Parameterised Electromagnetic Scattering Solutions for a Range of Incident Wave Directions

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Outline of the Presentation

The presentation will discuss

- Frequency domain variational statement;
- Arbitrary order $\boldsymbol{H}(\text{curl})$ conforming discretisation;
- Application to 2D scattering problems;
- The need for a reduced–order model;
- Reduced order model formulation;
- Construction of certainty bounds;
- Numerical examples.

Frequency Domain Formulation

Maxwells equations in the frequency domain reduce to

$$\text{curl}\,\frac{1}{\mu}\,\text{curl}\,\boldsymbol{E} - \omega^2\left(\epsilon - \mathrm{i}\frac{\sigma}{\omega}\right)\boldsymbol{E} = \boldsymbol{0}$$

$$\text{div}\,(\mathrm{i}\omega\epsilon + \sigma)\boldsymbol{E} = 0$$

with typical tangential boundary conditions

$$\begin{aligned} \boldsymbol{n} \times \boldsymbol{E} &= \boldsymbol{0} && \text{on } \Gamma_{PEC} \\ \boldsymbol{n} \times \text{curl}\,\boldsymbol{E} &= \boldsymbol{0} && \text{on } \Gamma_{PMC} \end{aligned}$$

Frequency Domain Formulation

Define

$$\boldsymbol{H}(\text{curl}\,\Omega) = \{\boldsymbol{v} \in (L_2(\Omega))^3; \text{curl}\,\boldsymbol{v} \in (L_2(\Omega))^3\}$$

$$\boldsymbol{H}_0(\text{curl}\,\Omega) = \{\boldsymbol{v} \in \boldsymbol{H}(\text{curl}\,\Omega), \boldsymbol{n} \wedge \boldsymbol{v} = \boldsymbol{0} \text{ on } \Gamma_{PEC}\}$$

(Kikuchi): Find $\boldsymbol{E} \in \boldsymbol{H}_0(\text{curl};\Omega)$, $p \in H_0^1(\Omega)$ such that

$$\begin{aligned} \left(\frac{1}{\mu}\text{curl}\,\boldsymbol{E}, \text{curl}\,\boldsymbol{W}\right)_\Omega - \omega^2\left(\left(\epsilon - \mathrm{i}\frac{\sigma}{\omega}\right)(\boldsymbol{E} + \nabla p), \boldsymbol{W}\right)_\Omega &= 0 \qquad \forall \boldsymbol{W} \in \boldsymbol{H}_0(\text{curl};\Omega) \\ \omega^2\left(\left(\epsilon - \mathrm{i}\frac{\sigma}{\omega}\right)\boldsymbol{E}, \nabla q\right)_\Omega &= 0 \qquad \forall q \in H_0^1(\Omega) \end{aligned}$$

where $H_0^1 = \{p \in H^1, p = 0 \text{ on } \Gamma_{PEC}\}$

Frequency Domain Formulation

For certain simulations with, $\omega > 0$ constant, the Lagrange multiplier $p \equiv 0$. Therefore use simplified variational statement: Find $\mathbf{E} \in \boldsymbol{H}_0(\text{curl};\Omega)$ such that

$$\left(\frac{1}{\mu}\text{curl } \boldsymbol{E}, \text{curl } \boldsymbol{W}\right)_\Omega - \omega^2 \left(\left(\epsilon - \mathrm{i}\frac{\sigma}{\omega}\right) \boldsymbol{E}, \boldsymbol{W}\right)_\Omega = 0 \qquad \forall \boldsymbol{W} \in \boldsymbol{H}_0(\text{curl};\Omega)$$

Discrete variational form: find $\boldsymbol{E}_H \in X_H \subset \boldsymbol{H}_0(\text{curl};\Omega)$ such that

$$\left(\frac{1}{\mu}\text{curl } \boldsymbol{E}_H, \text{curl } \boldsymbol{W}_H\right)_\Omega - \omega^2 \left(\left(\epsilon - \mathrm{i}\frac{\sigma}{\omega}\right) \boldsymbol{E}_H, \boldsymbol{W}_H\right)_\Omega = 0 \qquad \forall \boldsymbol{W}_H \in X_H$$

Construction of Ainsworth & Coyle's Edge Element Approximation

The edge degrees of freedom are chosen to be the weighted moments of the tangential component of the field on edge γ

$$\boldsymbol{E} \rightarrow \int_\gamma \omega_k \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r} \qquad k = 0, 1, \cdots, p$$

