Computational Electromagnetics
Laboratory

Finite Element Models for Eddy Current Effects in Windings:
Application to Superconductive Rutherford Cable

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Workshop “Coupled problems / model reduction”
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Overview

1. Introduction: field-circuit coupling
2. Voltage/current-driven branches
3. Circuit description
   + consistency check
   + partial loop/cutset transformations
   + topological changes
4. Field-circuit coupling
   + examples
5. Algebraic system properties
   + selection of iterative solution techniques
6. Superconductive Rutherford cable
   + adjacency eddy currents
   + cross-over eddy currents
7. Conclusions

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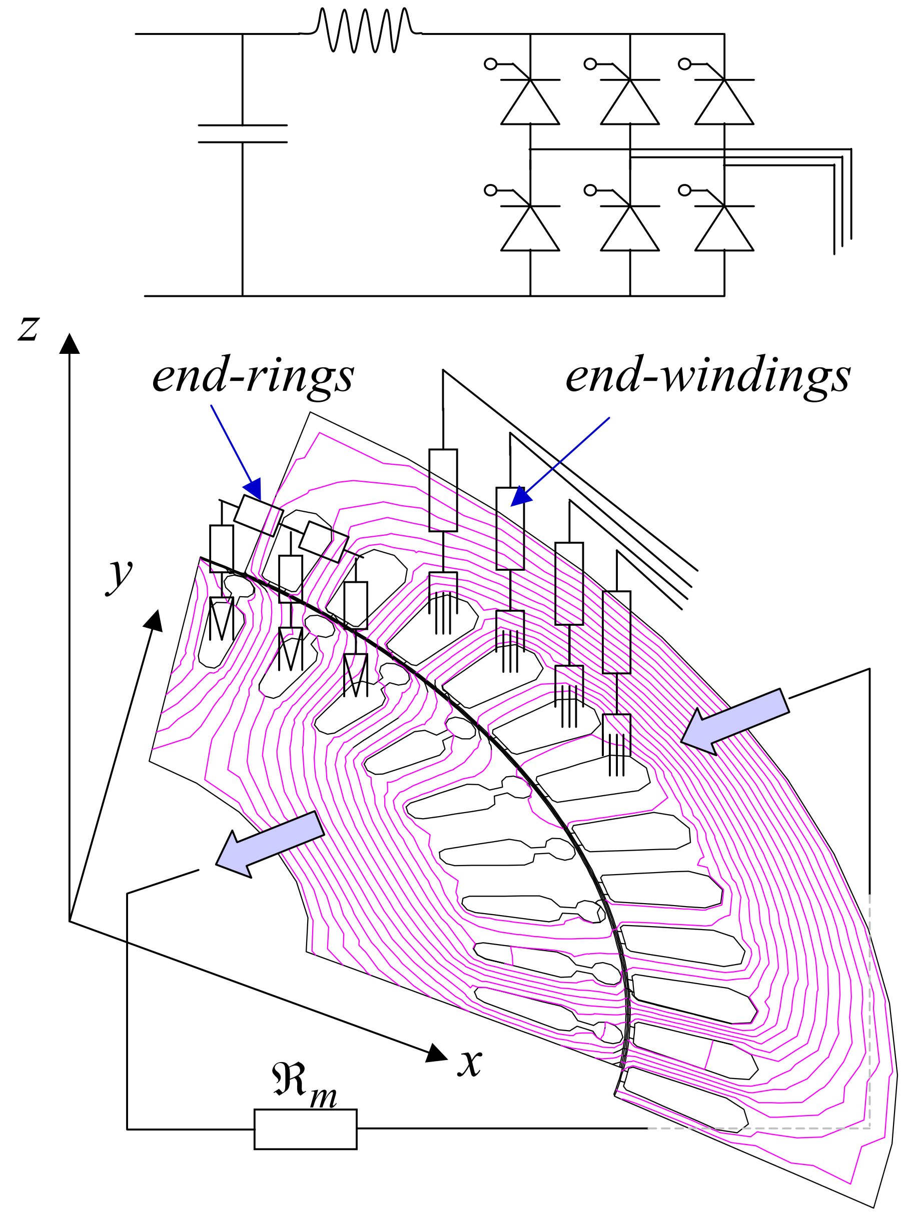
field-circuit coupling

FE/FIT model

- geometrical details
- ferromagnetic saturation (non-linear!!)
- (motional) eddy currents

circuit

- external sources/loads, (e.g. power electronic equipment)
- parts outside the FE model (e.g. end windings/rings)
- representing (linear) parts for which an equivalent circuit suffices (e.g. homopolar shaft flux)



Quasistatic formulation

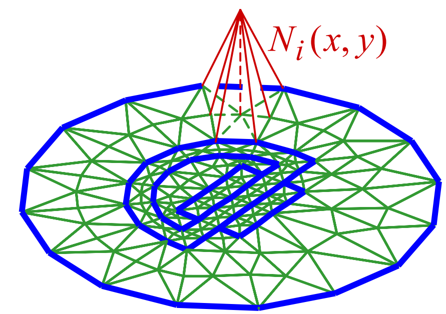
magnetic vector potential, pulsation, velocity, voltage

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla \times (\nu \nabla \times \mathbf{A}) - \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) + j\omega\sigma\mathbf{A} + \sigma \frac{\partial \mathbf{A}}{\partial t} = -\sigma \nabla V$$

reluctivity, conductivity

finite element



2D model $\mathbf{A} = (0, 0, A_z)$

$$-\frac{\partial}{\partial x}\left(\nu \frac{\partial A_z}{\partial x}\right) - \frac{\partial}{\partial y}\left(\nu \frac{\partial A_z}{\partial y}\right) + \sigma v_x \frac{\partial A_z}{\partial x} + \sigma v_y \frac{\partial A_z}{\partial y} + j\omega\sigma A_z + \sigma \frac{\partial A_z}{\partial t} = \frac{\sigma}{\ell_z} V$$

discretisation $A_z = \sum_j A_{z,j} N_j(x,y)$

$$\left[k_{ij} + m_{ij} + l_{ij}\right]\left[A_{z,j}\right] = \left[f_i\right]$$

large & sparse system of equations

$$k_{ij} = \int_\Omega \nu \left(\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right) \mathrm{d}\Omega$$

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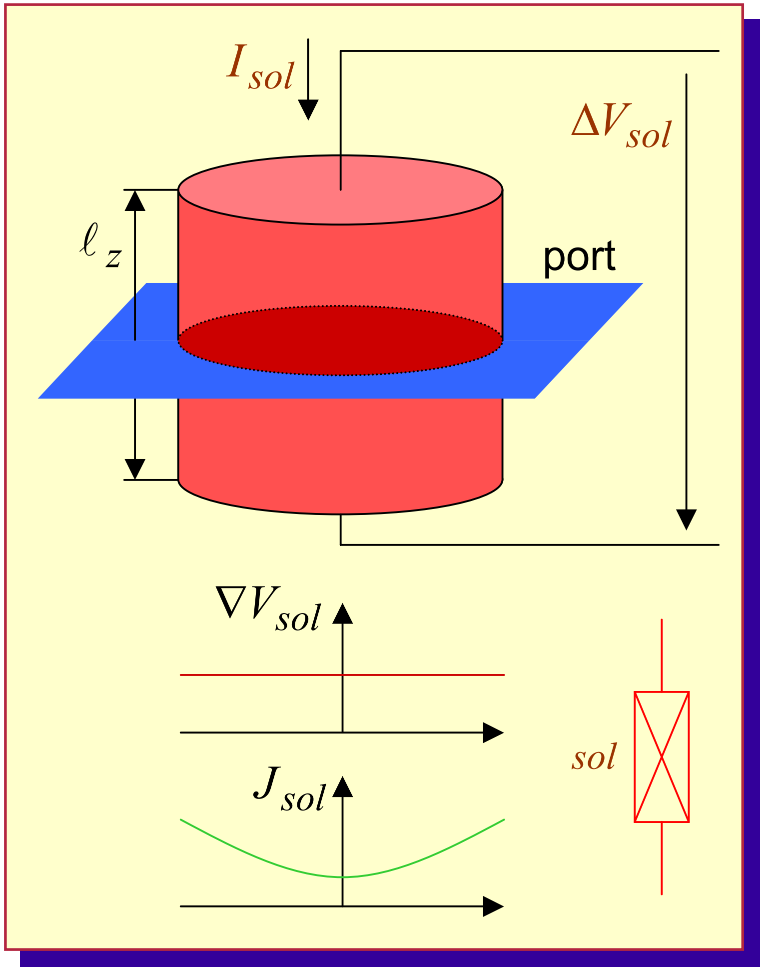
Solid (massive) conductor



"voltage-driven" conductor

no circuit equation if voltage drop is known
no fill-in in the FE matrix part if the voltage drop is an independent degree of freedom

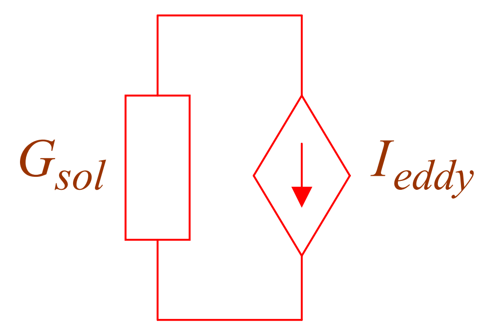
- magnetic equation

$$\nabla \times \left(\nu \nabla \times \mathbf{A}\right) + \sigma \frac{\partial \mathbf{A}}{\partial t} = \frac{\sigma}{\ell_z} \Delta V_{sol}$$

