Field-Circuit Coupling for Mechatronic Systems: Some Trends and Techniques

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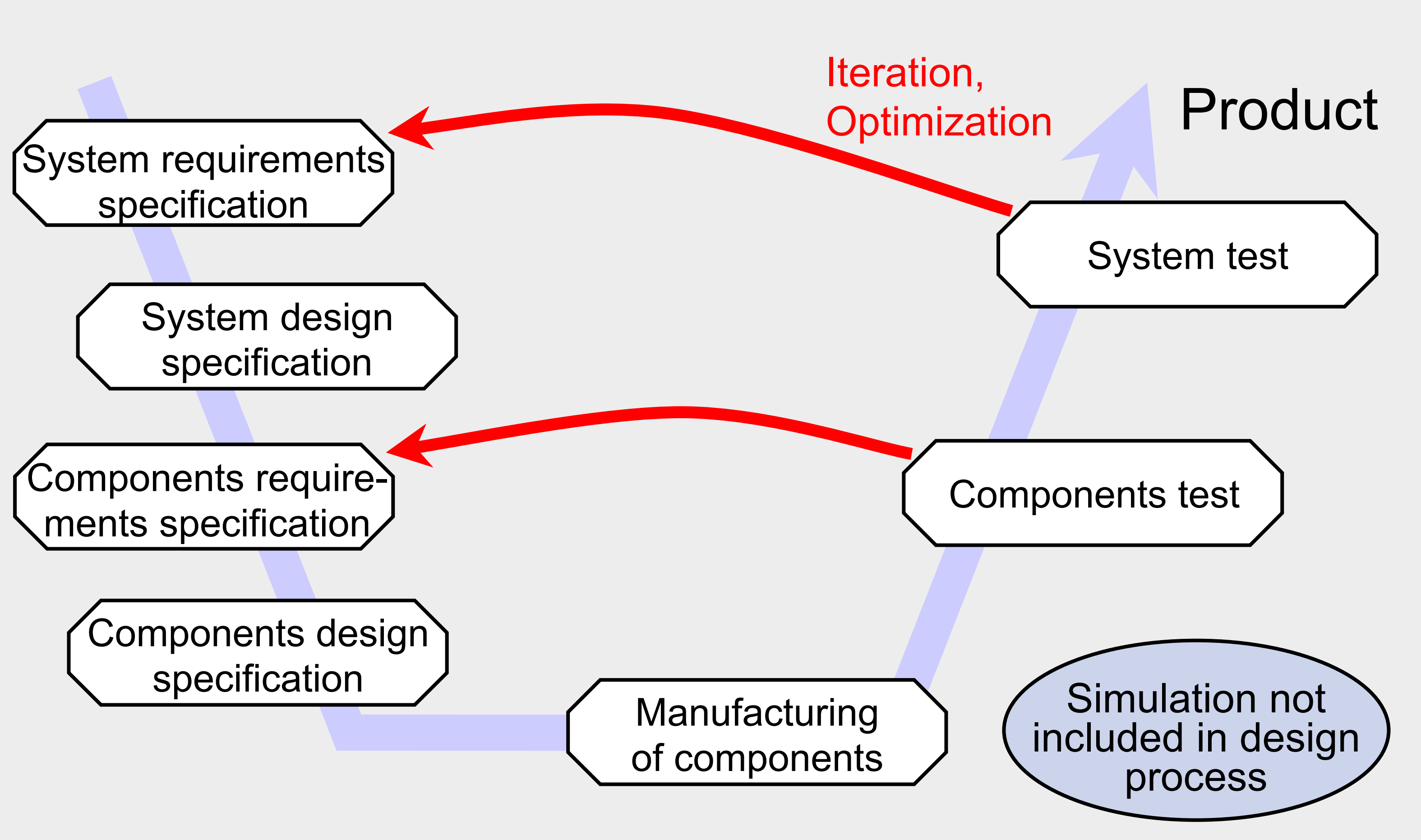
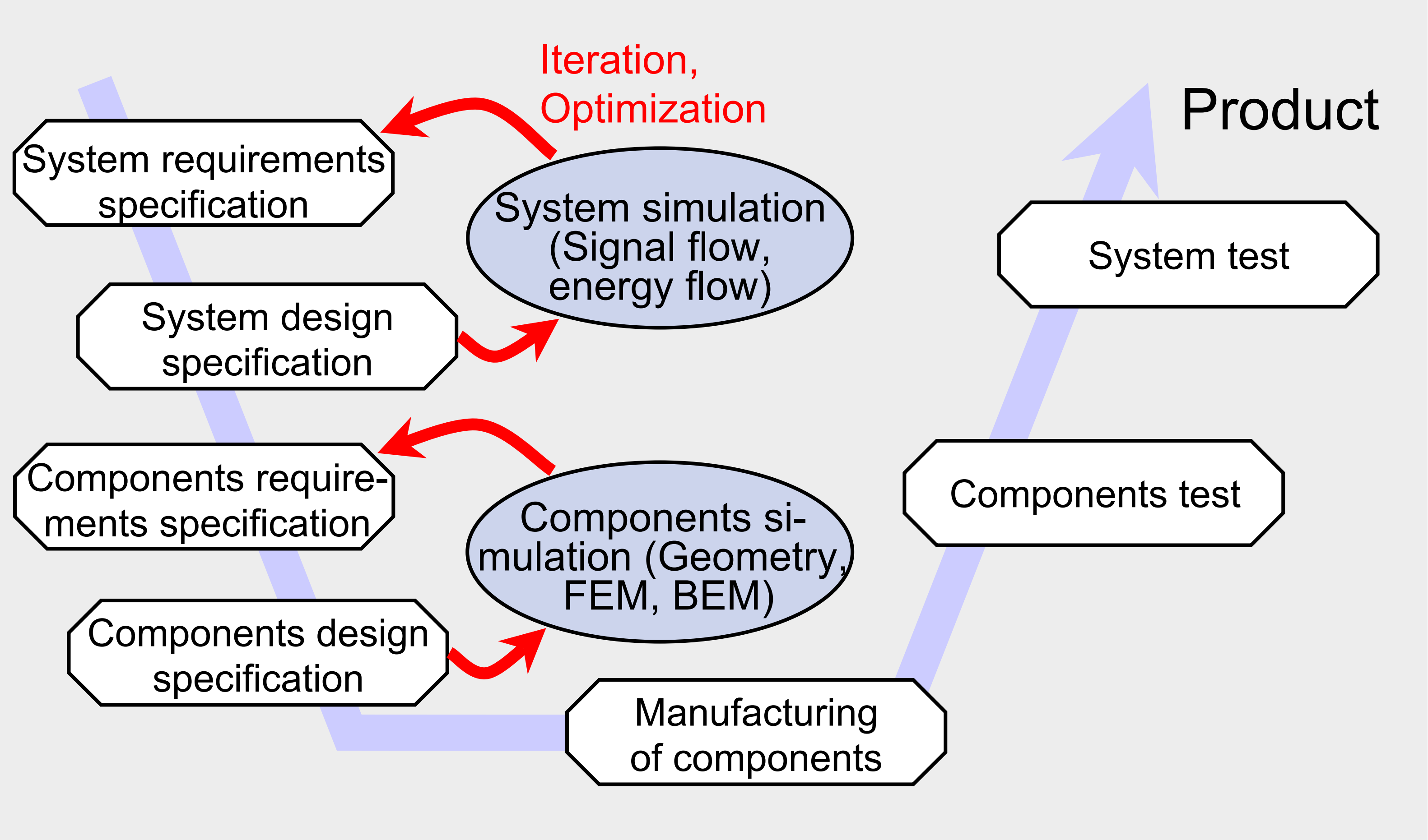
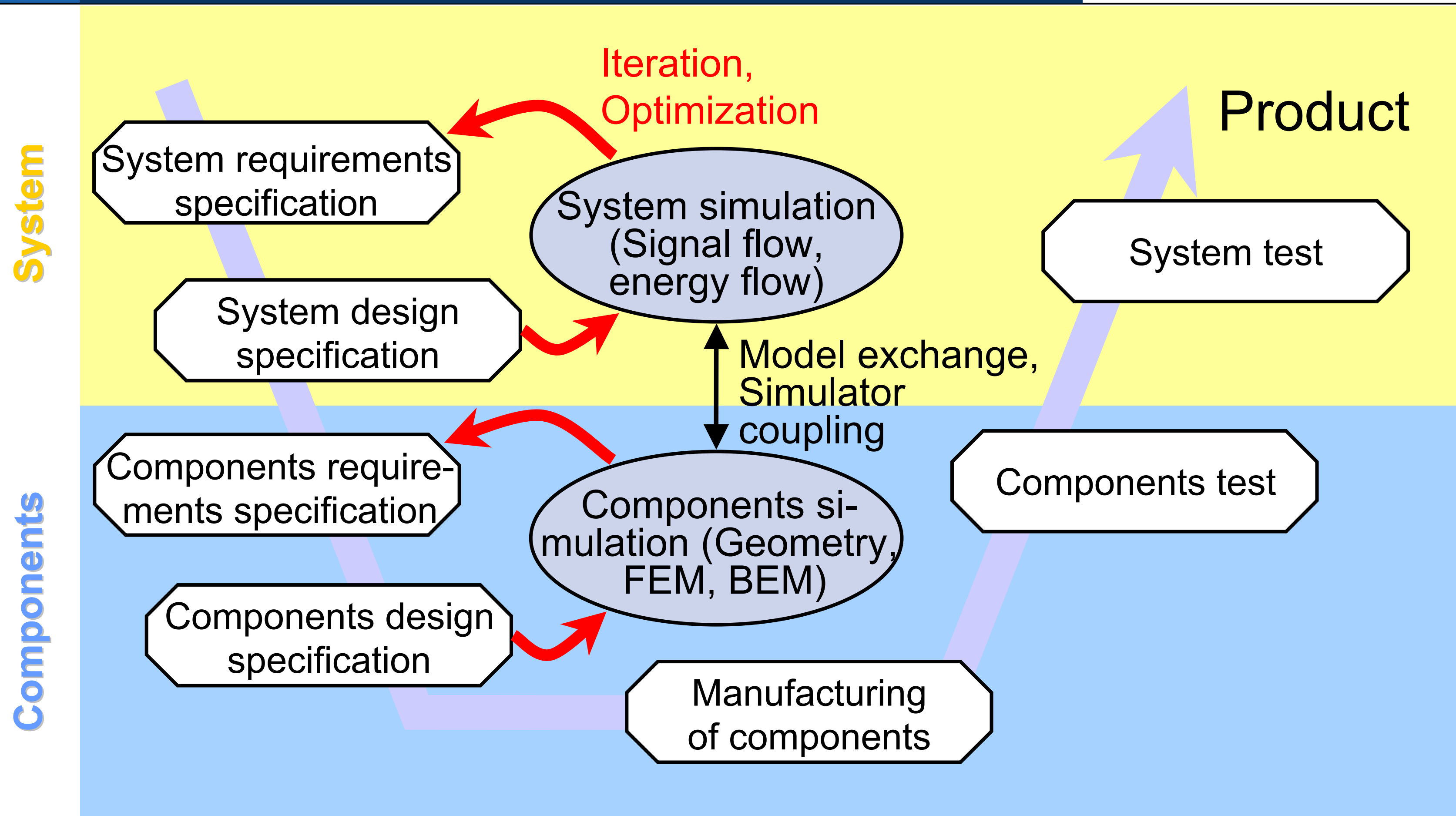
Outline

