- $$\widetilde{\boldsymbol{\mathcal{V}}}_0 := \{\mathbf{H}' \in \boldsymbol{\mathcal{V}}_0\,, \int_{\partial\Sigma} \mathbf{H}' \cdot d\mathbf{s} = 0\}$$

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H-based Current Excitation II

- variational formulation to prescribe current:

  Seek $\mathbf{H} \in I\widetilde{\mathbf{q}} + C^1(]0,T[, \widetilde{\boldsymbol{\mathcal{V}}}_0)$ such that for all $\mathbf{H}' \in \widetilde{\boldsymbol{\mathcal{V}}}_0$

$$\int_{\Omega_C} \frac{1}{\sigma} \,\mathbf{curl}\,\mathbf{H} \cdot \mathbf{curl}\,\mathbf{H}'\, d\mathbf{x} + \int_{\Omega} \partial_t(\mu \mathbf{H}) \cdot \mathbf{H}'\, d\mathbf{x} = 0\,.$$

- Recovery of voltage:

$$U = \frac{P}{I} = \int_{\Omega_C} \frac{1}{\sigma} \,\mathbf{curl}\,\mathbf{H} \cdot \mathbf{curl}\,\widetilde{\mathbf{q}}\, d\mathbf{x} + \int_{\Omega} \partial_t(\mu \mathbf{H}) \cdot \widetilde{\mathbf{q}}\, d\mathbf{x}\,.$$

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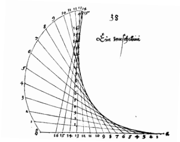


Inconsistencies

Eddy Current Model

Variational Formulations

Coupling Fields and Circuits

Generator Currents

Excitation by Contacts

Nonlocal Excitations

Summary

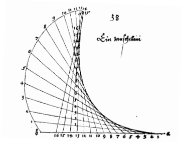
- “There are no sources in the model—and yet there is a non-zero current?!”
- One can show: *The nonlocal variational formulations violate (a weak variant of) the Faraday law along the cut*

$$\int_{\partial\Xi} \mathbf{E}_C \cdot d\mathbf{s} = - \int_{\Xi} \partial_t(\mu\,\mathbf{H}) \cdot \mathbf{n}\, dS\ ,$$

which plays the role of a compatibility condition for the RHSs of the exterior electric problem

$$\begin{aligned} \mathbf{n} \times \mathbf{E} &= \mathbf{n} \times \mathbf{E}_C && \text{on } \Gamma_C\ , \\ \mathbf{curl}\,\mathbf{E} &= -\partial_t(\mu\,\mathbf{H}) && \text{in } \Omega_I\ , \\ \operatorname{div}(\epsilon\,\mathbf{E}) &= 0 && \text{in } \Omega_I\ , \end{aligned}$$

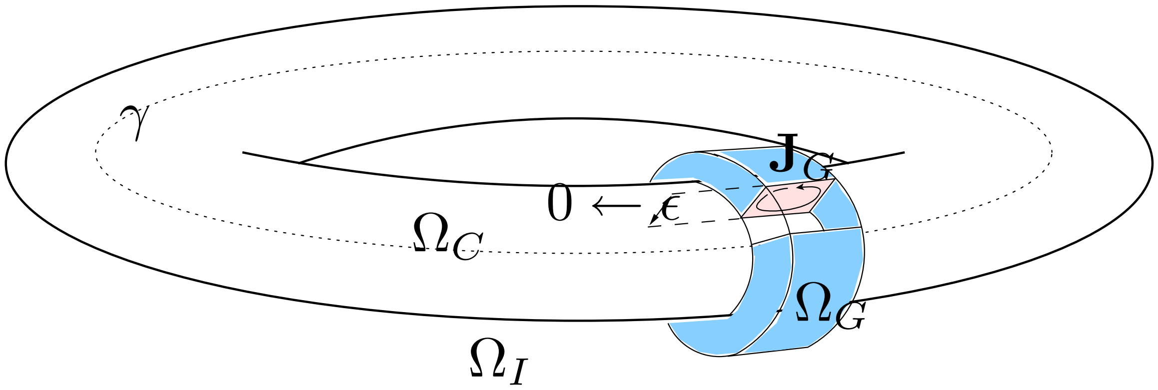
$$\int_{\Gamma_i} \epsilon\,\mathbf{E} \cdot \mathbf{n}\, dS = 0\ .$$