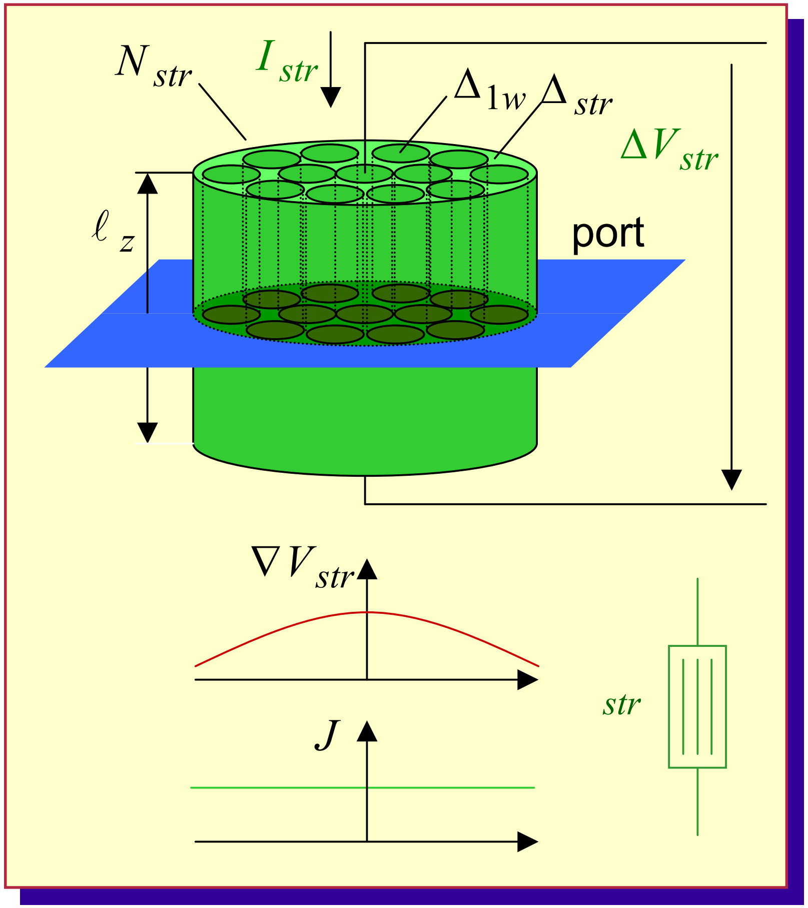
- total current $\quad I_{sol} = \int_{\Omega_{sol}} \mathbf{J} d\Omega$

$$I_{sol} = \underbrace{\underbrace{\int_{\Omega_{sol}} \frac{\sigma}{\ell_z} d\Omega}_{G_{sol}} \Delta V_{sol}}_{I_{source}} - \underbrace{\int_{\Omega_{sol}} \sigma \frac{\partial \mathbf{A}}{\partial t} d\Omega}_{I_{eddy}}$$

- equivalent scheme



Stranded (filamentary) conductor



"current-driven" conductor

no circuit equation if current is known
no fill-in in the FE matrix part if the current is an independent degree of freedom

- magnetic equation

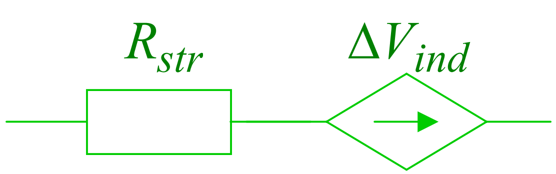
$$\nabla\times\left(\nu\nabla\times\mathbf{A}\right)=\frac{N_{str}}{\Delta_{str}}I_{str}$$

- total voltage

$$\Delta V_{str}=\frac{N_{str}\ell_z}{\Delta_{str}}\int_{\Omega_{str}}\left(-\nabla V\right)d\Omega$$

$$\Delta V_{str}=\underbrace{R_{str}I_{str}}_{\Delta V_{res}}+\underbrace{\frac{N_{str}\ell_z}{\Delta_{str}}\int_{\Omega_{str}}\frac{\partial\mathbf{A}}{\partial t}d\Omega}_{\Delta V_{ind}}$$

- equivalent scheme

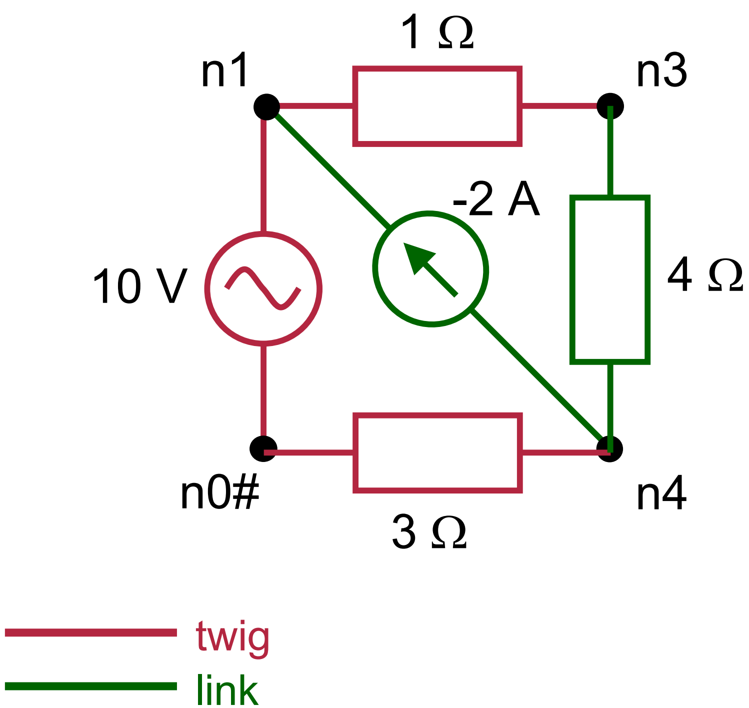


Coupling requirements

$\begin{bmatrix} K & B^T \\ B & C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}$	*(compacted) modified nodal analysis*	*(compacted) loop analysis*	*hybrid analysis*
unknowns	nodal voltages (+a few currents)	loop currents (+a few voltages)	twig voltages link currents
keep the FE matrix part unchanged ▪ sparsity ▪ preconditioners (multigrid) ▪ possible benefits thanks to structured grids (FIT)	no (yes)	no (yes)	yes
preserve symmetry ▪ Krylov subspace solvers for symmetric systems (CG, MINRES, QMR) ▪ storage	yes	yes	yes
preserve positive definiteness ▪ solvers (CG) ▪ preconditioners (IC)	yes (no)	yes (no)	no [yes]

Circuit description (1)

1. Trace a tree through the circuit



starting from the circuit node n0#, the twigs are selected in the order

1. voltage source 10 V
2. resistor 1 Ω
3. resistor 3 Ω

Priority list

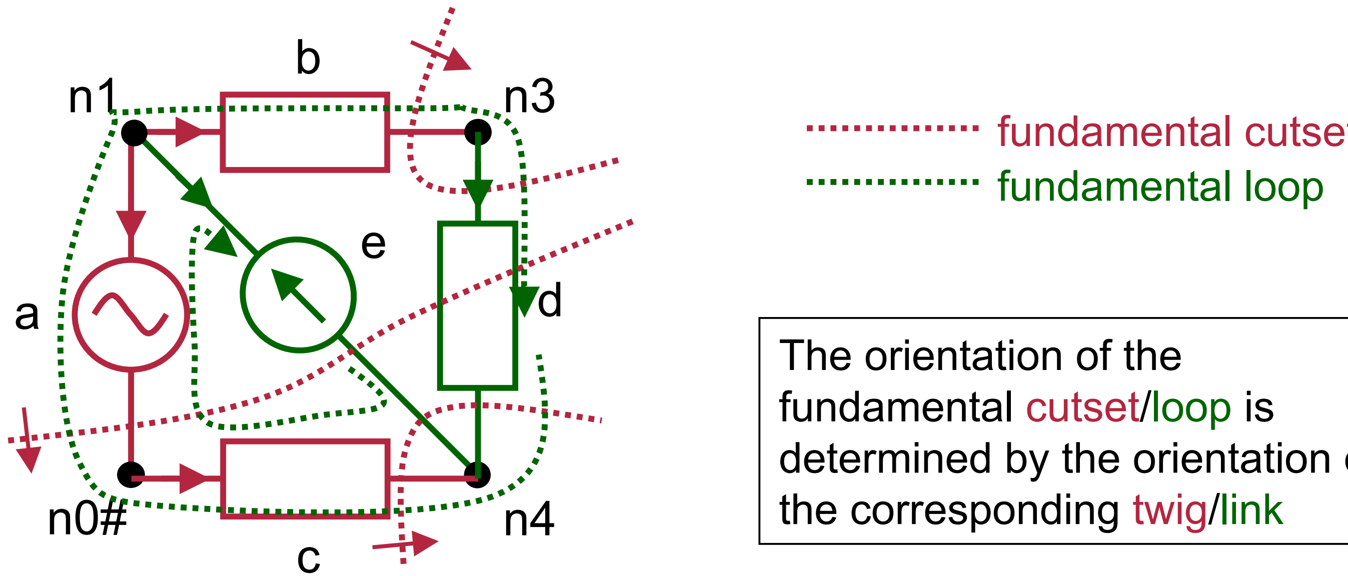
highest priority, preferably twig

- voltage sources
- solid conductors (coupled)
- capacitors (largest capacitance first)
- resistors (largest conductance first)
- inductors (smallest inductance first)
- stranded conductors (coupled)
- current sources

smallest priority, preferably link

Circuit description (2)

2. Determine fundamental cutsets and fundamental loops



The orientation of the fundamental cutset/loop is determined by the orientation of the corresponding twig/link

A fundamental cutset is formed by 1 twig and the unique set of links completing the set of branches which would upon removal result in two disconnected circuit parts.