- The Design Process
- Electric Circuit Elements
- Coils and de Rham Cohomology
- Setting of the Field-Circuit Problem
- Direct Coupling
  - mastered by Field Simulator
  - mastered by Circuit Simulator
- Summary


Iteration,
Optimization
Product
System requirements specification
System design specification
Components require-ments specification
Components design specification
Manufacturing of components
System test
Components test
Simulation not included in design process


Iteration,
Optimization
Product
System requirements specification
System simulation (Signal flow, energy flow)
System test
System design specification
Components require-ments specification
Components si-mulation (Geometry, FEM, BEM)
Components test
Components design specification
Manufacturing of components

The Design Process



Main Categories of Field-Circuit Coupling

Equivalent Circuit

- System simulation on network level
- Field simulation to obtain parameters [14]

Direct Coupling

- Field and circuit equations are collected in one overall matrix

Indirect Coupling

- Keep both simula-tions separated
- Communication via coupling matrices [3]

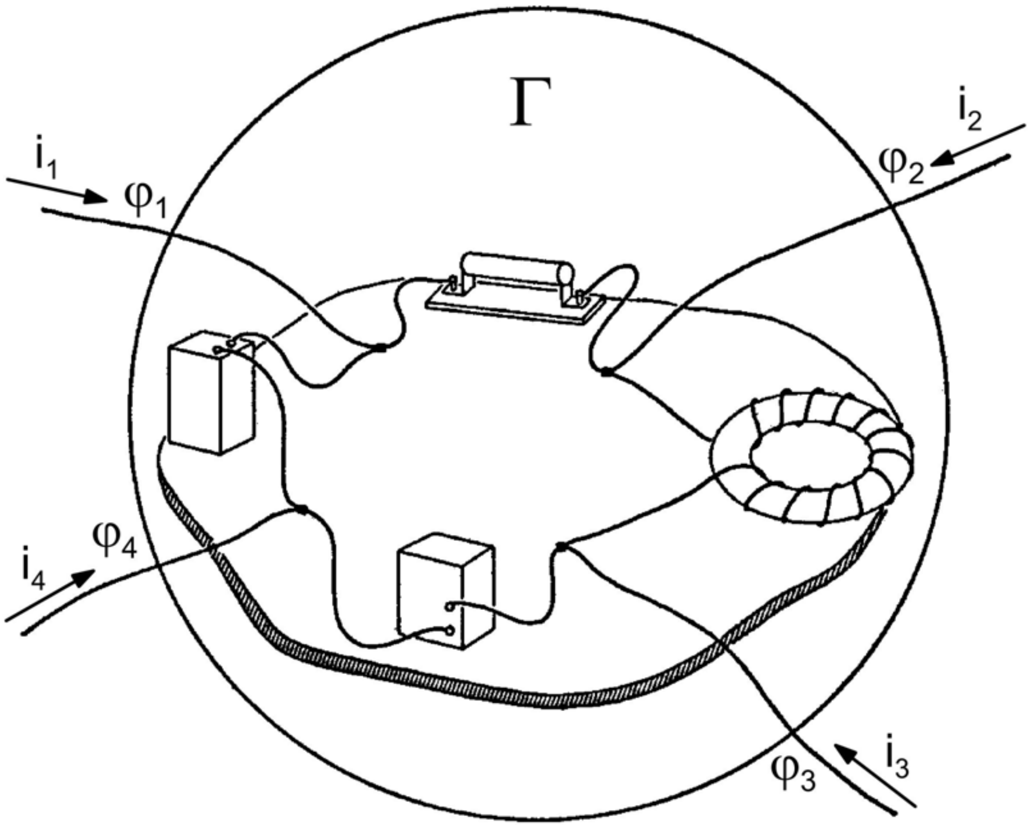
Mastered by Field Simulator

- FE matrix augmented by circuit‘s contribution [18]

Mastered by Circuit Simulator

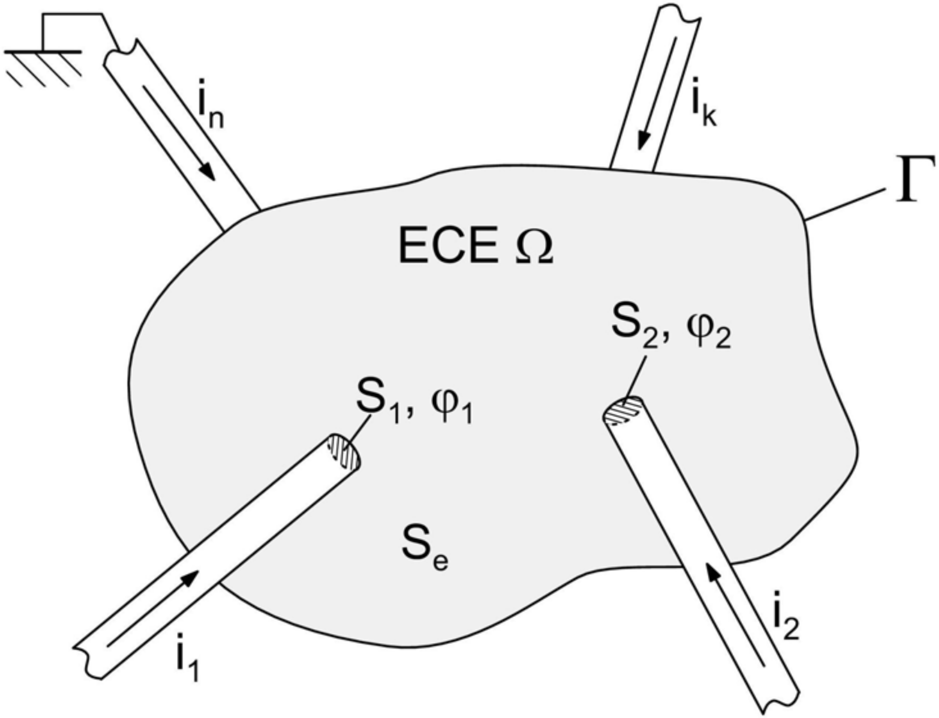
- FE equations represented as a multiport device [19]

The Circuit Concept of Voltage



- Voltmeters measure the line integral of the electric field along the path formed by the connecting leads.
- There are implicit limitations on the use of voltmeters: Indication should not depend appreciably on the exact position of the leads.