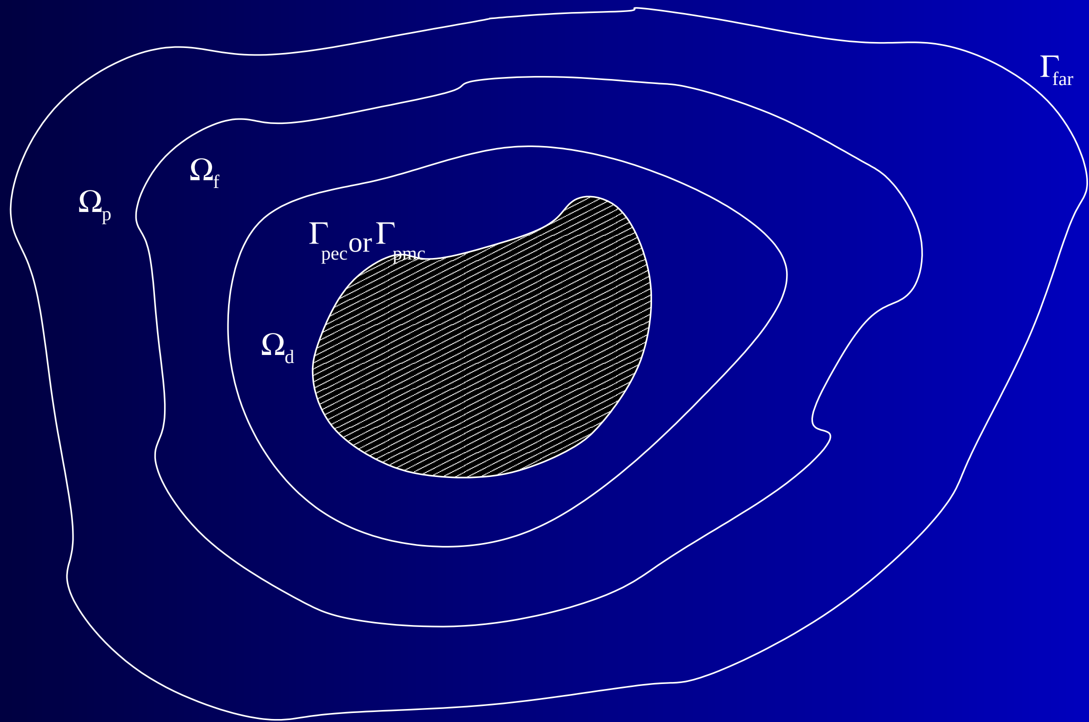
When the edge is parameterized by $s \in (-1, +1)$ then ω_k is chosen to be the k^{th} degree Legendre polynomial L_k.

The interior degrees of freedom have no compatibility condition on the interface. These are chosen to complete the polynomial space.

Ainsworth, Coyle Hierarchic hp-edge element families for Maxwell's equations in hybrid quadrilateral/triangular meshes. Comp. Meth. Appl. Mech. Eng. 2001;190:6709–6733.

2D Electromagnetic Scattering Problems

$$\boldsymbol{E} = \boldsymbol{E}^i + \boldsymbol{E}^s \qquad \Gamma = \Gamma_{PEC} + \Gamma_{PMC} + \Gamma_{FAR} \qquad \Omega = \Omega_d + \Omega_f + \Omega_p$$



Ledger et al. Arbitrary order edge elements for electromagnetic scattering simulations using hybrid meshes and a PML, Int.J Num. Meth. Eng. 2002;55:339–358.

Formulation for Scattering Problems

Find $\boldsymbol{E}_H^s$ in X_H^D

$$\mathcal{A}(\boldsymbol{E}_H^s, \boldsymbol{W}_H) = \ell(\boldsymbol{W}_H) \qquad \forall \boldsymbol{W}_H \in X_H$$

where

$$\begin{aligned} \mathcal{A}(\boldsymbol{E}_H^s, \boldsymbol{W}_H) &= \left(\frac{1}{\mu} \text{curl } \boldsymbol{E}_H^s, \text{curl } \boldsymbol{W}_H \right)_\Omega - \omega^2 \left(\left(\epsilon - \mathrm{i}\frac{\sigma}{\omega} \right) \boldsymbol{E}_H^s, \boldsymbol{W}_H \right)_\Omega \\ \ell(\boldsymbol{W}_H) &= \left(\boldsymbol{n} \times \text{curl } \boldsymbol{E}^i, \boldsymbol{W}_H \right)_{\Gamma_{PMC}} - \mathcal{A}(\boldsymbol{E}^i, \boldsymbol{W}_H) \end{aligned}$$

$$\begin{aligned} X_H^D \subset \boldsymbol{H}_D(\text{curl}) &= \{ \boldsymbol{v} \in \boldsymbol{H}(\text{curl}),\ \boldsymbol{n} \times \boldsymbol{v} = -\boldsymbol{n} \times \boldsymbol{E}^i \text{ on } \Gamma_{PEC} \text{ and } \boldsymbol{n} \times \boldsymbol{v} = \boldsymbol{0} \text{ on } \Gamma_{FAR} \} \\ X_H \subset \boldsymbol{H}_0(\text{curl}) &= \{ \boldsymbol{v} \in \boldsymbol{H}(\text{curl}),\ \boldsymbol{n} \times \boldsymbol{v} = \boldsymbol{0} \text{ on } \Gamma_{PEC} \text{ and } \boldsymbol{n} \times \boldsymbol{v} = \boldsymbol{0} \text{ on } \Gamma_{FAR} \} \end{aligned}$$