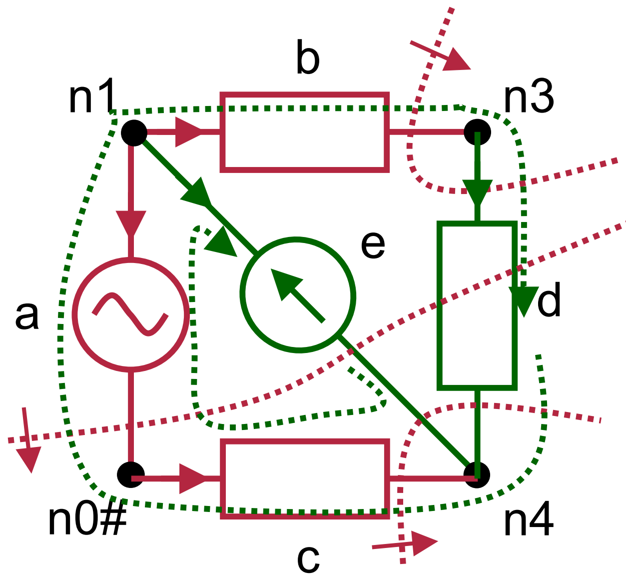
A fundamental loop consists of 1 link and the unique path through the tree closing the loop.

Property: priority(twig) ≥ priority(branch),
$\forall$branch ∈ fundamental cutset

Property: priority(link) ≤ priority(branch),
$\forall$branch ∈ fundamental loop

Circuit description (3)

3. Construct the fundamental cutset and fundamental loop matrices



fundamental cutset matrix

$$D = \left[\begin{array}{ccc:cc} 1 & & & 1 & 1 \\ & 1 & & -1 & \\ & & 1 & 1 & 1 \end{array}\right]$$

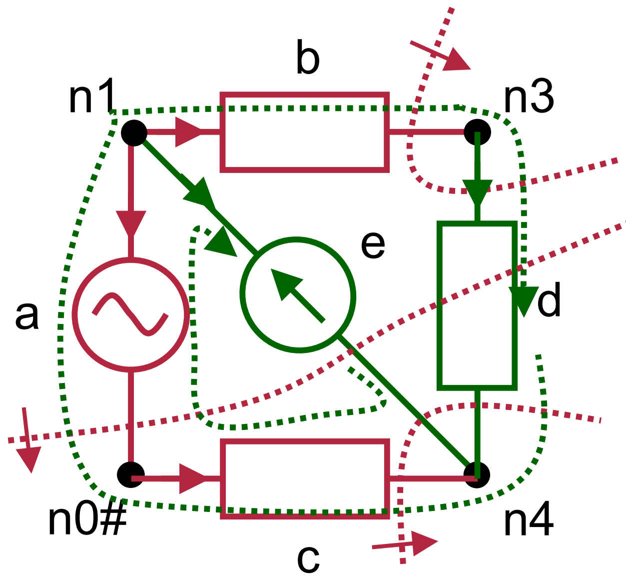
fundamental loop matrix

$$B = \left[\begin{array}{ccc:cc} -1 & 1 & -1 & 1 & \\ -1 & & -1 & & 1 \end{array}\right]$$

remark: $B_{\mathrm{ln,tw}} = -D_{\mathrm{tw,ln}}^{\mathrm{T}}$

Circuit description (4)

4. Partition the fundamental incidence matrices



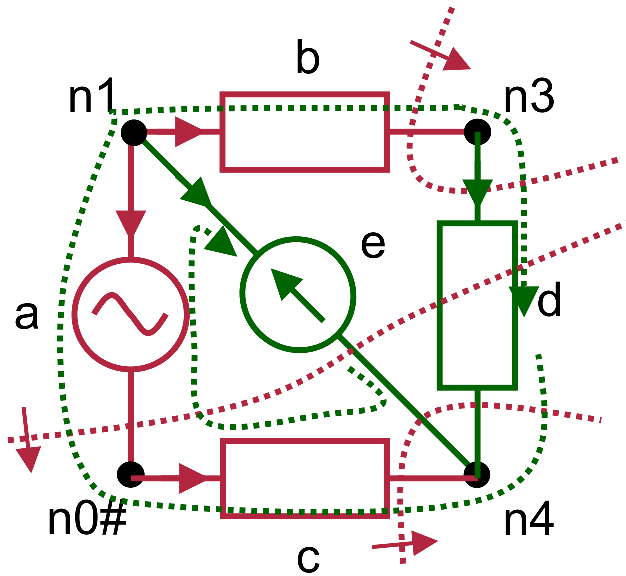
$$D = \left[\begin{array}{ccc|cc} 1 & & & 1 & 1 \\ & 1 & & -1 & \\ & & 1 & 1 & 1 \end{array}\right]$$

$D_{\mathrm{twv,lno}}$, $D_{\mathrm{twv,lni}}$, $D_{\mathrm{two,lni}}$, $D_{\mathrm{two,lno}}$

twv	twigs at which the voltage is known (voltage sources)
two	twigs at which an unknown voltage is assigned (“*free twigs*”)
twu	eliminated twigs (“*eliminated twigs*”) (not in this example)
lnu	eliminated links (“*eliminated links*”) (not in this example)
lno	links at which an unknown current is assigned (“*free links*”)
lni	links at which the current is known (current sources)

Circuit description (5)

5. Write impedance/admittance matrices and voltage/current vectors



1. admittance matrix for the free twigs

$$Y_{\mathrm{two}} = \begin{bmatrix} 1/1 & \\ & 1/3 \end{bmatrix}$$

2. impedance matrix for the free links

$$Z_{\mathrm{lno}} = [4]$$

3. voltage vector for the voltage sources

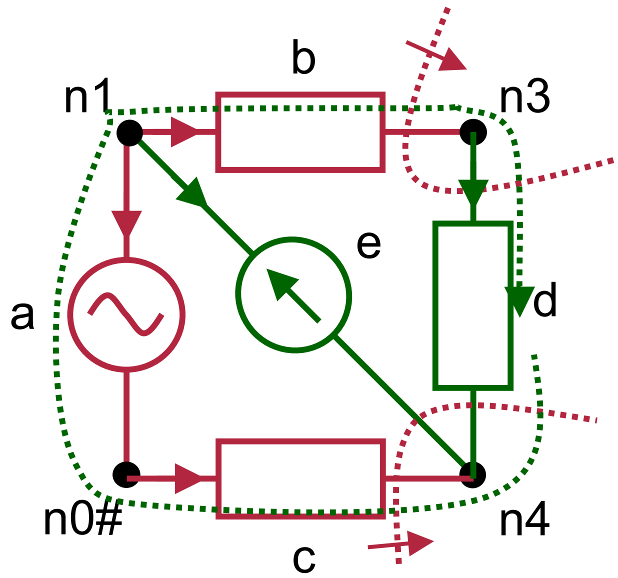
$$v_{\mathrm{twv}} = [10]$$

4. current vector for the current sources

$$i_{\mathrm{lni}} = [2]$$

Circuit description (6)

6. Write system of equations



intuitive approach:

1. write the Kirchhoff current law for each fundamental cutset associated with a free twig

2. write the Kirchhoff voltage law for each fundamental loop associated with a free link

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0.333 & 1 \\ -1 & 1 & -4 \end{bmatrix} \begin{bmatrix} v_b \\ v_c \\ i_d \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} Y_{\text{two}} & D_{\text{two,lno}} \\ -B_{\text{lno,two}} & -Z_{\text{lno}} \end{bmatrix} \begin{bmatrix} v_{\text{two}} \\ i_{\text{lno}} \end{bmatrix} = \begin{bmatrix} -D_{\text{two,lni}} i_{\text{lni}} \\ B_{\text{lno,twv}} v_{\text{twv}} \end{bmatrix}$$

remark: symmetric because $B_{\text{lno,two}} = -D^{\text{T}}_{\text{two,lno}}$

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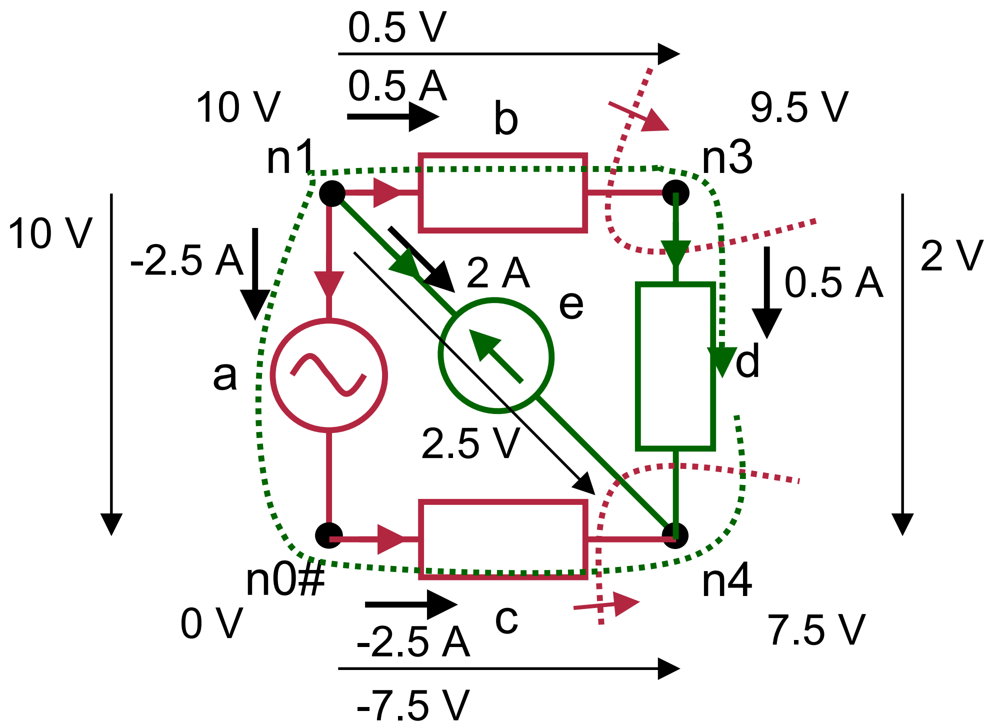
Circuit description (7)