Physical Interpretation of Nonlocal Excitations

- Eddy Current Model
- Variational Formulations
- Coupling Fields and Circuits
- Generator Currents
- Excitation by Contacts
- Nonlocal Excitations
- Summary

- There is no electric field $\mathbf{E} \in \boldsymbol{H}(\mathbf{curl};\Omega)$ that matches $\mathbf{H}$!
- Allow jumps $[\mathbf{E} \times \mathbf{n}]_{\Gamma_C} \neq 0 \Rightarrow \quad \mathbf{E} \notin \boldsymbol{H}(\mathbf{curl};\Omega)$!
- nonlocal excitations can be interpreted as idealized thin coils with zero exterior magnetic field:



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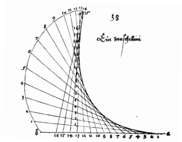


A-based Nonlocal Excitations

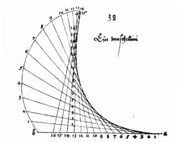
variational formulations for voltage and current excitation:

- $\mathbf{p}$: representative of co-homology group $\mathcal{H}^1(\Omega_C, \mathbb{R})$ (situation b), or a gradient field (a), extended by zero in Ω_I, $\mathbf{E} = -\partial_t \mathbf{A} - U\widetilde{\mathbf{p}}$
- Seek $\mathbf{A} \in C^1(]0,T[, \boldsymbol{H}_0(\mathbf{curl};\Omega))$ such that for all $\mathbf{A}' \in \boldsymbol{H}_0(\mathbf{curl};\Omega)$

$$\int_\Omega \frac{1}{\mu}\,\mathbf{curl}\,\mathbf{A}\cdot\mathbf{curl}\,\mathbf{A}'\,d\mathbf{x} + \int_{\Omega_C} \sigma\,\partial_t\mathbf{A}\cdot\mathbf{A}'\,d\mathbf{x} = -U\int_{\Omega_C}\sigma\mathbf{p}\cdot\mathbf{A}'\,d\mathbf{x}\,.$$

- Seek $\mathbf{A} \in C^1(]0,T[, \boldsymbol{H}_0(\mathbf{curl};\Omega))$ and $U \in C^1(]0,T[,\mathbb{R})$ such that for all $\mathbf{A}' \in \boldsymbol{H}_0(\mathbf{curl};\Omega)$

$$\begin{aligned}
\int_\Omega \frac{1}{\mu}\,\mathbf{curl}\,\mathbf{A}\cdot\mathbf{curl}\,\mathbf{A}'\,d\mathbf{x} + \int_{\Omega_C} \sigma\,\partial_t\mathbf{A}\cdot\mathbf{A}'\,d\mathbf{x} + U\int_{\Omega_C}\sigma\,\mathbf{p}\cdot\mathbf{A}'\,d\mathbf{x} &= 0\\
\int_{\Omega_C}\sigma\,\partial_t\mathbf{A}\cdot\mathbf{p}\,d\mathbf{x} \qquad + U\int_{\Omega_C}\sigma\,|\mathbf{p}|^2\,d\mathbf{x} &= I\,.
\end{aligned}$$



Summary

- coupling eddy currents and circuit equations by conservation of power and current
- define voltage by power
- dual variational formulations in case of
  - given generator current distributions
  - contacts at the boundary with known normal component of the current densities or PEC type contacts
  - nonlocal excitations
- in case of nonlocal excitations Faraday's law will be violated...
- ...but if we abandon tangent continuity of $\mathbf{E}$ we find an interpretation of nonlocal sources as inductive excitation by thin coils

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