Output of Interest: RCS

The far field pattern (RCS) is a measure of the scattered wave in the far field. Its distribution is given by

$$\sigma(\boldsymbol{E}_H^s;\phi) = \mathcal{L}^{\mathcal{O}}(\boldsymbol{E}_H^s;\phi)\overline{\mathcal{L}^{\mathcal{O}}(\boldsymbol{E}_H^s;\phi)}$$

where

$$\mathcal{L}^{\mathcal{O}}(\boldsymbol{E}_H^s;\phi) = \int_{\Gamma_c} (\boldsymbol{n}\times\boldsymbol{E}_H\cdot\boldsymbol{V} - \boldsymbol{n}\wedge \text{curl}\ \boldsymbol{E}_H^s\cdot\boldsymbol{Y})\,\mathrm{d}\Gamma$$

and

$$\{\boldsymbol{V},\boldsymbol{Y}\} = \{-[0,0,1]^T, \frac{1}{\mathrm{i}\omega}[\sin\phi, -\cos\phi, 0]^T\}\exp\{\mathrm{i}\omega(x'\cos\phi + y'\sin\phi)\}$$

Why Use a Reduced Order Model?

An engineer designing components may wish to make small modifications to a design and investigate the change in an “output”. Variables may include:

- Changes in geometry;
- Changes in frequency;
- Changes in material parameters;
- Changes in incidence direction.

Each change requires a new computation, and for many changes this may be too expensive.

Reduced Order Model Description

- Off–line stage
  - N_θ Complete scattering solutions for incidences $\theta_1, \cdots, \theta_{N_\theta}$
  - N_ϕ Complete adjoint solutions for viewing angles $\phi_1, \cdots, \phi_{N_\phi}$
- On–line stage
  - For a new incident angle θ the scattering width is rapidly predicted.
  - Confidence bounds ensure reliability in output prediction.

Detailed Off-Line Description

N_θ and N_ϕ are prescribed by the user. We currently use equally spaced angles in both cases.

- Find $\boldsymbol{E}_H^s(\theta_i) \in X_H^D$, $i = 1, 2, \cdots, N_\theta$

$$\mathcal{A}(\boldsymbol{E}_H^s(\theta_i), \boldsymbol{W}) = \ell\,(\boldsymbol{W}; \theta) \qquad \forall \boldsymbol{W} \in X_H$$

- Find $\boldsymbol{\Psi}_H(\phi_i) \in X_H$, $i = 1, 2, \cdots, N_\phi$

$$\mathcal{A}(\boldsymbol{W}, \boldsymbol{\Psi}_H(\phi_i)) = -\mathcal{L}^{\mathcal{O}}(\boldsymbol{W}; \phi) \qquad \forall \boldsymbol{W} \in X_H$$

The solutions $\boldsymbol{E}_H^s(\theta_i)$, $i = 1, 2, \cdots, N_\theta$ and $\boldsymbol{\Psi}_H(\phi_i)$, $i = 1, 2, \cdots, N_\phi$ are stored and reused in the on–line stage.

Detailed On-Line Description

- Define

$$W_{N_\theta}^{\text{pr}} = \text{span}\{\boldsymbol{E}_H^s(\theta_i); i = 1, \cdots, N_\theta\} \qquad W_{N_\phi}^{\text{du}} = \text{span}\{\boldsymbol{\Psi}_H(\phi_i); i = 1, \cdots, N_\phi\}$$

- For a new θ, find $\boldsymbol{E}_{N_\theta}^s(\theta) \in W_{N_\theta}^{\text{pr}} \subset X_H^D$

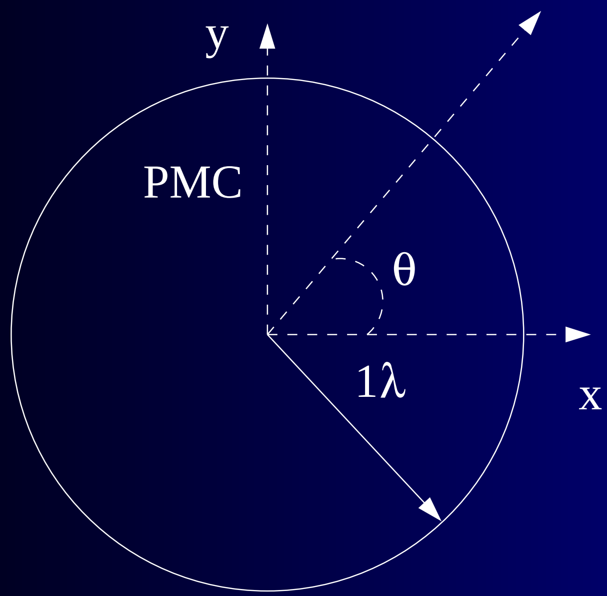
$$\mathcal{A}(\boldsymbol{E}_{N_\theta}^s, \boldsymbol{W}) = \ell(\boldsymbol{W}) \qquad \forall \boldsymbol{W} \in W_{N_\theta}^{\text{pr}}$$