7. Solve system of equations & propagate the circuit solution



solution

$$\begin{bmatrix} v_b \\ v_c \\ \hdashline i_d \end{bmatrix} = \begin{bmatrix} 0.5 \\ -7.5 \\ \hdashline 0.5 \end{bmatrix}$$

twig currents:

$$i_{two} = -D_{two,lno} i_{lno} - D_{two,lni} i_{lni}$$
$$i_{twv} = -D_{twv,lno} i_{lno} - D_{twv,lni} i_{lni}$$

link voltages

$$v_{lno} = -B_{lno,two} v_{two} - B_{lno,twv} v_{twv}$$
$$v_{lni} = -B_{lni,two} v_{two} - B_{lni,twv} v_{twv}$$

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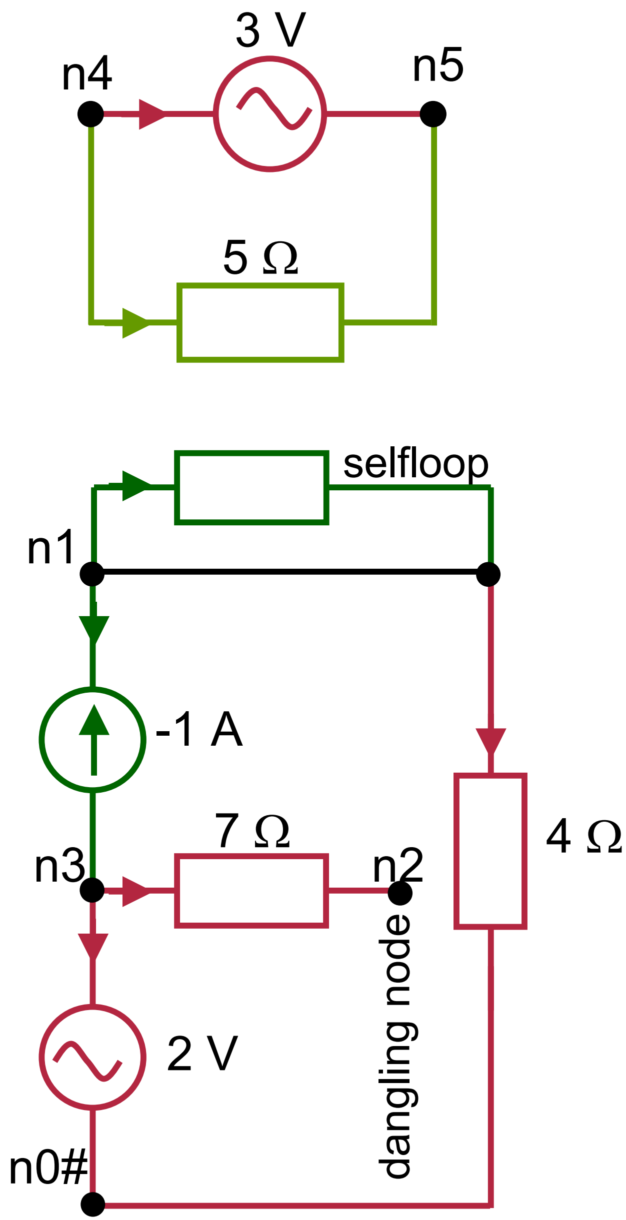
Particularities



1. Distinct circuit parts

2. Dangling nodes

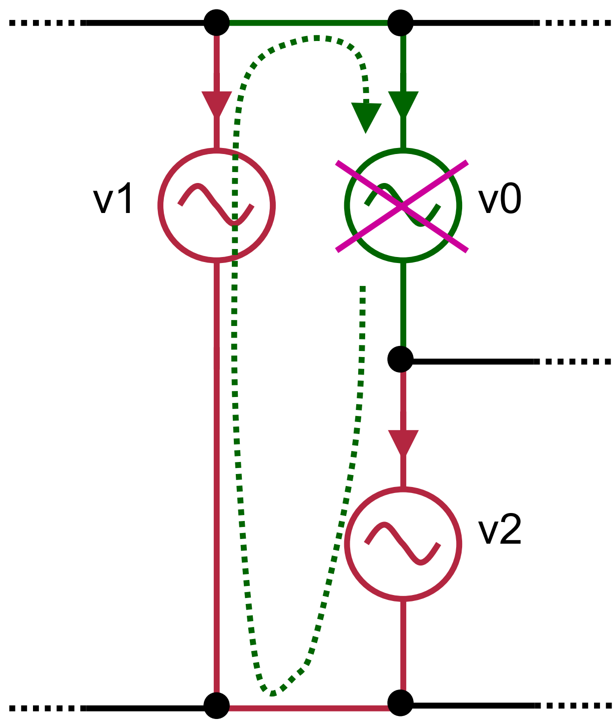
a branch to a dangling node
always a twig
associated fundamental cutset
only contains the twig

3. Self-loops

a self-loop
is always a link
the associated fundamental loop
only contains the link

Consistency check (1)

1. Fundamental loop consisting of voltage sources



Problem: a voltage source is necessarily selected as link

Treatment: check the Kirchhoff voltage law in the associated fundamental loop

` e.g. v0-v1+v2 = 0 ??

if valid

omit the voltage source link

if not valid

the circuit has no solution

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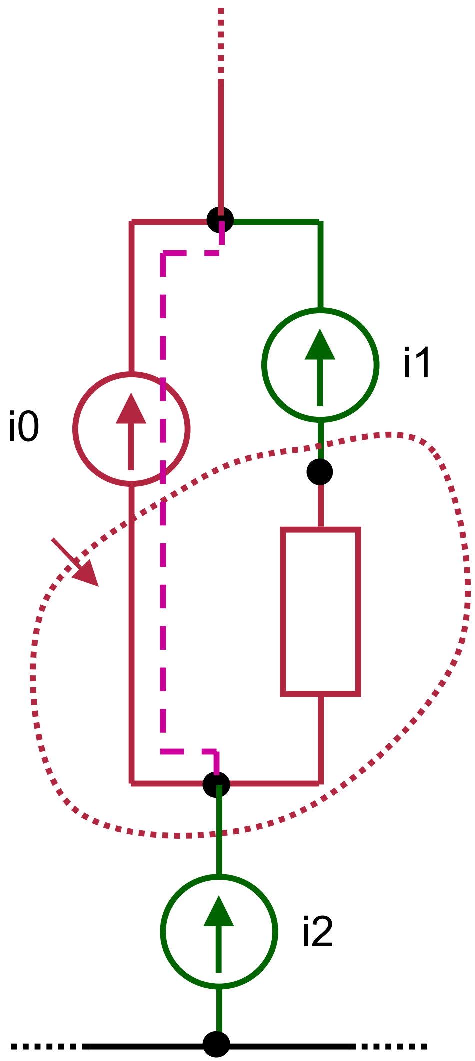
Consistency check (2)

2. Fundamental cutset consisting of current sources



Problem: a current source is necessarily selected as twig

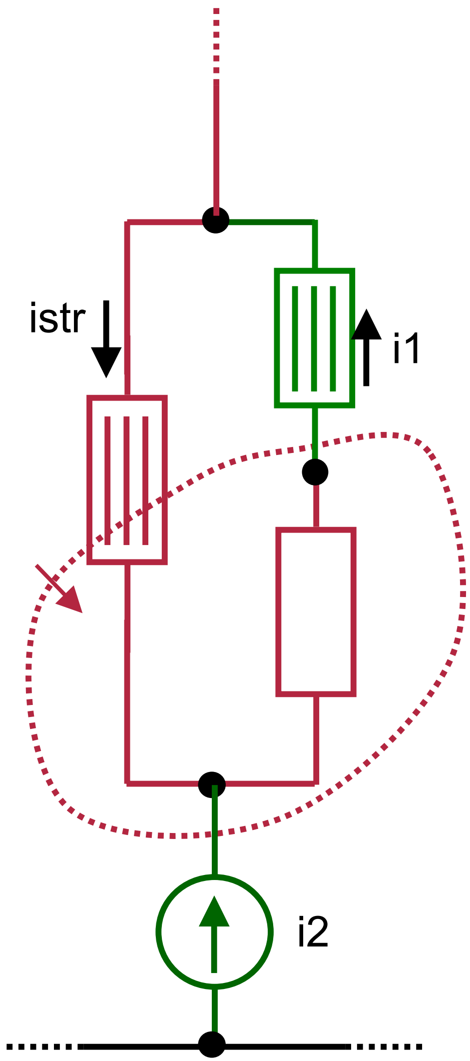
Treatment: check the Kirchhoff current law in the associated fundamental cutset
e.g. i0 + i1 - i2 = 0 ??

if valid
replace the current source twig by a short-circuit connection

if not valid
the circuit has no solution

Partial transformation (1)

1. Stranded conductor being selected as twig



Problem: a stranded conductor (current-driven branch) is necessarily selected as twig

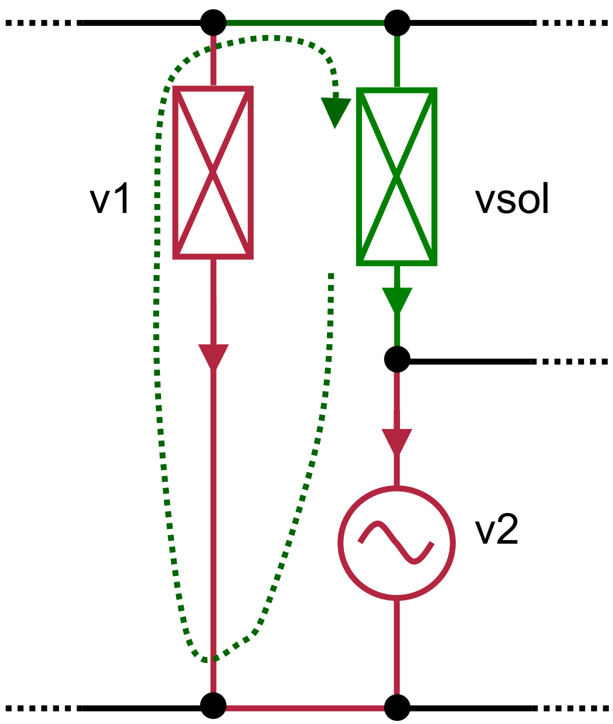
Property: priority(twig) ≥ priority(branch), ∀branch ∈ associated fundamental cutset