Definition of an Electric Circuit Element (ECE)



(1) $\exists \underline{\varphi} \in \mathcal{F}^0(\Gamma): \ \mathrm{t}\underline{E} = \mathrm{d}\,\underline{\varphi}$

(2) $\underline{\varphi} = \underline{\varphi}_k = \text{const}$ on S_k,

$S_k \ldots$ terminal connectors

(3) $\mathrm{t}\,\mathrm{d}\,\underline{H} = \mathrm{t}(\underline{j} + \partial_t \underline{D})$

$= 0$ on S_e,

$S_\mathrm{e} = \Gamma \setminus \sum S_k$

… insulating boundary

see: Munteanu & Ioan 2001 [14]

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- Terminal currents i_k and voltages u_k

$$i_k = -\int_{\partial S_k} \mathrm{t}\underline{H}, \qquad k = 1, \ldots, n$$

$$\rightarrow \quad \sum_{k=1}^{n} i_k = 0 \qquad \text{Kirchhoff's current law}$$

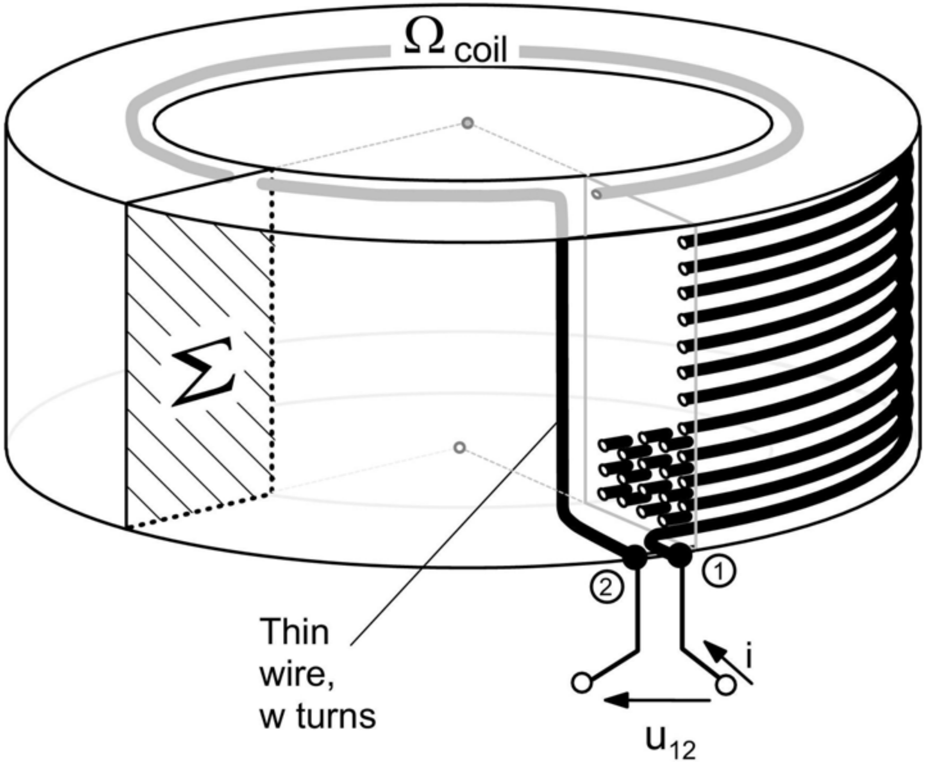
$$u_k = \underline{\varphi}_k - \underline{\varphi}_n, \qquad k = 1, \ldots, n \qquad \text{Kirchhoff's voltage law}$$

- Received power

$$p_\Gamma(t) = -\int_\Gamma \mathrm{t}(\underline{E} \wedge \underline{H}) = \sum_{k=1}^{n-1} u_k i_k$$

$\rightarrow$ $\int \mathrm{t}(\underline{E} \wedge \underline{H}) + \sum u_k i_k$ gives a measure for the approximation of the ECE conditions (1) - (3)

Stranded Conductor



- Winding density

$$\tau = \frac{w}{\Sigma}\, \mathrm{d}\Sigma$$

$$\int_{\Sigma} \tau = w$$

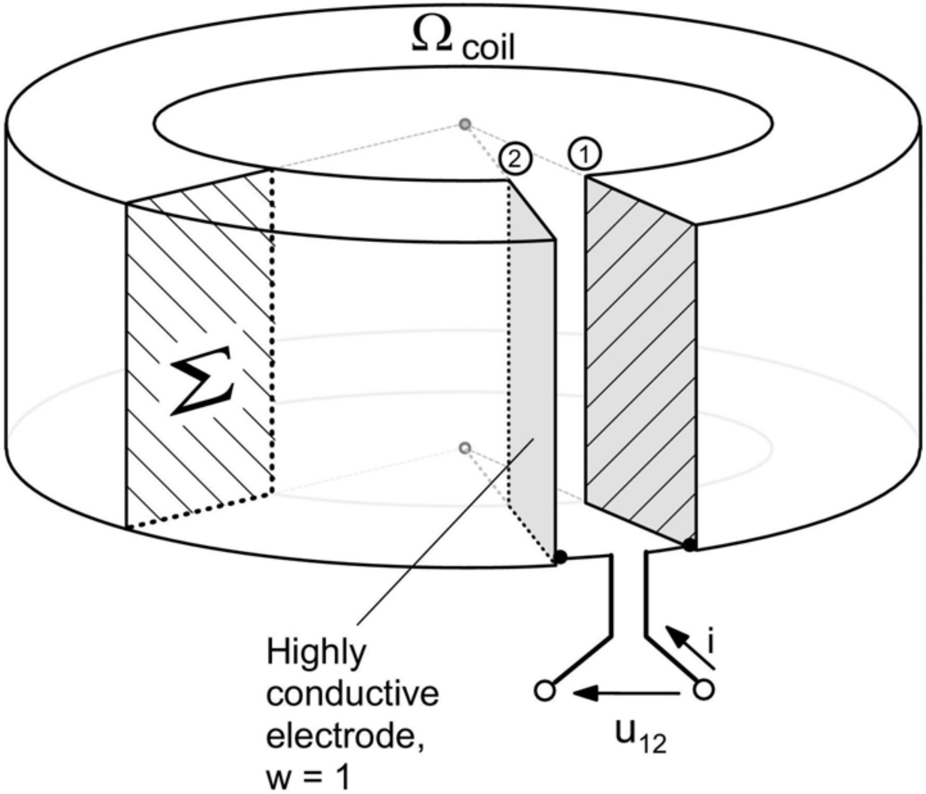
- Induced voltage

$$u_{\mathrm{ind}} = u_{12}\Big|_{i=0}$$

$$= \int_{\mathrm{wire}} \partial_t \underline{A}$$

$$= \int_{\Omega_{\mathrm{Coil}}} \partial_t \underline{A} \wedge \tau$$

Solid Conductor



- Assume $\sigma = \text{const}$ throughout Ω_{Coil}
- Apply a DC voltage U to the coil $\rightarrow\ I,\ \underline{j}_{\text{S}}$
- Let $\tau = \dfrac{\underline{j}_{\text{S}}}{I}$ where $\displaystyle\int_{\Sigma} \tau = w$
- Note that

$$\mathrm{d}\tau = 0, \quad \delta\tau = 0, \quad \mathrm{t}\tau = 0,$$

i.e. τ is a normal harmonic form.