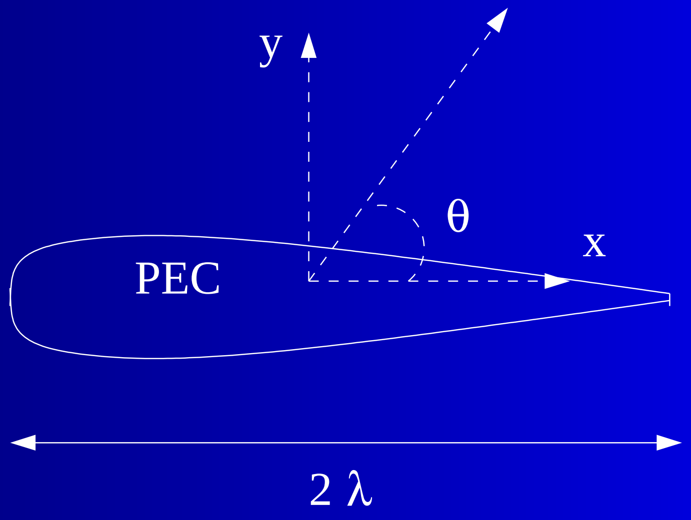
- For each ϕ, find, $\boldsymbol{\Psi}_{N_\phi}(\phi) \in W_{N_\phi}^{\text{du}} \subset X_H$ and $s_N(\theta, \phi) \in \mathbb{C}$

$$\mathcal{A}(\boldsymbol{W}, \boldsymbol{\Psi}_{N_\phi}) = -\mathcal{L}^{\mathcal{O}}(\boldsymbol{W}) \qquad \forall \boldsymbol{W} \in W_{N_\phi}^{\text{du}}$$

$$s_N = \mathcal{L}^{\mathcal{O}}(\boldsymbol{E}_{N_\theta}^s) - \left[\ell\left(\boldsymbol{\Psi}_{N_\phi}\right) - \mathcal{A}(\boldsymbol{E}_{N_\theta}^s, \boldsymbol{\Psi}_{N_\phi})\right] \qquad \sigma_N = s_N \overline{s_N}$$

