technische universiteit eindhoven

Reduction and Realization Techniques in Modelling of Passive Electronic Structures

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Funny example of ROM:

An image consists of matrix (or 3)

Singular value decomposition:

$$\mathbf{U}\Sigma\mathbf{V}^T = \mathbf{A}$$

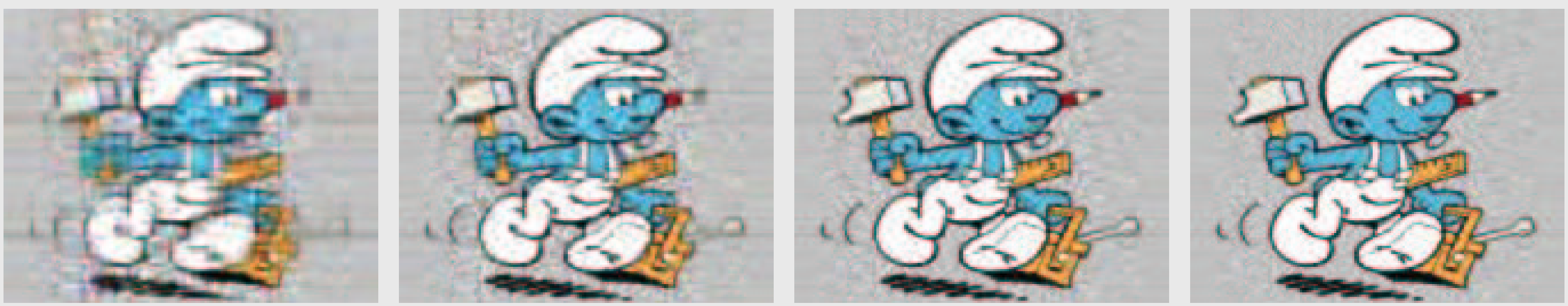
Truncate a certain amount of singular values.

Example:

139 singular values

Example:

The largest 10, 20, 30 and 40 singular values



Overview of my talk

- System formulation
- What is ROM?
- Krylov subspace methods
- Orthogonalization
- Realization

Electronic structures

- Passive structures:
  - Interconnects
  - Analog parts of chips
  - Coupled with digital part
- Many models can be represented by RLC-networks:
  - TLM
  - PEEC
  - EFIE integral model
  - FIT
  - FDTD (spatially discretized)

System formulation

DAE system:

$$\begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} \end{pmatrix} \frac{d}{dt} \begin{pmatrix} \mathbf{v}(t) \\ \mathbf{i}(t) \end{pmatrix} = -\begin{pmatrix} \mathbf{G} & -\mathbf{P}^T \\ \mathbf{P} & \mathbf{R} \end{pmatrix} \begin{pmatrix} \mathbf{v}(t) \\ \mathbf{i}(t) \end{pmatrix} + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{L}^T \begin{pmatrix} \mathbf{v}(t) \\ \mathbf{i}(t) \end{pmatrix}$$

Or:

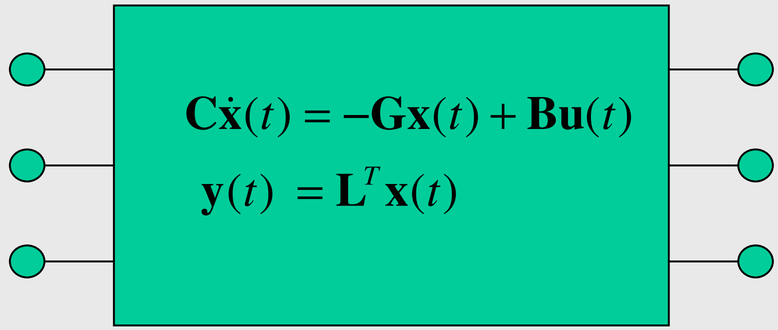
$$\mathbf{C}\dot{\mathbf{x}}(t) = -\mathbf{G}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{L}^T \mathbf{x}(t)$$

For instance: voltage-in-current-out

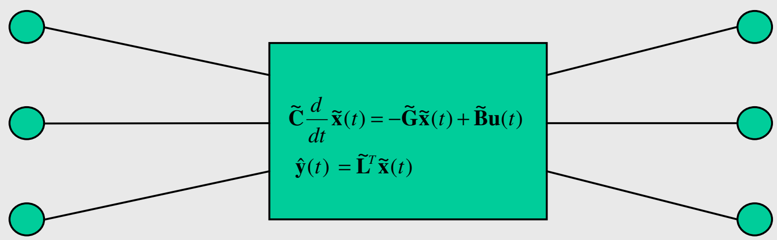
What is ROM?