- $\left(\frac{\Sigma}{w}\right)$ can be regarded as a basis for the relative 2-cycles of Ω_{Coil} (mod $\partial\Omega_{\text{Coil}}$).
- (τ) is the dual basis, since $\displaystyle\int_{\frac{\Sigma}{w}} \tau = 1$

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- The current density $\underline{j}$ is divergence-free ($\mathrm{d}\,\underline{j} = 0$) and has zero trace ($\mathbf{t}\underline{j} = 0$)

$\rightarrow$ de Rham decomposition $\underline{j} = \underbrace{\mathrm{d}\,\underline{T}}_{\text{eddy current}} + \underbrace{i\boldsymbol{\tau}}_{\text{loop current}}, \quad \mathbf{t}\underline{T} = 0$

with the current vector potential $\underline{T}$ and the terminal current $i = \int_{\underline{\Sigma}_w} \underline{j}$

- Power delivered by the source

$$p(t) = i(t) \cdot u_{12}(t)$$

$$= i(t) \cdot \left(i(t) \cdot \frac{1}{\kappa} \int_{\Omega_{\mathsf{Coil}}} *\boldsymbol{\tau} \wedge \boldsymbol{\tau} + \frac{\mathrm{d}}{\mathrm{d}t} \int_{\Omega_{\mathsf{Coil}}} \underline{A} \wedge \boldsymbol{\tau} \right)$$

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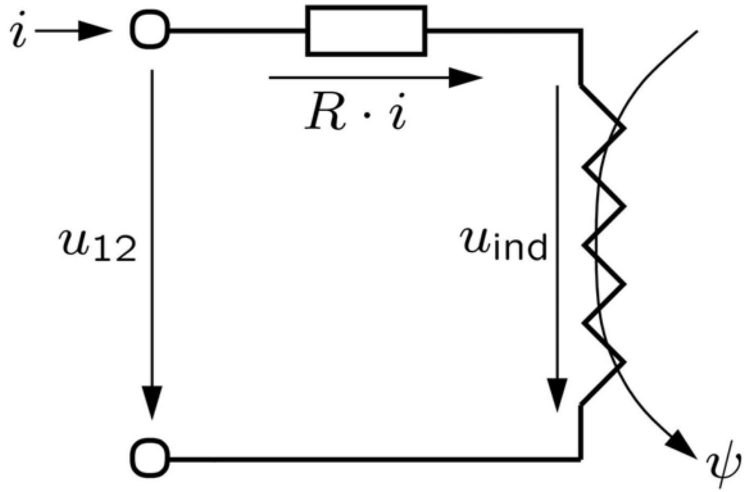
- Equivalent circuit diagram

$$u_{12}(t) = R \cdot i(t) + \frac{\mathrm{d}\psi}{\mathrm{d}t}$$

$$R = \frac{1}{\kappa} \int_{\Omega_{\mathrm{Coil}}} *\boldsymbol{\tau} \wedge \boldsymbol{\tau}$$

$$\psi = \int_{\Omega_{\mathrm{Coil}}} \underline{A} \wedge \boldsymbol{\tau}$$



- Circuit parameters R, ψ computable from a suitable basis of the space of normal harmonic forms.

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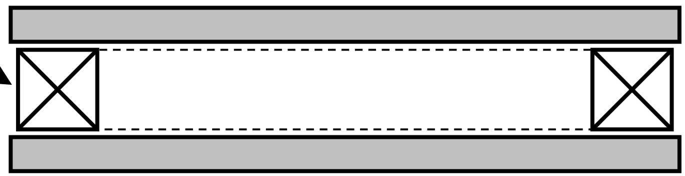
A Simple Example

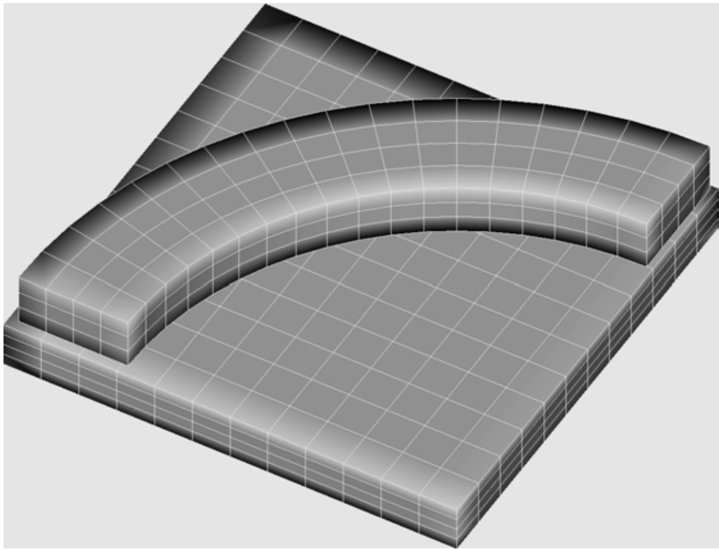
Leonard & Rodger 1988 [11]