Scattering Examples

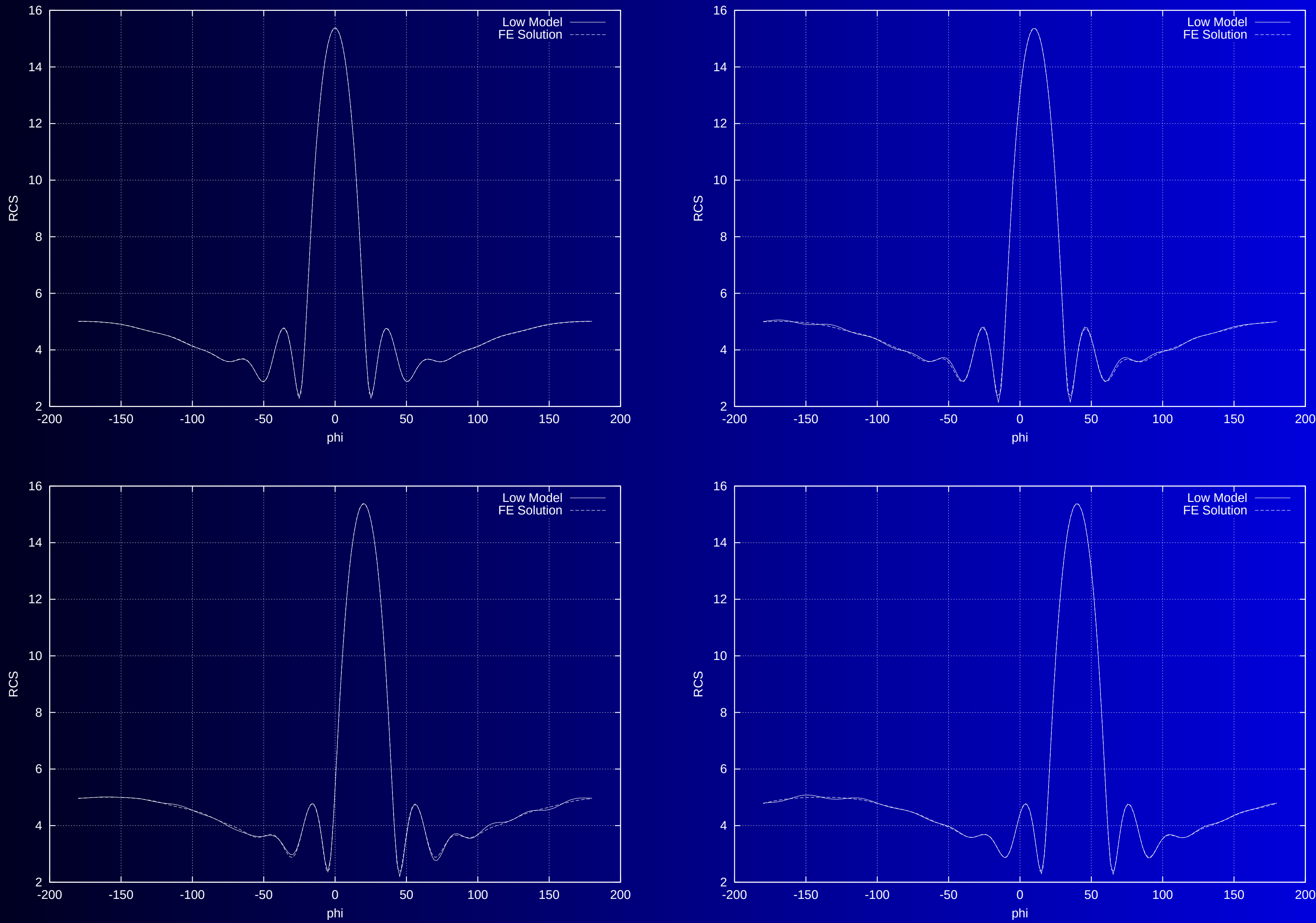




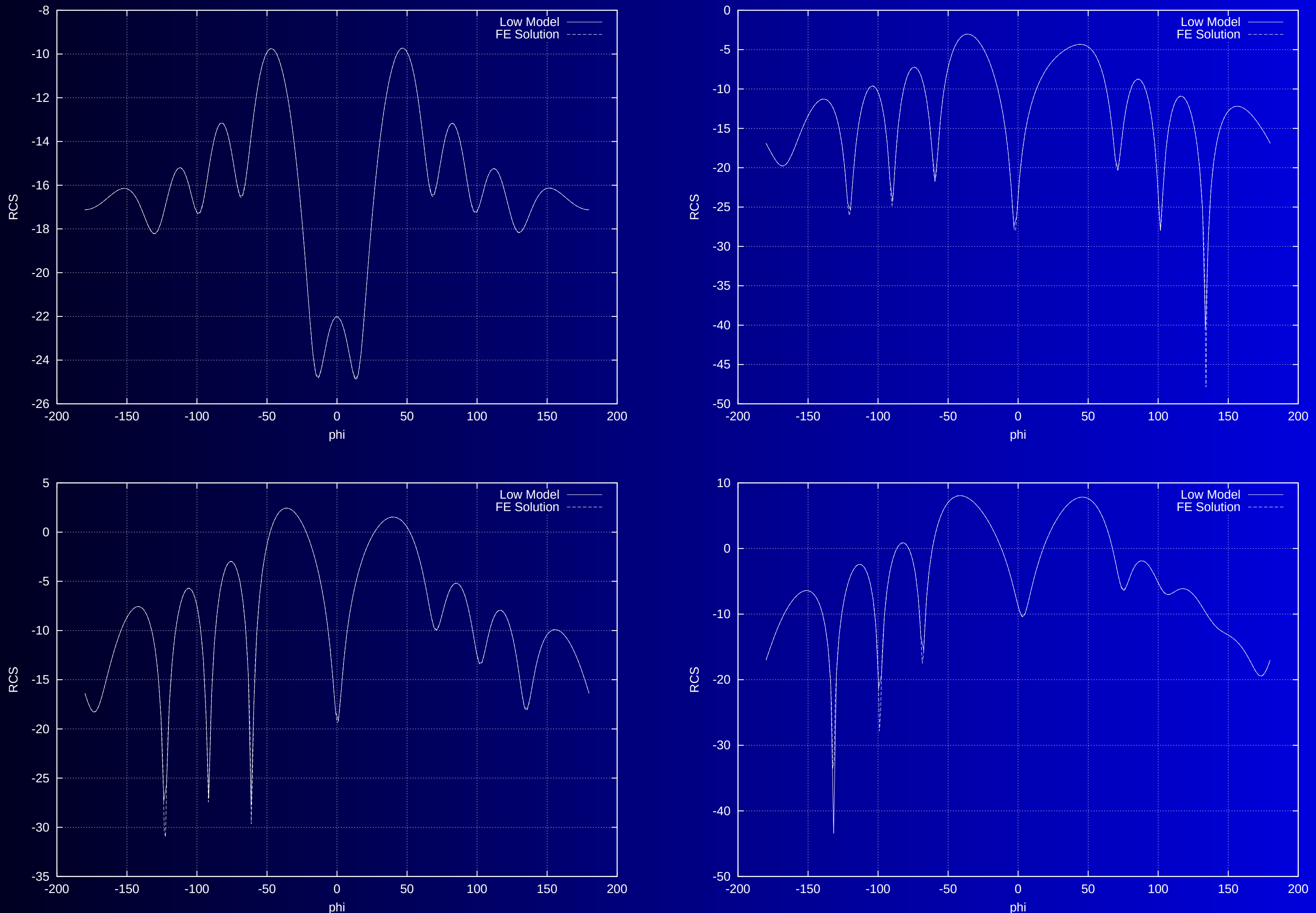
For each case

- N_θ and N_ϕ are specified and off–line solutions created;
- The RCS for a range of new θ values is computed.