Treatment: apply the Kirchhoff current law in the associated fundamental cutset to express the stranded-conductor current in terms of link currents
e.g. istr = i1 + i2

~ small and independent Schur complements

Partial transformation (2)

2. Solid conductor being selected as link



Problem: a solid conductor (voltage-driven branch) is necessarily selected as link

Property: priority(link) ≤ priority(branch), ∀branch ∈ associated fundamental loop

Treatment: apply the Kirchhoff voltage law in the associated fundamental loop to express the solid-conductor voltage in terms of twig voltage
e.g. vsol = v1 + v2

~ small and independent Schur complements

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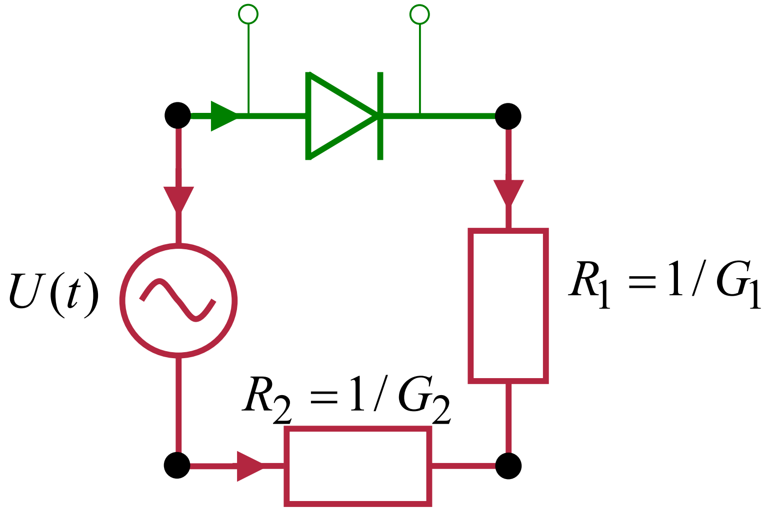
Topological changes (1)

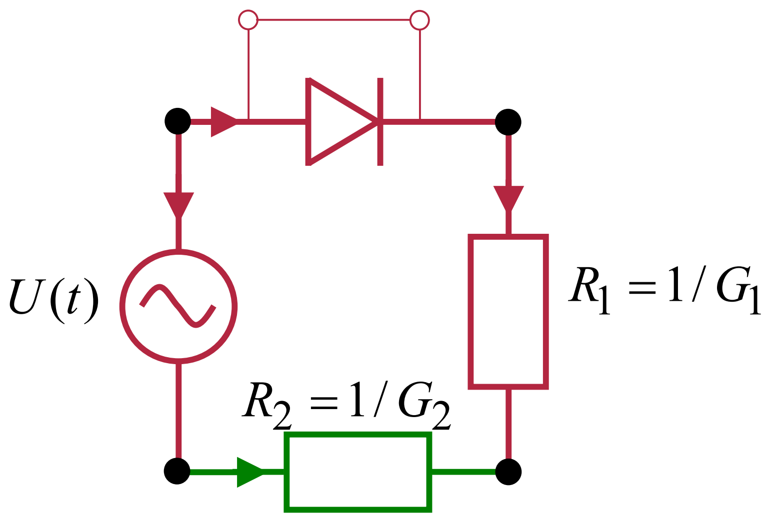
1. Switching elements closes (switch, diode, thyristor,…)



Problem: the priority of a branch increases during (transient) simulation

$$\begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

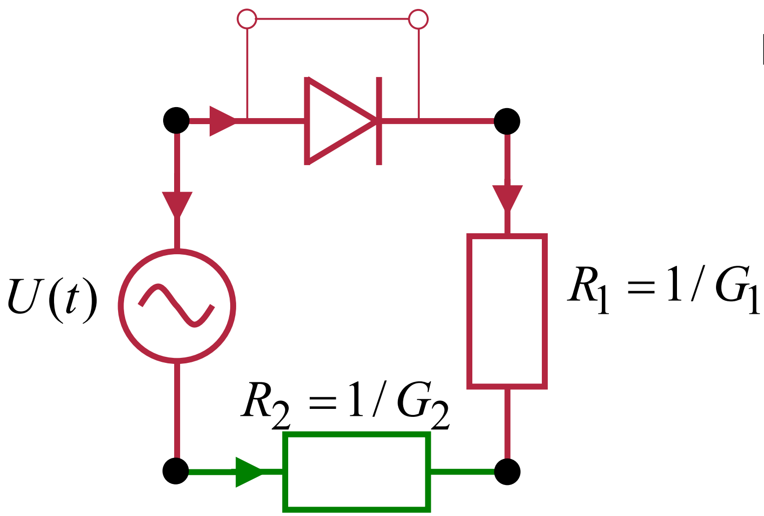
Treatment: consider associated fundamental loop and possibly change link/twig-mode with the branch with the lowest priority



$$\begin{bmatrix} G_1 & 1 \\ 1 & -R_2 \end{bmatrix}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ U \end{bmatrix}$$

Topological changes (2)

1. Switching element opens (switch, diode, thyristor,…)

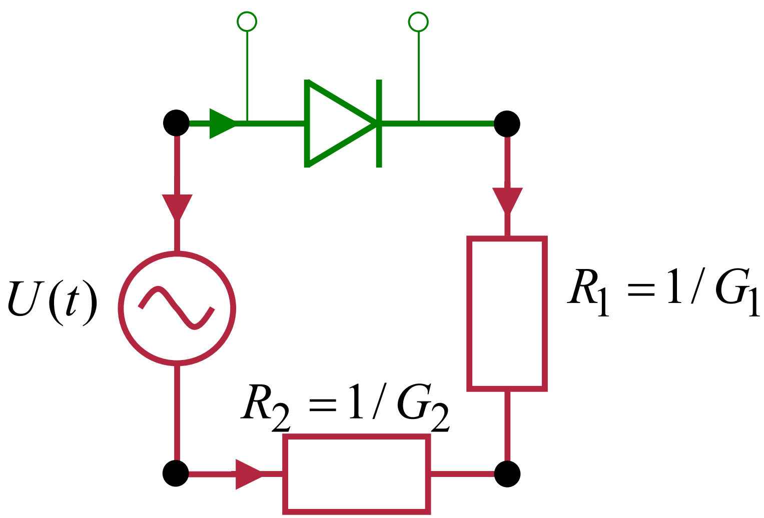


Problem: the priority of a branch decreases during (transient) simulation

$$\begin{bmatrix} G_1 & 1 \\ 1 & -R_2 \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ U \end{bmatrix}$$

Treatment: consider associated fundamental cutset and possibly change link/twig-mode with the branch with the highest priority

$$\begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



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Field-circuit coupling (3)

KCL applied to the fundamental cutsets

$$i_t + D i_l = 0$$

KVL applied to the fundamental loops

$$v_l + B v_t = 0$$

magnetodynamic PDE

reluctivity, magnetic vector potential, conductivity

$$\nabla \times \left(\nu \nabla \times \mathbf{A}\right) + j\omega\sigma\mathbf{A} = -\sigma \nabla V$$

FE equations:

cutset equations:

loop equations:

$$\left[\begin{array}{c|cc} k_{ij} + j\omega l_{ij} & q_{it} & -p_{il} \\ \hline q_{sj} & \chi G_m & \chi D \\ -p_{kj} & -\chi B & -\chi R_m \end{array}\right] \left[\begin{array}{c} A_{zj} \\ \hline v_t \\ i_l \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline \chi I_{app} \\ -\chi V_{app} \end{array}\right]$$

symmetrisation factor χ

& $B = -D^T$

symmetric !

$$i_t = G_m v_t - j\omega q_{mj} A_{zj}$$

twig branch relations

$$v_l = R_m i_l + j\omega p_{mj} A_{zj}$$

link branch relations

Algebraic solution (1)