Large (RLC-)circuit



replaced by

Small circuit



(with approximately same behavior)

Demands

- Behavior approximated well:
  - For a fixed set of inputs
  - Up to a maximum frequency
- Gain in computational time
- Passivity preservation!

System formulation

$$\mathbf{C}\dot{\mathbf{x}}(t) = -\mathbf{G}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{L}^T\mathbf{x}(t)$$

Transform with Laplace: $s\mathbf{C}\mathbf{X}(s) = -\mathbf{G}\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s)$

$$\mathbf{Y}(s) = \mathbf{L}^T\mathbf{X}(s)$$

Transfer function: $\mathbf{H}(s) = \mathbf{L}^T(\mathbf{G} + s\mathbf{C})^{-1}\mathbf{B}$

Direct relation between input and output

Approximation, in frequency domain

Krylov-subspace methods

PRIMA (Odabasioglu, Celik) and PVL (Feldmann, Freund): $\mathbf{H}(s) = \mathbf{L}^T(\mathbf{G} + s\mathbf{C})^{-1}\mathbf{B} = \mathbf{L}^T(\mathbf{I} - (s - s_0)\mathbf{A})^{-1}\mathbf{R}$

$$\mathcal{K}_n(\mathbf{b}, \mathbf{A}) = [\mathbf{b}, \mathbf{A}\mathbf{b}, \ldots, \mathbf{A}^{n-1}\mathbf{b}]$$

with: $\mathbf{A} = -(\mathbf{G} + s_0\mathbf{C})^{-1}\mathbf{C}$

$$\mathbf{R} = (\mathbf{G} + s_0\mathbf{C})^{-1}\mathbf{B}$$

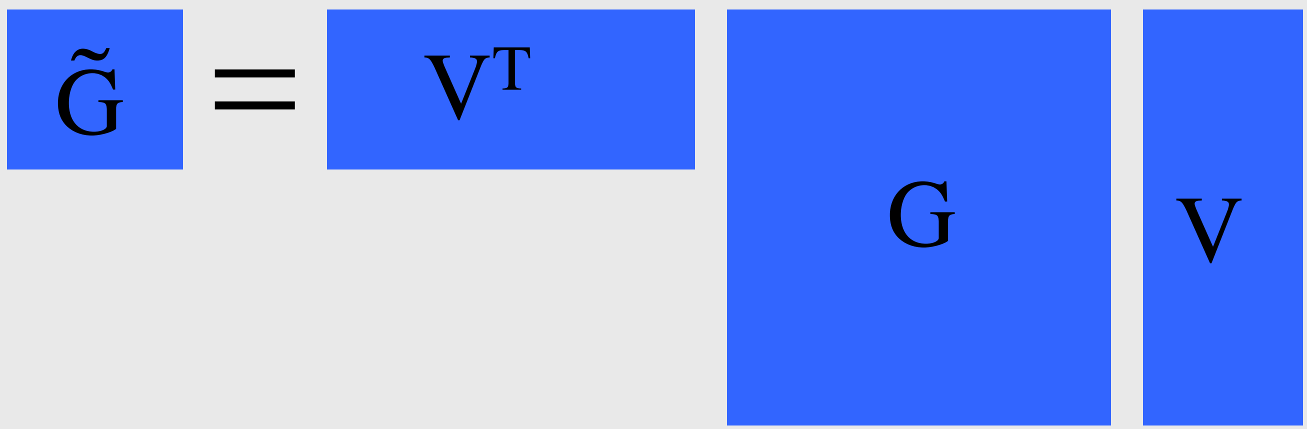
Krylov-subspace methods (2)

Defined by:

$$\mathcal{K}_n(\mathbf{b}, \mathbf{A}) = [\mathbf{b}, \mathbf{A}\mathbf{b}, \ldots, \mathbf{A}^{n-1}\mathbf{b}]$$

Orthonormal basis: **V**

Projection:



Krylov-subspace methods(3)

SVD-Laguerre (Knockaert, De Zutter):