Scattering by 2λ PMC Cylinder $\theta = 0,\ 10,\ 20,\ 40$



Scattering by 2λ PEC NACA $\theta = 0,\ 10,\ 20,\ 40$



Construction of Certainty Bounds 1

Consider the following residuals

- $R_E(\boldsymbol{W}) = \ell(\boldsymbol{W}) - \mathcal{A}(\boldsymbol{E}_{N_\theta}, \boldsymbol{W})$;
- $R_\Psi(\boldsymbol{W}) = \mathcal{L}^{\mathcal{O}}(\boldsymbol{W}) - \mathcal{A}(\boldsymbol{W}, \boldsymbol{\Psi}_{N_\phi})$.

whose discretised equivalents $\boldsymbol{R}^E$ and $\boldsymbol{R}^\Psi$ can be evaluated. It can be shown that certainty bounds on the reduced–order model output can be constructed using

$$|s_H - s_N| \leq \frac{\|\boldsymbol{R}^\Psi\| \cdot \|\boldsymbol{R}^E\|}{\min \mu_i} \qquad \Delta\sigma = (|s_H - s_N|^2)$$

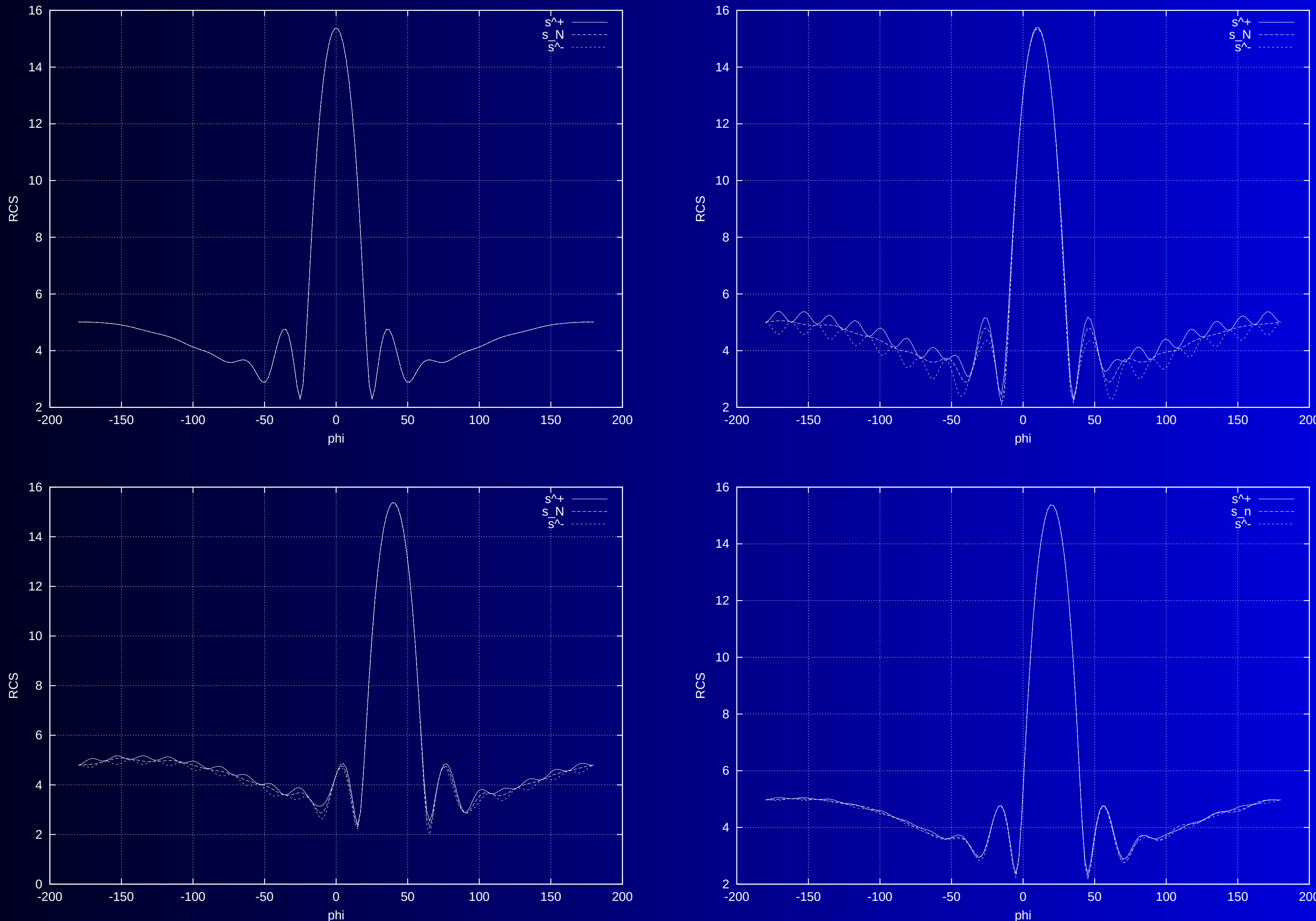
Construction of Certainty Bounds 2

where

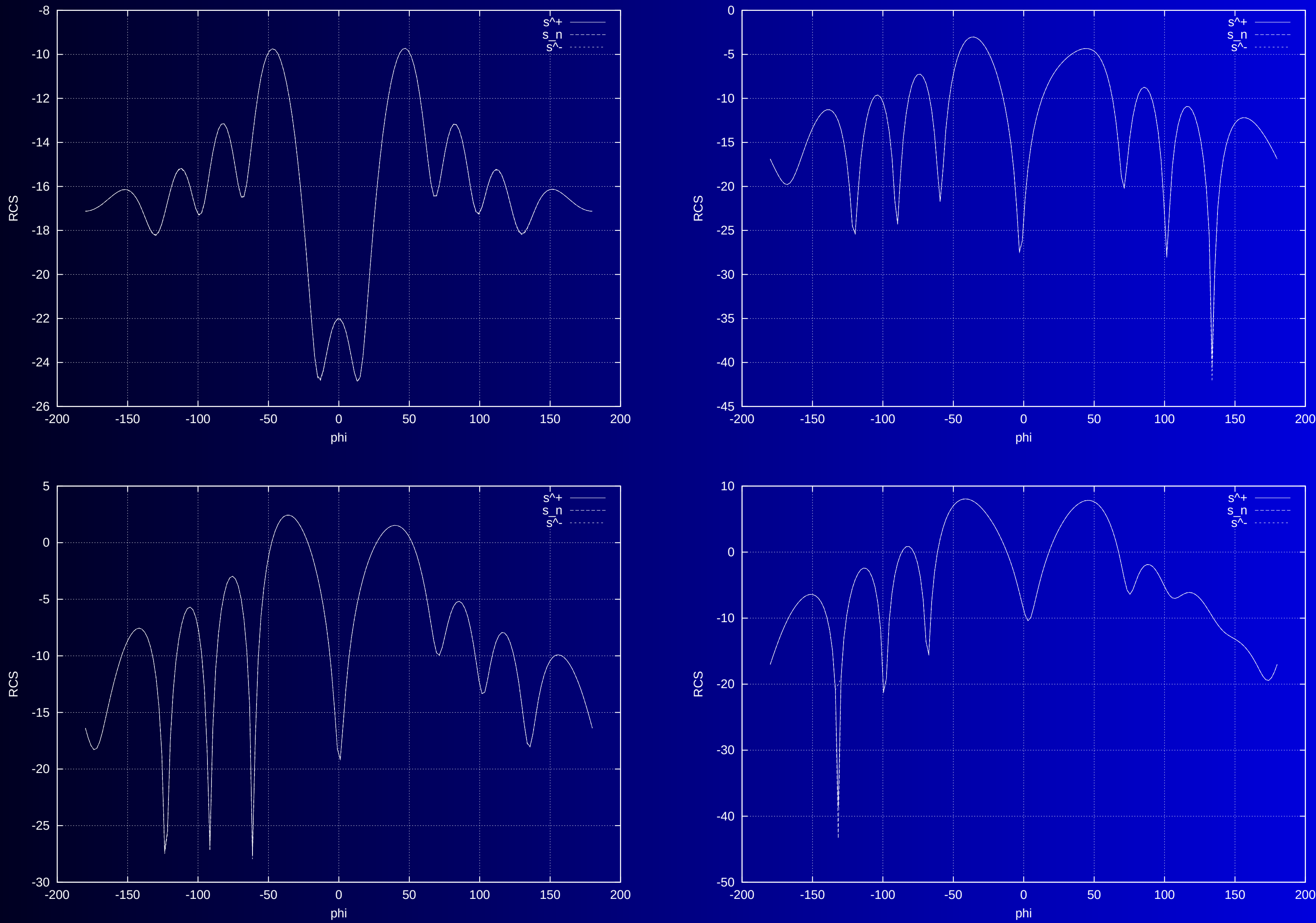
- $\| \boldsymbol{R}^{\Psi} \|$ denotes the Euclidean norm of $\boldsymbol{R}^{\Psi}$;
- μ_i denote the singular values of the matrix $\mathbf{A}$ (discretised $\mathcal{A}$);