Circular coil, stranded conductor



Square aluminium plates

1/8 of the problem, used for computer model



coil

plate

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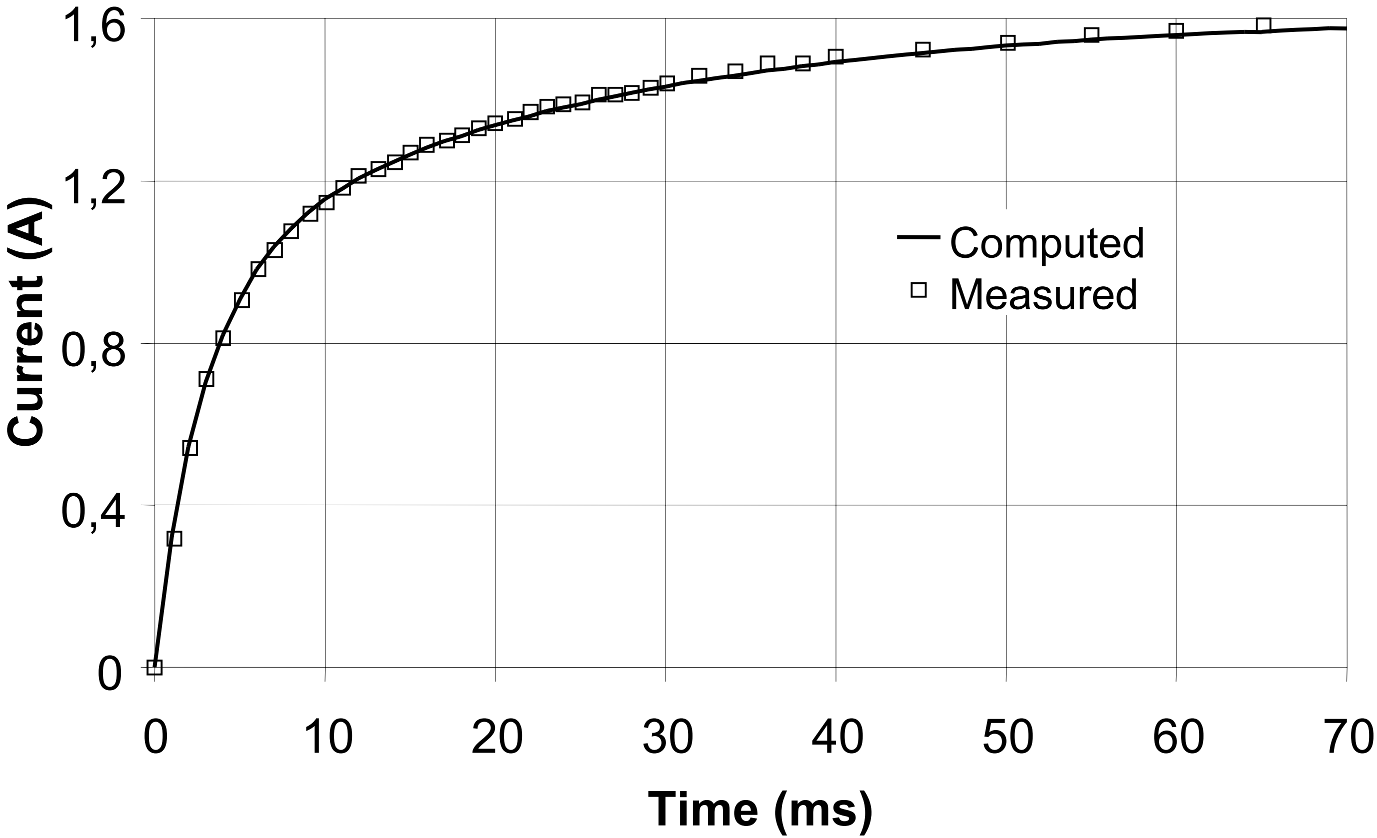
Response to a Step Voltage



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Basic Equations

- Fundamental equation of the eddy current problem
  $$\mathrm{d}\,\nu * \mathrm{d}\,\underline{A} + \sigma * \partial_t \underline{A} = \underline{j}_{\mathrm{S}}$$
- Circuit equation for stranded conductor
  $$\frac{\mathrm{d}\psi}{\mathrm{d}t} + R \cdot i = u_{12}$$
- Winding density
  $$\boldsymbol{\tau} = \frac{\underline{j}_{\mathrm{S}}}{i}, \qquad \mathrm{d}\,\boldsymbol{\tau} = 0$$
- Flux linkage
  $$\psi = \int_{\Omega_{\mathrm{Coil}}} \underline{A} \wedge \boldsymbol{\tau}$$

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Field-Circuit Coupling

$$\left.\begin{aligned} \mathrm{d}\,\nu * \mathrm{d}\,\underline{A} + \sigma * \partial_t \underline{A} - i \cdot \boldsymbol{\tau} = 0 \\ \int_{\Omega_{\mathrm{Coil}}} (\partial_t \underline{A}) \wedge \boldsymbol{\tau} + R \cdot i = u_{12} \end{aligned}\right\}$$

PDE

- Space discretisation: Galerkin Edge-FEM
- Time discretisation: Implicit Euler
- Non-linear solver: Newton-Raphson

$$\begin{pmatrix} [J] & -\{U\} \\ -\{U\}^T & -\Delta t R \end{pmatrix} \begin{pmatrix} \{\delta A\} \\ \delta i \end{pmatrix} = \begin{pmatrix} \{F_\delta\} \\ e_\delta \end{pmatrix}$$

Linear system

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Slightly More General Problem: *p*>1 Coils

- Linear system

$$\begin{pmatrix} [J] & -[U] \\ -[U]^T & -\Delta t[R] \end{pmatrix} \begin{pmatrix} \{\delta A\} \\ \{\delta i\} \end{pmatrix} = \begin{pmatrix} \{F_\delta\} \\ \{e_\delta\} \end{pmatrix}$$

$[J] \in \mathbb{R}^{n \times n}$, s.p.d.

$[U] \in \mathbb{R}^{n \times p}$

$[R] \in \mathbb{R}^{p \times p}$, s.p.d.