Coupled system matrix
- symmetric/indefinite

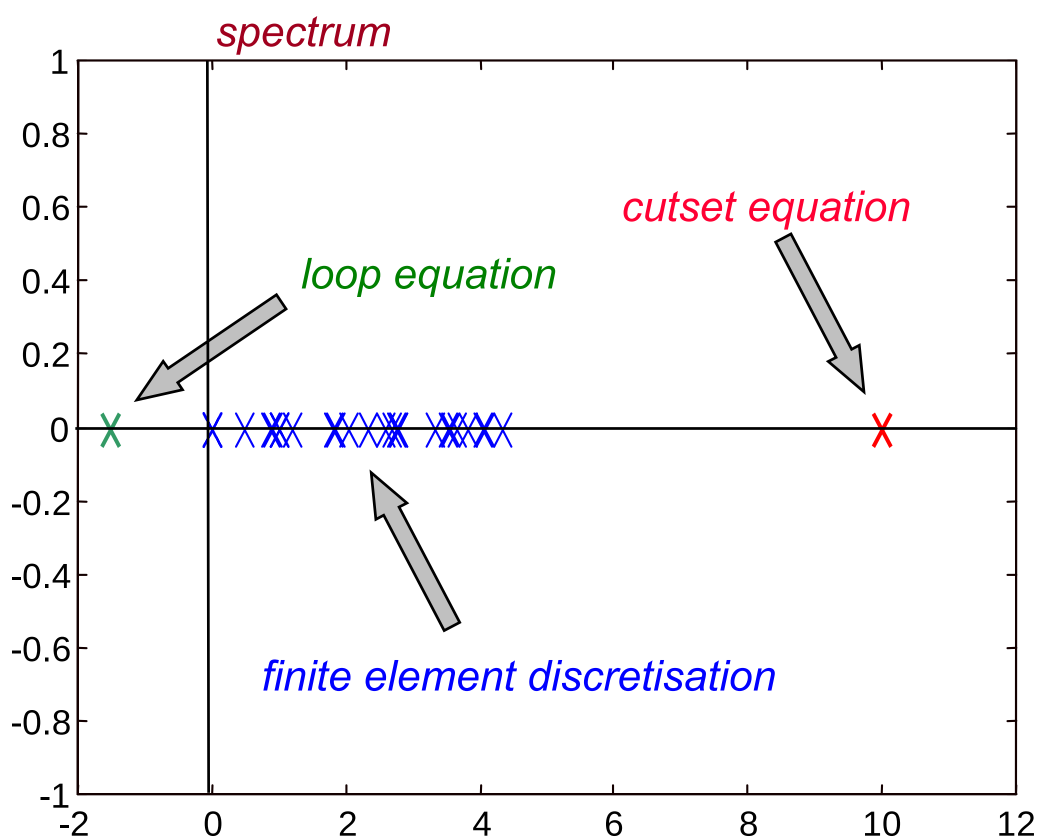
FE/FIT part
- sparse (no fill-in)
- changes due to non-linearities

Circuit part
- dense
- changes due to non-linearities & switching

Coupling blocks
- dense
- does not change

#positive-eigenvalues
= #FE/FIT-dofs + #cutset-equations

#negative-eigenvalues
= #loop-equations



Algebraic solution (2)

	solver	preconditioner
$\begin{bmatrix} K & B^T \\ B & C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}$	MINRES QMR MINRES/QMR	|SSOR| SSOR BJac (AMG,LU)
Schur complement 1 $\left(K - B^T C^{-1} B\right) x = f - B^T C^{-1} g$	CG(LU)	? SSOR/AMG ? DD
Schur complement 2 $\left(C - B K^{-1} B^T\right) y = g - B K^{-1} f$	LU(AMGCG)	? LU ? DD

& Domenico Lahaye: Algebraic multigrid for field-circuit coupled systems

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Algebraic solution (3)

Algebraic multigrid for field-circuit coupled systems
(Dr. Domenico Lahaye, CRS4, Cagliari, Italy)

fine grid system:

$$\begin{bmatrix} K & B^T \\ B & C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}$$

prolongation: $P_{H \to h}$

restriction: $R_{h \to H} = P_{H \to h}^T$

coarse grid system:

$$\begin{bmatrix} P_{H \to h}^T & \\ & I \end{bmatrix} \begin{bmatrix} K_H & B_H^T \\ B_H & C \end{bmatrix} \begin{bmatrix} P_{H \to h} & \\ & I \end{bmatrix} \begin{bmatrix} x_H \\ y_H \end{bmatrix} = \begin{bmatrix} P_{H \to h}^T & \\ & I \end{bmatrix} \begin{bmatrix} f \\ g \end{bmatrix}$$

smoother: block-Jacobi with

1. Gauss-Seidel for the FE/FIT unknowns
2. no smoothing or an exact solution for the circuit unknowns

Algebraic solution (4)

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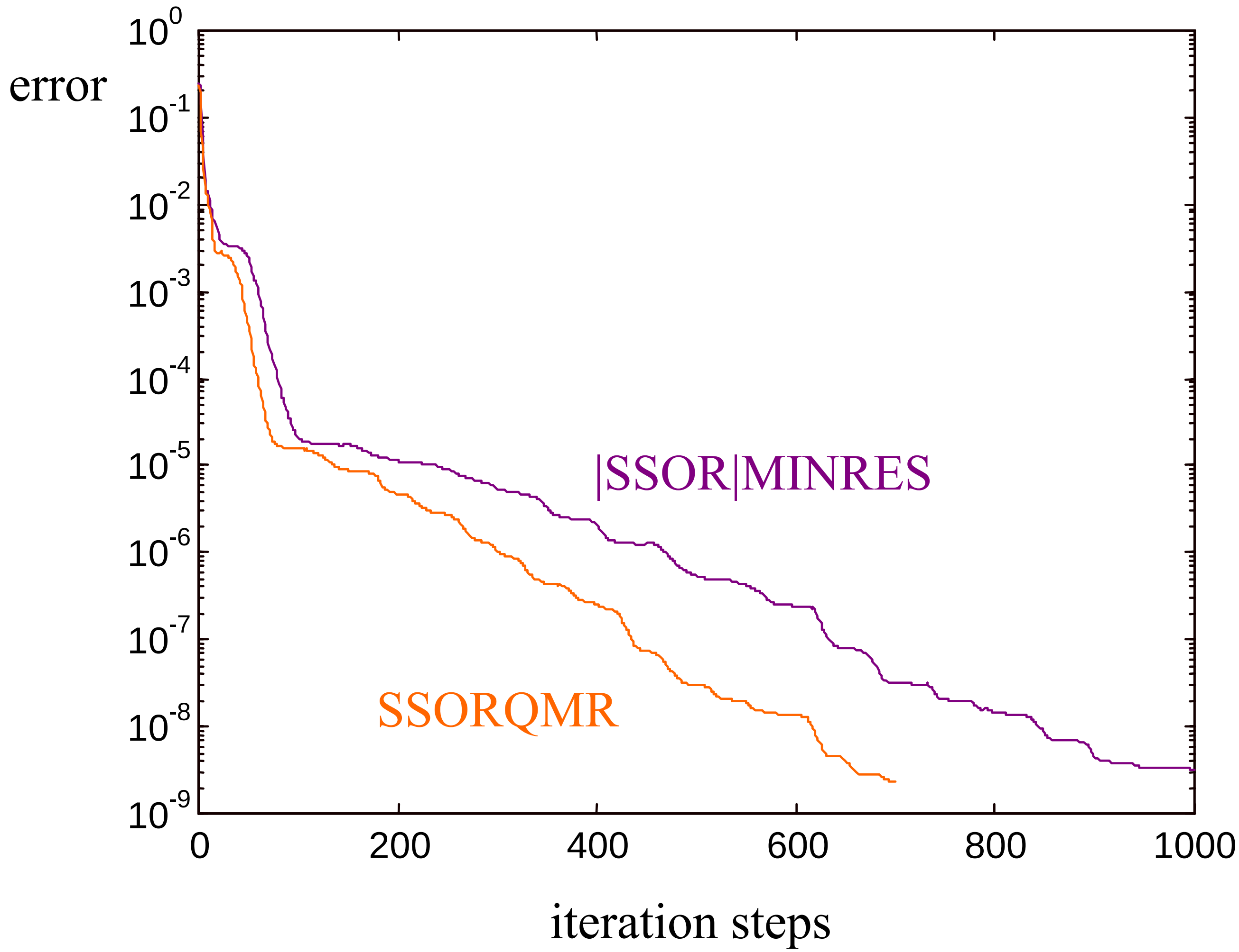
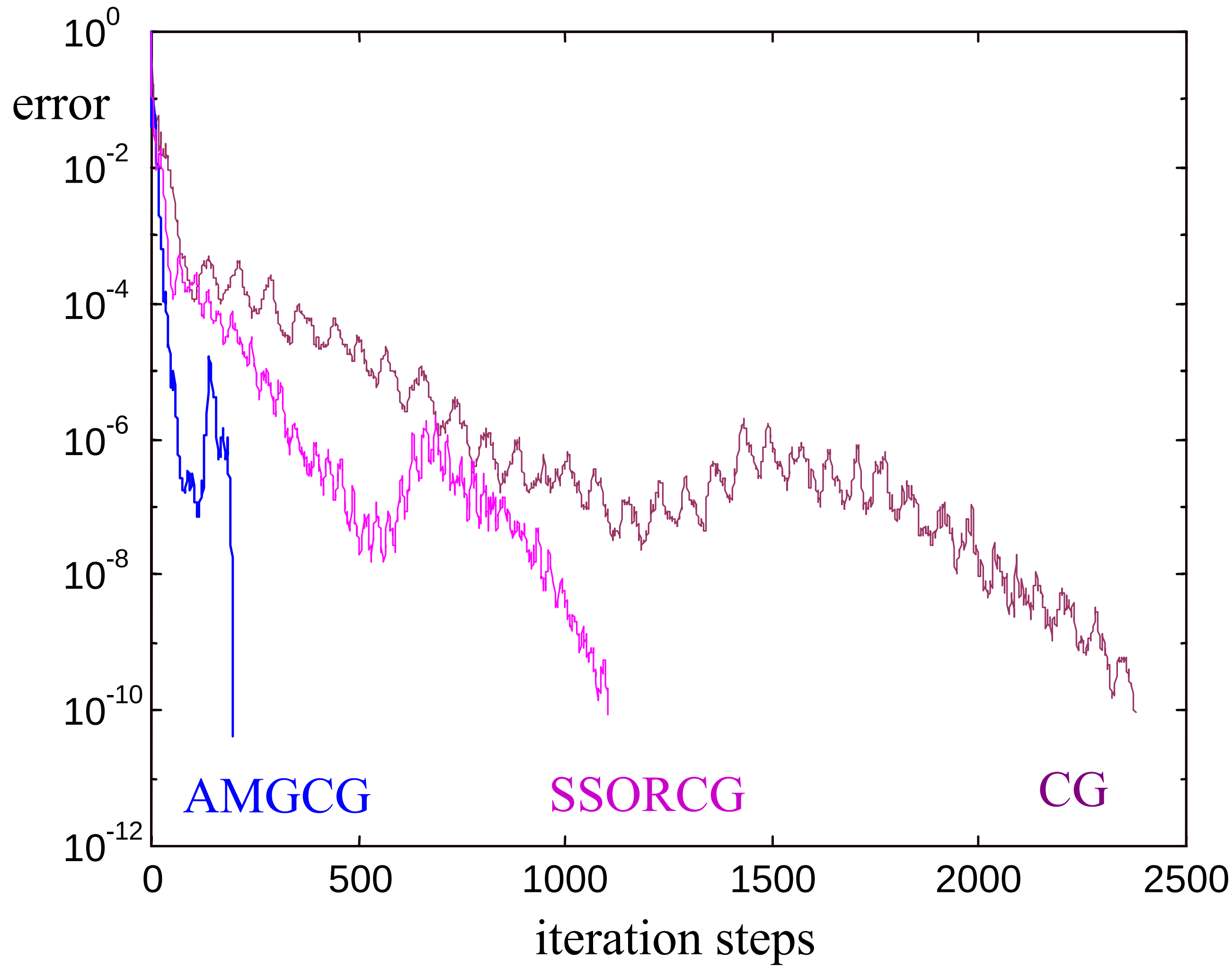
Performance of an indefinite and definite preconditioner