Laguerre expansion:

$$\mathbf{H}(s) = \mathbf{L}^T(\mathbf{G}+s\mathbf{C})^{-1}\mathbf{B} =$$

$$\frac{2\alpha}{s+\alpha}\mathbf{L}^{\mathbf{T}}\sum_{n=0}^{\infty}\left((\mathbf{G}+\alpha\mathbf{C})(\mathbf{G}-\alpha\mathbf{C})\right)^n(\mathbf{G}+\alpha\,\mathbf{C})^{-1}\mathbf{B}\left(\frac{s-\alpha}{s+\alpha}\right)^n$$

Krylov-space:

$$\mathbf{b} = (\mathbf{G}+\alpha\,\mathbf{C})^{-1}\mathbf{B} \qquad \mathbf{A} = (\mathbf{G}+\alpha\,\mathbf{C})^{-1}(\mathbf{G}-\alpha\,\mathbf{C})$$

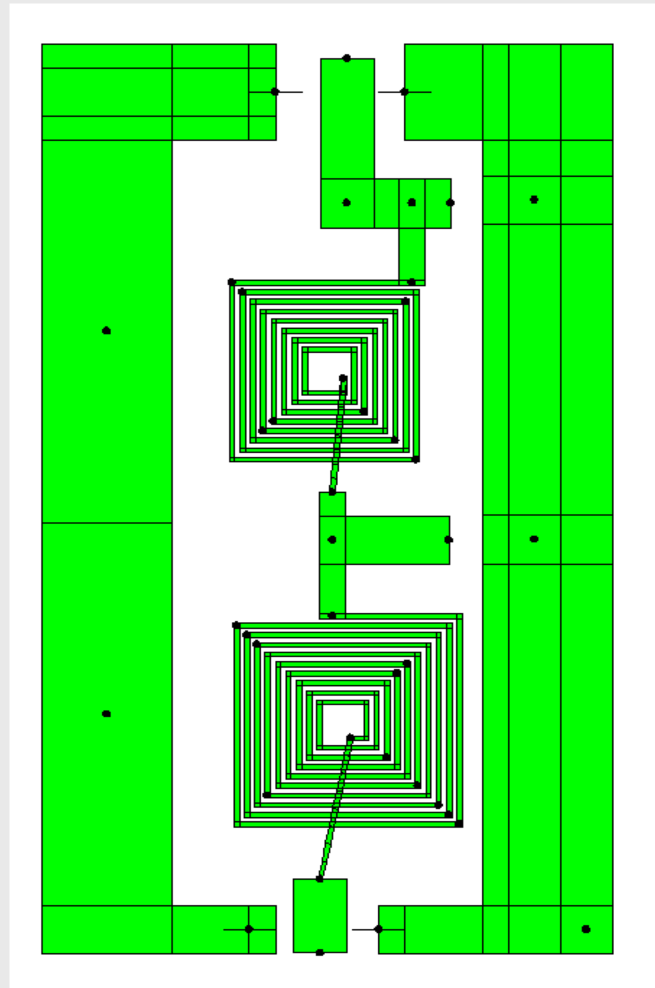
Example

PCB

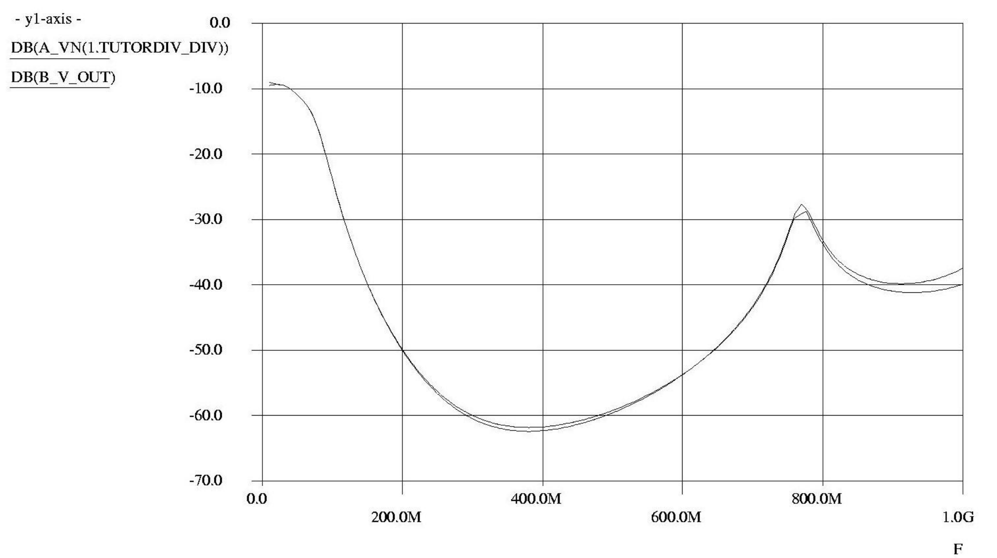
Original size 695 by 695

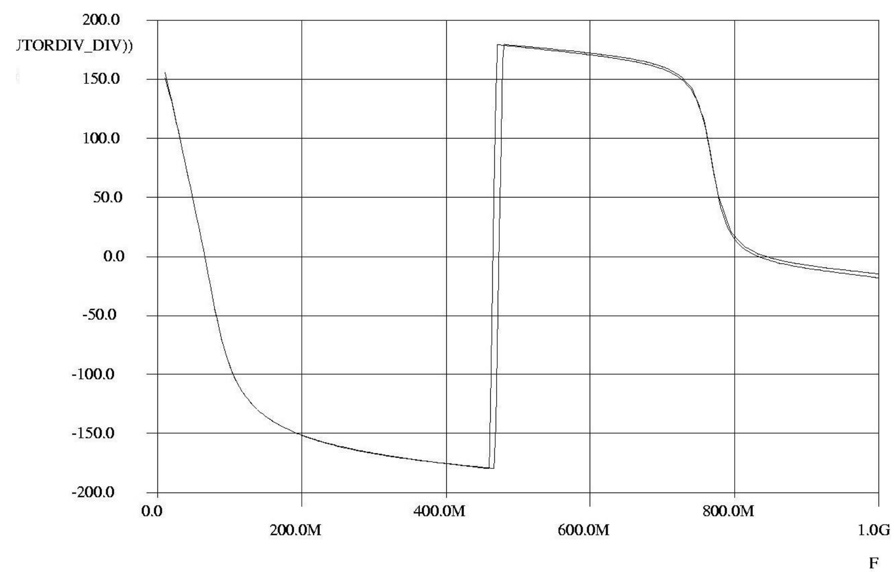
Reduced to 70 by 70

Behavior up to 1GHz

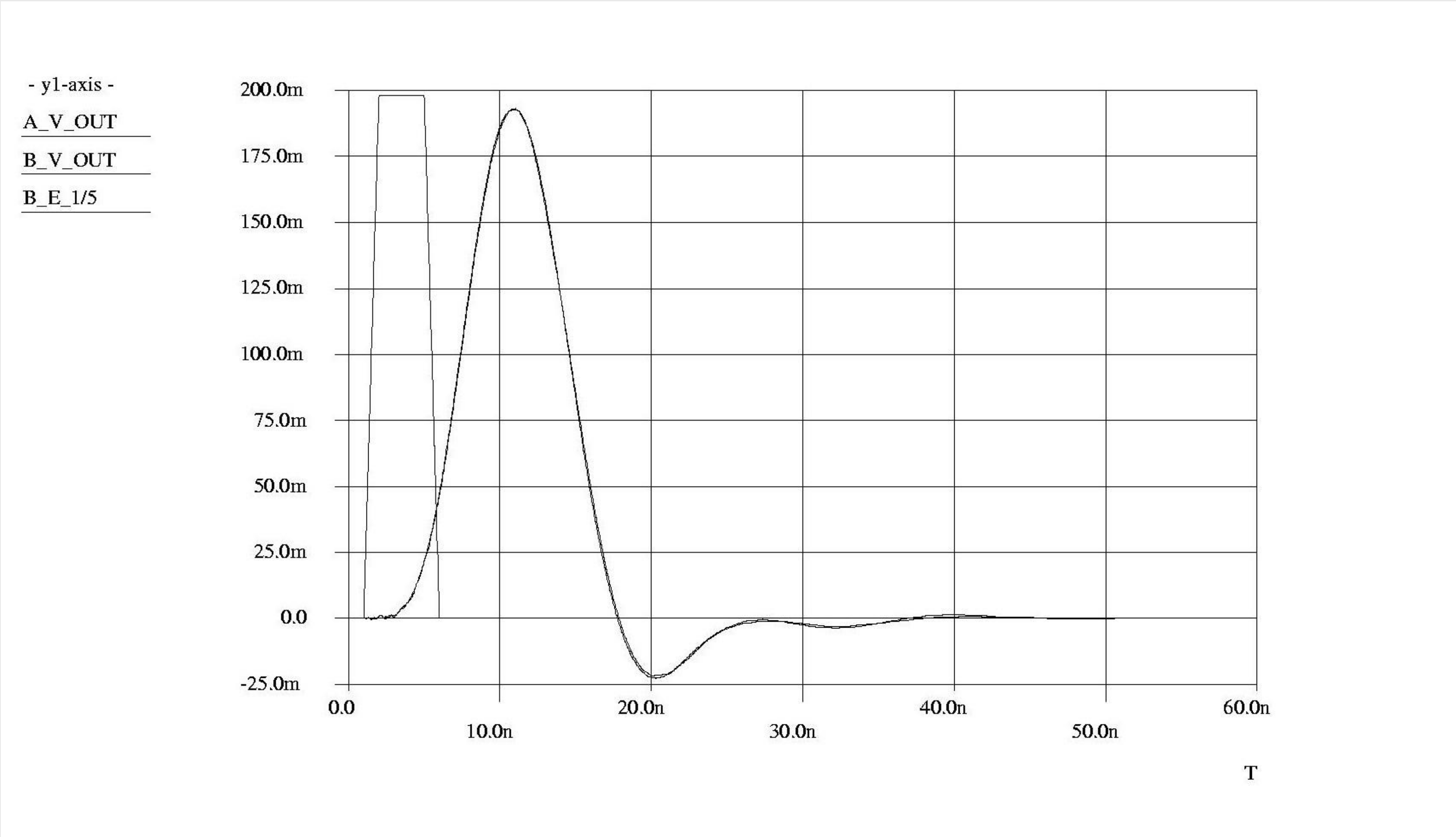


Example (AC analysis)





Example (transient)



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Orthogonalization