$p \ll n$

- Original system: Sparse, symmetric but indefinite ☹

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The Schur Complement System

- Idea: Eliminate $\{\delta i\}$ by taking the Schur complement (Fetzer & Kurz 1998, De Gersem et.al. 2000 [8])

$$[\tilde{J}] = [J] + [U]\frac{1}{\Delta t}[R]^{-1}[U]^T \qquad \text{s.p.d.}$$

- Resulting system

$$\begin{pmatrix} [\tilde{J}] & 0 \\ -[U]^T & -\Delta t[R] \end{pmatrix} \begin{pmatrix} \{\delta A\} \\ \{\delta i\} \end{pmatrix} \begin{pmatrix} \{\tilde{F}_\delta\} \\ \{e_\delta\} \end{pmatrix}$$

$$\{\tilde{F}_\delta\} = \{F_\delta\} - [U]\frac{1}{\Delta t}[R]^{-1}\{e_\delta\}$$

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Solution Strategy

$$[\tilde{J}] = [J] + [U]\frac{1}{\Delta t}[R]^{-1}[U]^T$$

Field Jacobian...

- s.p.d.
- sparse

...augmented by circuit's contribution

- symmetric, non-negative
- rank *p*

- As $[\tilde{J}]$ is s.p.d., the PCG method is the best choice for iterative solution.
- The Schur complement needs not to be explicitely formed to deal with matrix-by-vector products.
- The low rank perturbation does not much affect the original convergence rate. (Kurz & Rischmüller 2002)

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What about More General Circuits?

- Situation more involved for general circuits: Stranded conductors, solid conductors, capacitors, inductors, resistors and sources joined together.

- Topological analysis (tree-cotree decomposition, Signal Flow Graph) yields a symmetric indefinite system with sparse FE matrix. (De Gersem et.al. 1998 [9])

$$\left(\begin{array}{c|cc} [J] & [Q] & -[P] \\ \hline [Q]^T & \Delta t[Y] & \Delta t[D] \\ -[P]^T & \Delta t[D]^T & -\Delta t[Z] \end{array}\right) \begin{pmatrix} \{\delta A\} \\ \{\delta u_{\mathrm{tw}}\} \\ \{\delta i_{\mathrm{ln}}\} \end{pmatrix} = \begin{pmatrix} \{F_\delta\} \\ \{e_\delta\} \end{pmatrix}$$

$[U]$ (upper right block), $[U]^T$ (lower left block), $\Delta t[R]$ (lower right block)

- Elimination of the circuit part $[R]$ results in a s.p.d. Schur complement

The Schur Complement System

- Eliminate $\{\delta A\}$ by taking the Schur complement

$$[\tilde{R}] = [R] + \frac{1}{\Delta t}[U]^T[J]^{-1}[U]^T \qquad \text{s.p.d.}$$

- Resulting system

$$\begin{pmatrix} [J] & -[U] \\ 0 & -\Delta t[\tilde{R}] \end{pmatrix} \begin{pmatrix} \{\delta A\} \\ \{\delta i\} \end{pmatrix} \begin{pmatrix} \{F_\delta\} \\ \{\tilde{e}_\delta\} \end{pmatrix}$$

$$\{\tilde{e}_\delta\} = \{e_\delta\} + [U]^T[J]^{-1}\{F_\delta\}$$

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➔ The second equation can be cast into the form

$$[R]\{i\}_{k+1}^{t+\Delta t} + \frac{1}{\Delta t}[L]\{\Delta i\}_{k+1} + \{v\}_{k}^{t+\Delta t} = \{u\}^{t+\Delta t}$$

$$[L] = [U]^{T}[J]^{-1}[U]$$ . . . inductance matrix

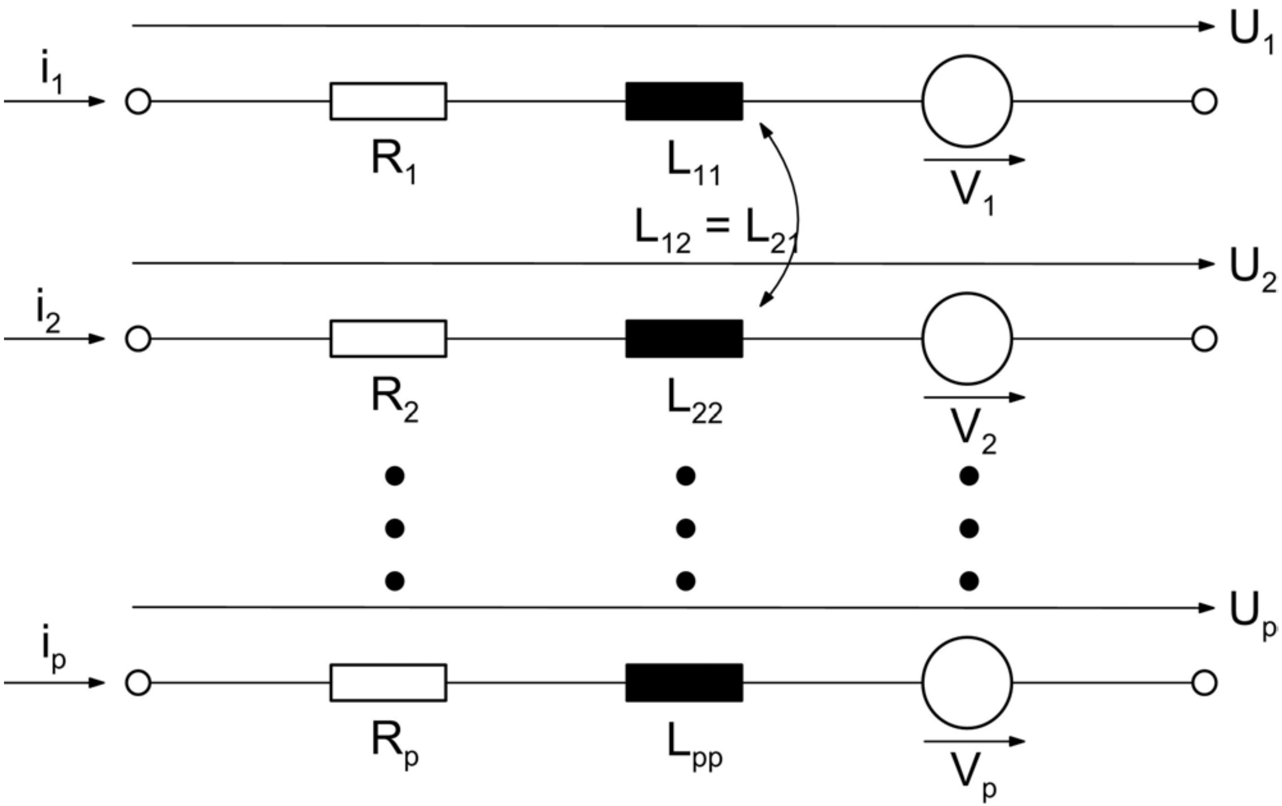
$$\{v\}_{k}^{t+\Delta t} = \frac{1}{\Delta t}[U]^{T}\left(\{\Delta A\}_{k+1}\right)_{\{i\}=\text{const}}$$ . . . open circuit voltage that would be induced in the coils if the currents were kept constant

k . . . index referring to the Newton-Raphson method

Δ . . . indicates Euler increments during Δt

Equivalent Multiport Device [19]