Algebraic solution (5)

Preconditioning of the Schur-complement system



Algebraic solution (6)

Iteration counts and computation times of the iterative system solution for 1 time step of a transient simulation.

	iteration steps	computation time (s)
MINRES	1997	59.80
QMR	1330	40.23
GMRES	1031	159.71
|SSOR|MINRES	476	11.03
SSORQMR	320	7.41
SSORGMRES	312	38.13
JAC(SSOR,LU)GMRES	325	53.91
JAC(AMG,LU)QMR	245	5.23
CG*	2376	30.06
SSORCG*	1105	17.32
AMGCG*	199	9.28

MINRES, QMR: *without preconditioning* (QMR, GMRES also bracketed)

|SSOR|MINRES, SSORQMR, SSORGMRES: *non-symmetric ↔ symmetric solver*

JAC(SSOR,LU)GMRES, JAC(AMG,LU)QMR: *block preconditioning*

CG*, SSORCG*, AMGCG*: *Schur complement*

|SSOR|MINRES, SSORQMR: *indefinite ↔ definite preconditioning*

& Domenico Lahaye: Algebraic multigrid for field-circuit coupled systems

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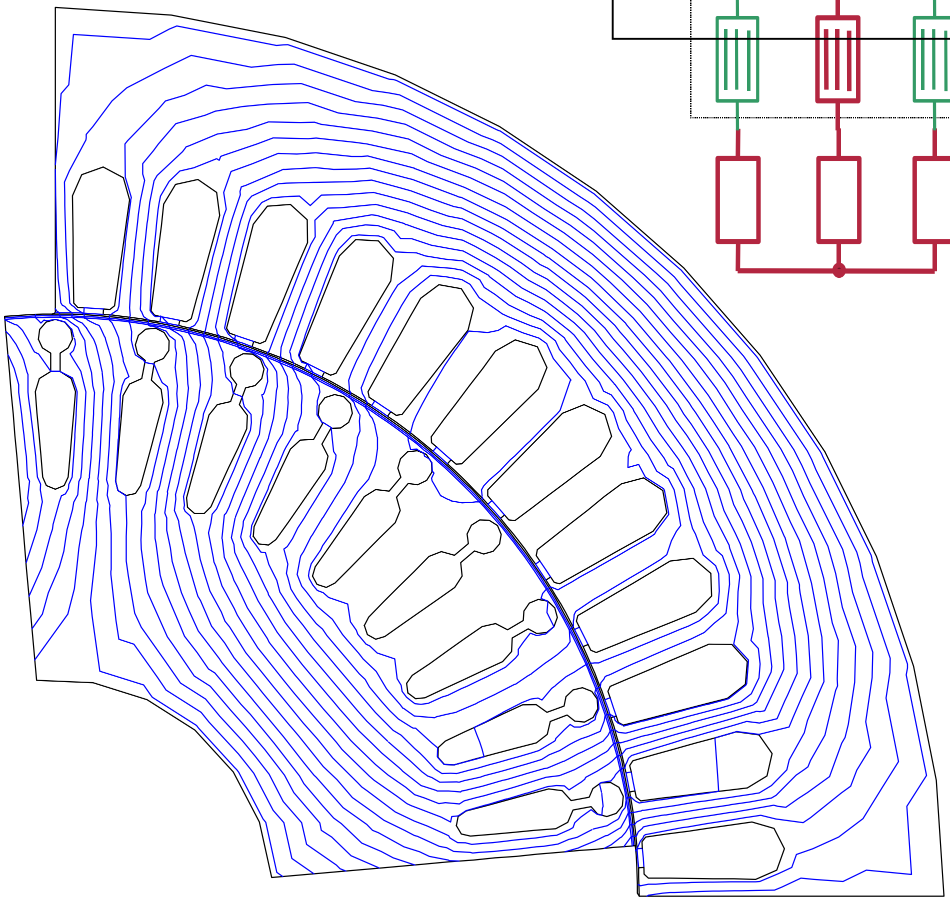
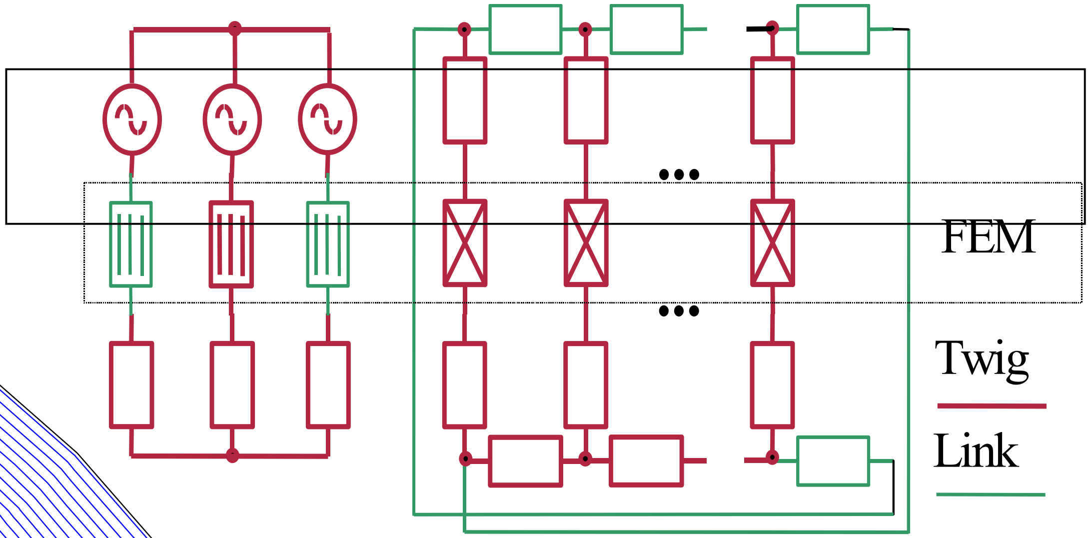
Examples (1)