Krylov spaces

Orthogonalize while building up:

$$\mathcal{K}_n(\mathbf{B}, \mathbf{A}) = [\mathbf{B}, \mathbf{AB}, ..., \mathbf{A}^{n-1}\mathbf{B}]$$

Extra care for multiple columns of B

Choices:

– which columns are generated when?

– what is orthogonalized against what?

Every column in *B* has its own Krylov-space.

Krylov spaces

Which columns are generated when?

- Columns of B separately
- B in blocks

What is orthogonalized against what?

- Afterwards (eg. with SVD)
- During
  - Against all
  - Against column of same Krylov space

Modified Gram-Schmidt

Sometimes re-orthogonalization is necessary.

```
for i=1..j
   h = v_i^T w;
   w = w – h v_i;
end
w = w/|w|;
```

Rule of thumb:

re-orthogonalize if more than 75% is removed

Krylov spaces

- Mixing of Krylov spaces
  - essential information can be lost
  - spurious information can occur
- Preserve the shape of the Hessenberg matrix
- *Block Arnoldi* (as in PRIMA) is a right way and an efficient way

Realization

Realization

Projection: $\tilde{\mathbf{x}} = \mathbf{V}^T \mathbf{x}$

Physical meaning is lost

Given an arbitrary system, find a circuit:

$$\tilde{\mathbf{C}} \frac{d}{dt} \tilde{\mathbf{x}}(t) = -\tilde{\mathbf{G}} \tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}} \mathbf{u}(t)$$

$$\hat{\mathbf{y}}(t) = \tilde{\mathbf{L}}^T \tilde{\mathbf{x}}(t)$$

AC and Transient analysis

Need for realization

Transient analysis can be done, via frequency domain result (IFFT) ..

… or via general solution and a convolution integral.

All these methods are expensive and specific for one input.

A circuit can be coupled with the rest of circuit.

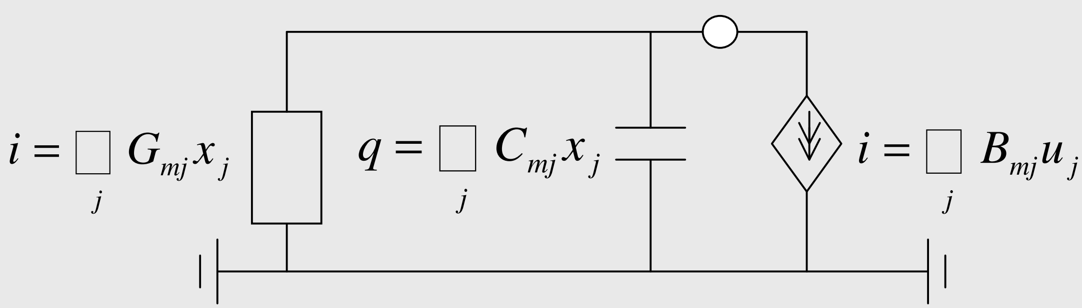
Realization (2)