➔ Circuit realisation of the second Schur complement equation



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How Do We Get the Parameters?

Assuming that the field solver is able to compute the coils' flux linkages:

- Freeze the field Jacobian at its current value $[J] = [J]_k^t$
- Keep the currents constant and have the solver compute the flux increments $\Delta\psi_{0,\nu}$ during the time step Δt,

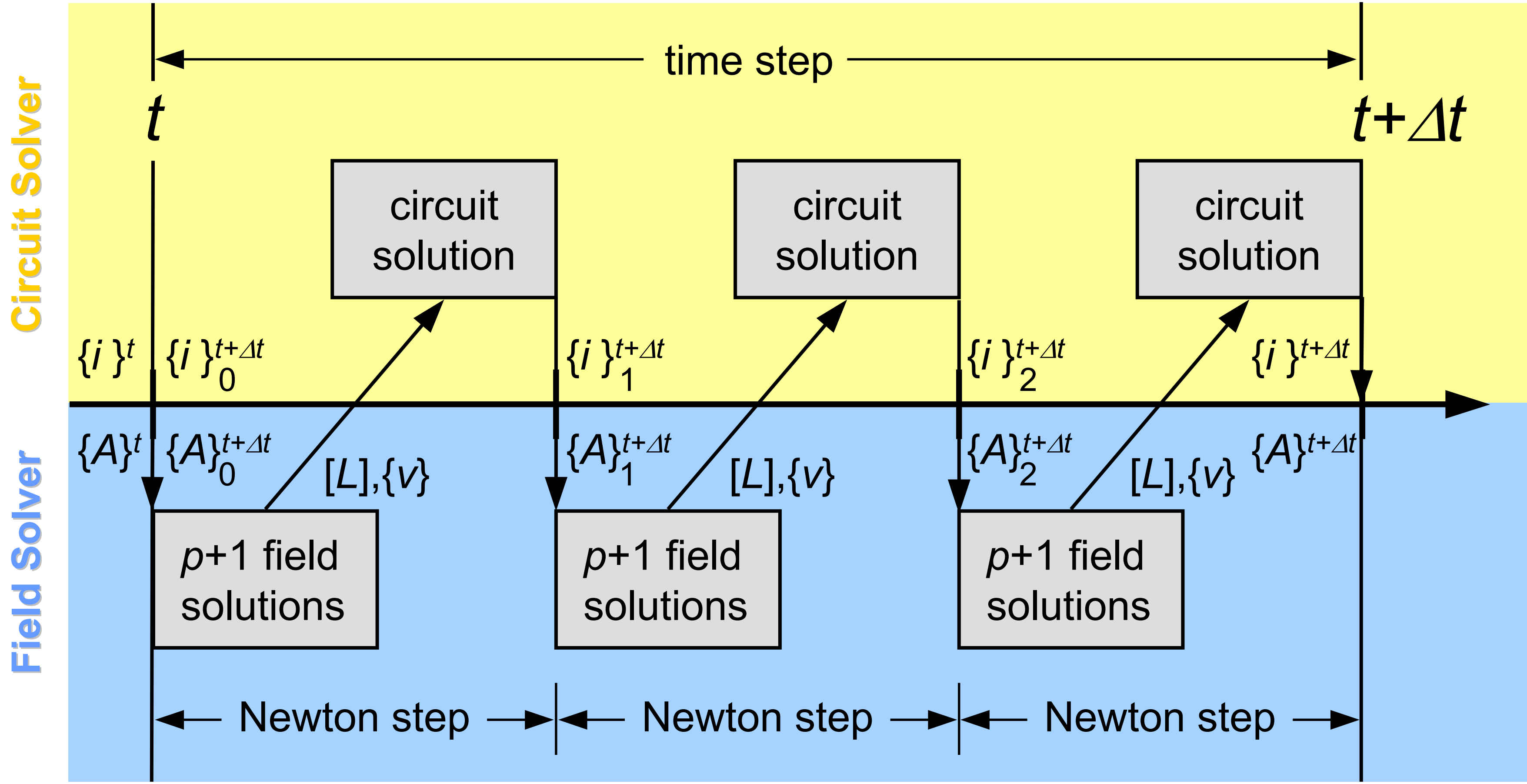
$$\text{let} \quad v_\nu = \frac{\Delta\psi_{0,\nu}}{\Delta t}, \qquad \nu = 1, \ldots, p.$$

- Increase successively each current i_μ by one unit and have the solver compute the flux increments $\Delta\psi_{\mu,\nu}$ in coil number ν,

$$\text{let} \quad L_{\mu,\nu} = \Delta\psi_{\mu,\nu} - \Delta\psi_{0,\nu}, \qquad \mu, \nu = 1, \ldots, p.$$

see: Demerdash & Nehl 1999 [6], Mc Dermott, Zhou & Gilmore 1997 [13].

Solution Strategy



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Summary

- Field-circuit coupling can create a link between simulation on the system and the components level of mechatronic systems.
- The Electric Circuit Element provides a rigorous definition of terminal voltages and currents, according to Kirchhoff's laws.
- Direct coupling yields one overall matrix, which collects the field and the circuit equations.
- The solution process can be mastered either by the field or by the circuit simulator.
- Solution strategies for the resulting Schur complement systems have been discussed in both cases.