Ledger et. al. Parmaterised electromagnetic scattering solutions for a range of incident wave angles, Comp. Meth. Appl. Mech. Eng. submitted 2003

Certainty Bounds for 2λ PMC Cylinder $\theta = 0,\ 10,\ 20,\ 40$



Certainty Bounds for 2λ PEC NACA $\theta = 0,\ 10,\ 20,\ 40$



Convergence of the Bounds

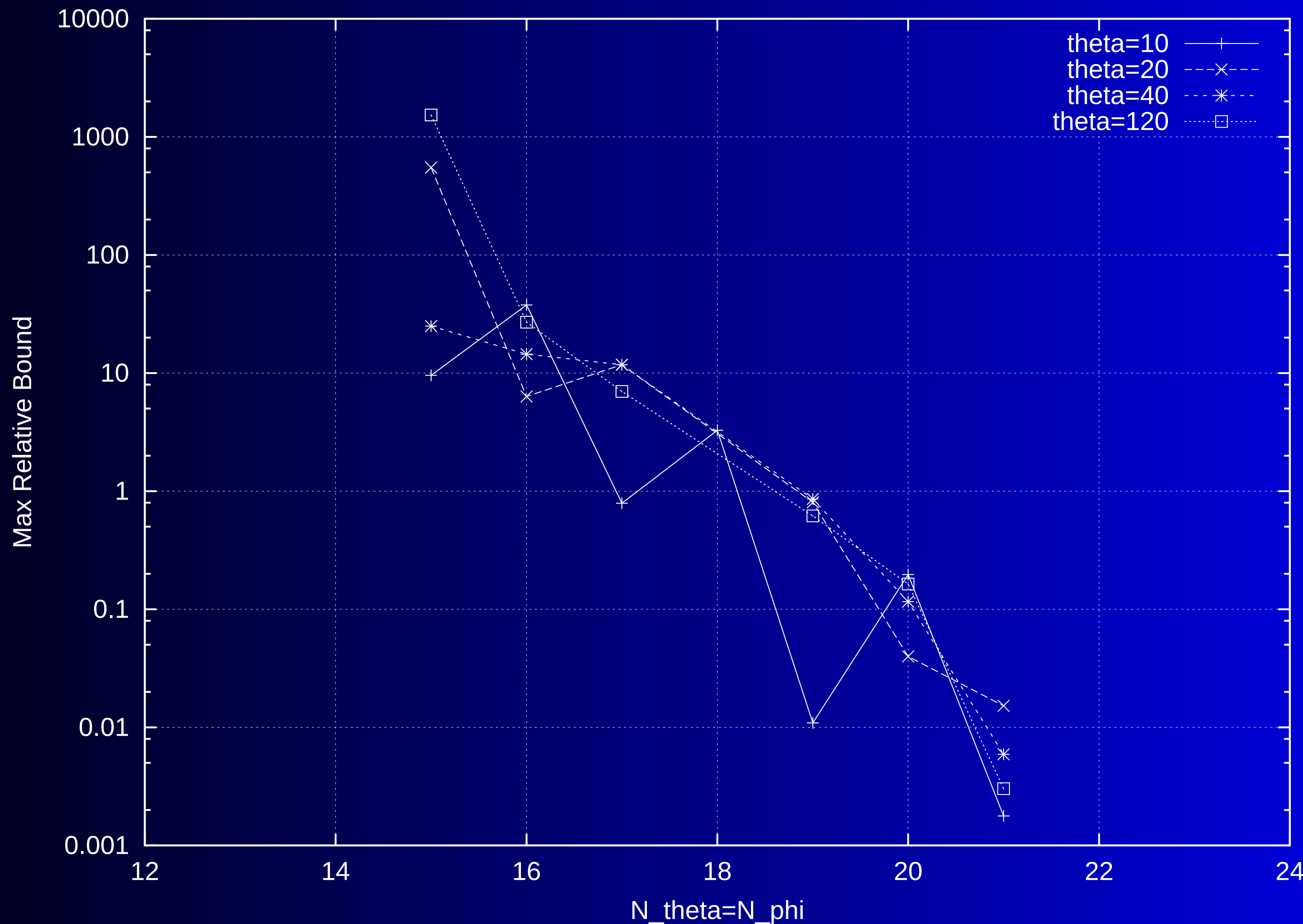
The magnitude of the bound gap is reduced by either

- Increasing N_θ;
- Increasing N_ϕ;

Best computational efficiency obtained by simultaneously increasing both.

The convergence of the bounds with increasing N_θ and N_ϕ is exponential in nature

Convergence of Max-Bound gap for 2λ PMC Cylinder



Conclusions

This presentation has shown

- Higher order edge element approach to 2D–EM scattering problems;
- Reduced–order model which enables computational efficient calculation of scattering width for new incidence directions;
- Construction of confidence bounds which ensure reliability in the predictions.

Extensions are possible to other parameters.

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