Define a circuit with *q* internal nodes

State space vector: node voltages

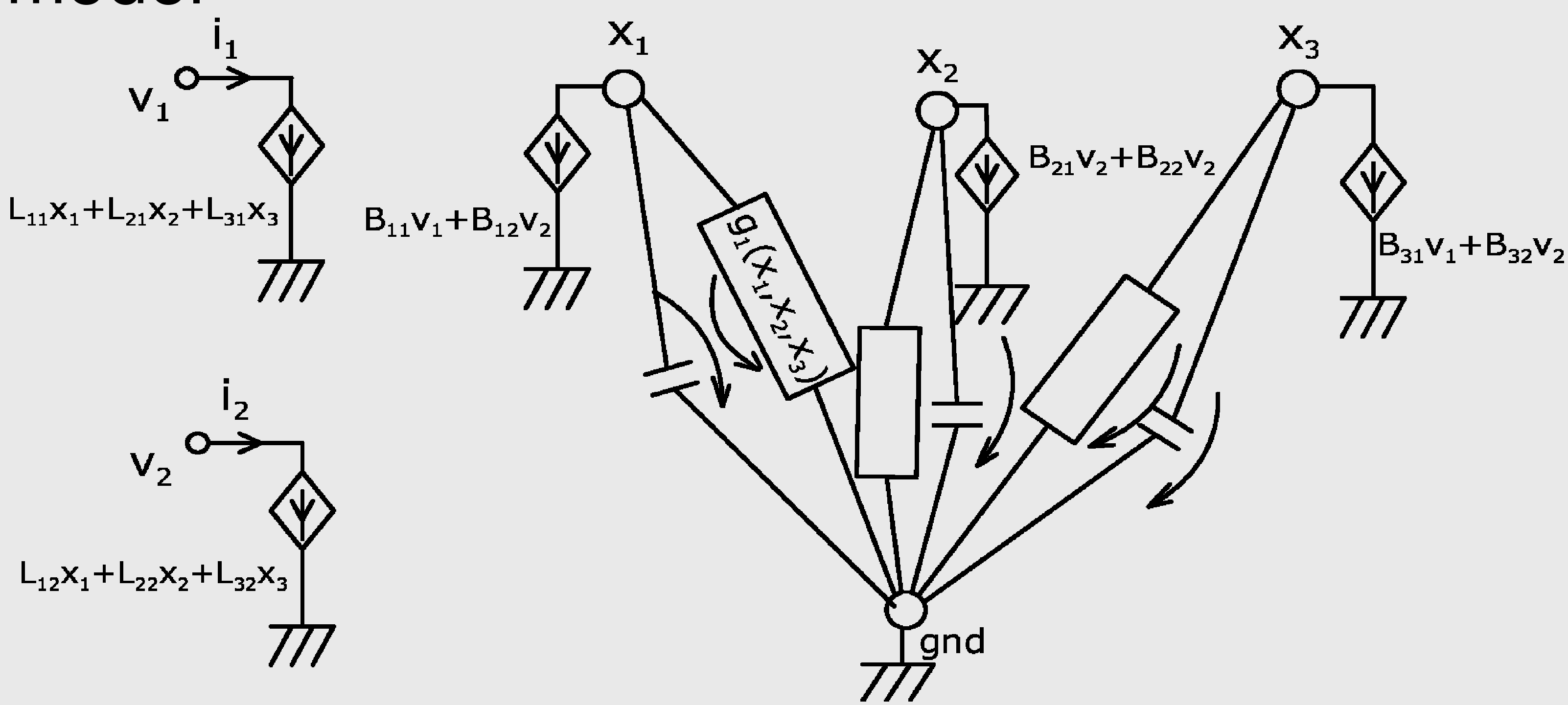
Rows: KCL’s for every node

=> Circuit with *q* nodes:



Realization (3)

Output is defined as sources to the terminals of the model



Results

- From large model to small model
- Able to combine reduced model with components and other models
  - AC analysis and stable transient analysis
- Future work:
  - non-linear MOR
  - Parametrized MOR

My website, email and …

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Thank you for your attention!