Time-harmonic simulation of a three-phase induction machine

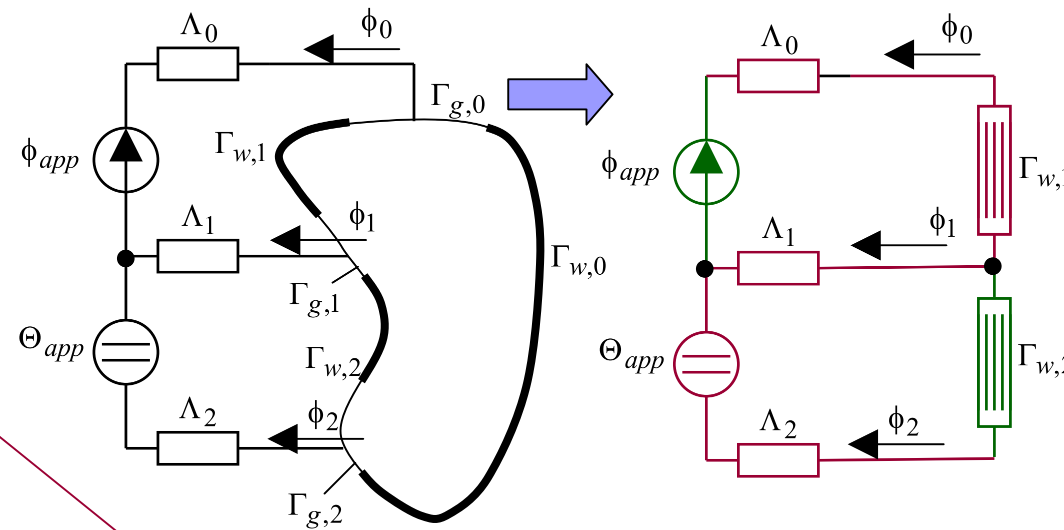


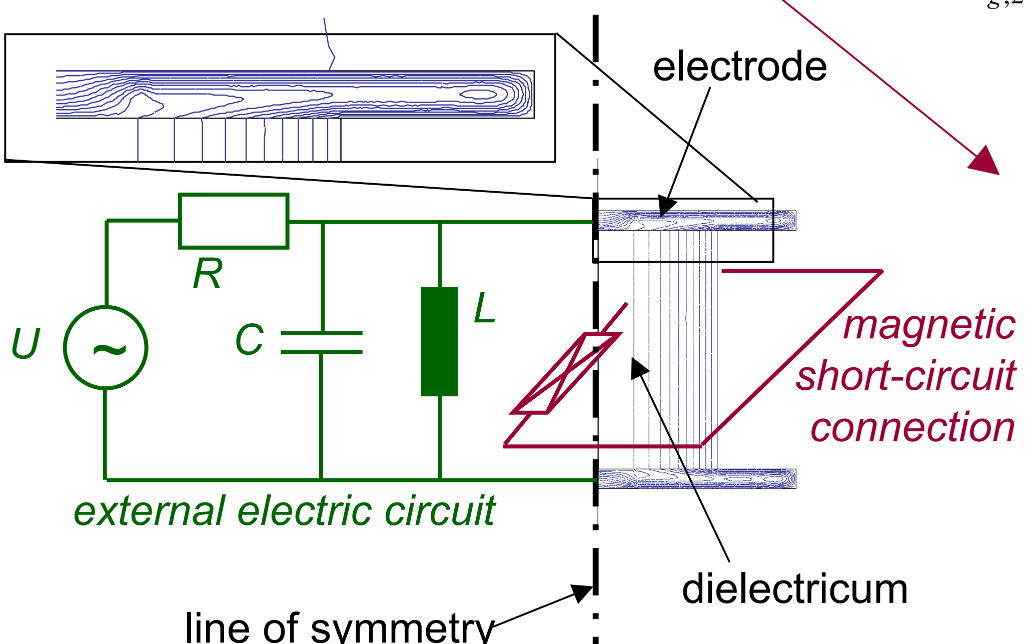


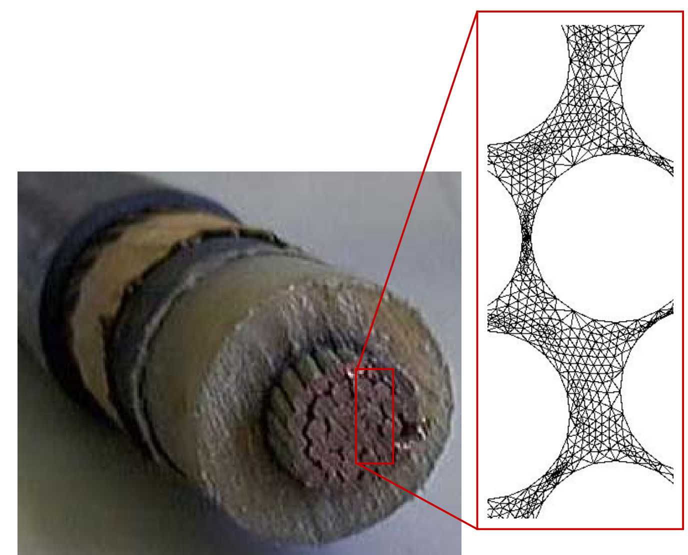
- three-phase voltage excitation
- end-winding resistances & inductances
- end-rind resistances & inductances

Examples (2)

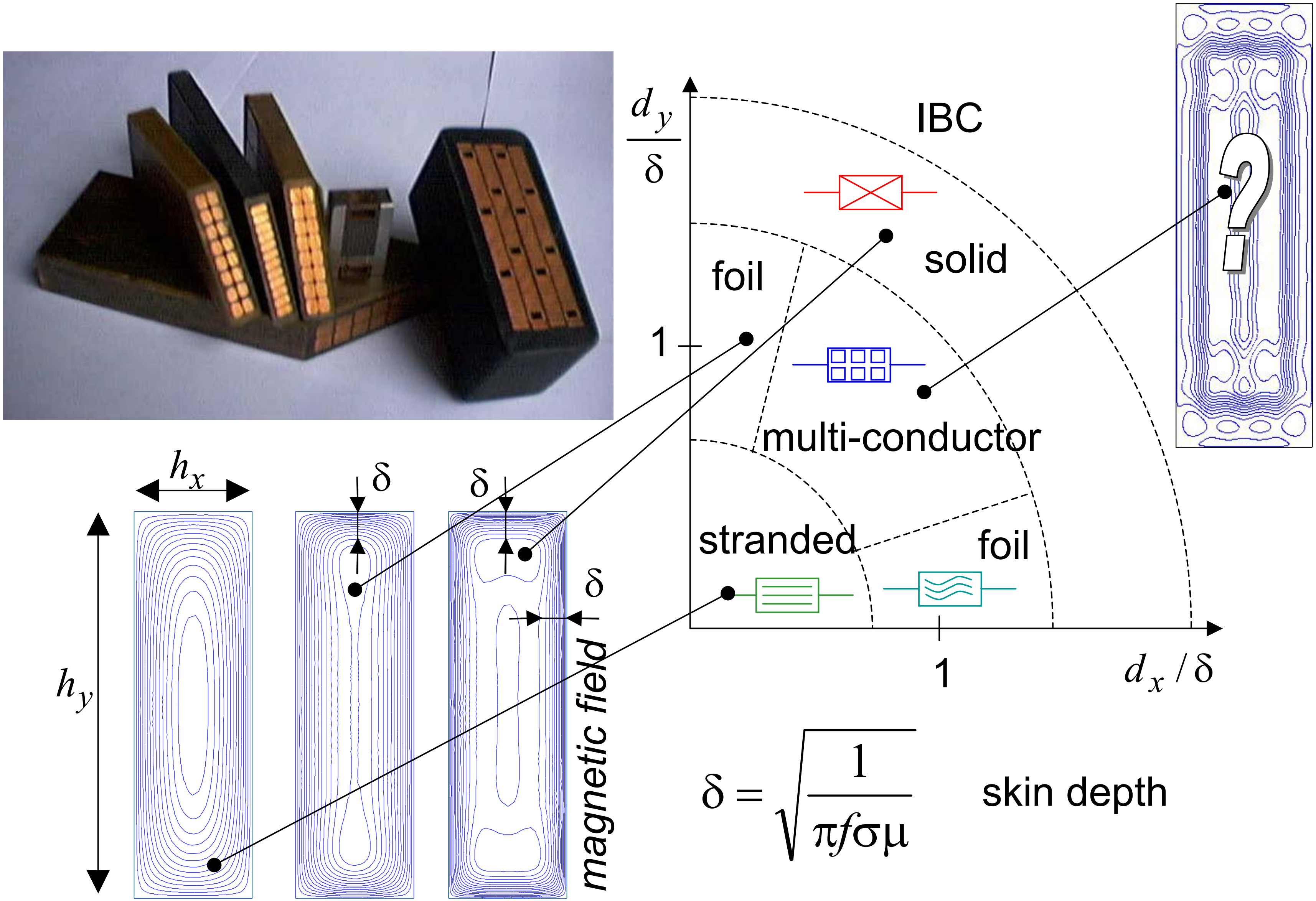
1. magnetic field + magnetic circuit
2. magnetic field + analytical model + electric circuit
3. electrokinetic field + magnetic circuit + electric circuit







More general conductor systems ?



$$\delta = \sqrt{\frac{1}{\pi f \sigma \mu}}$$ skin depth

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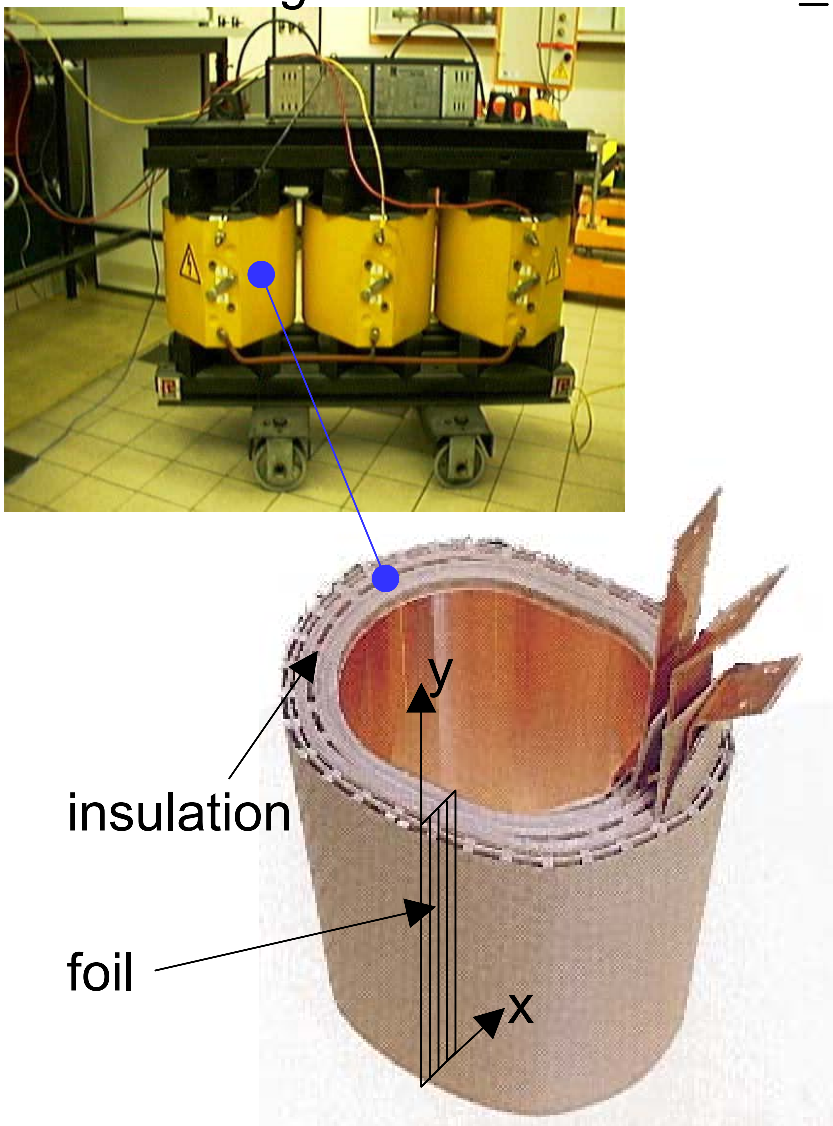
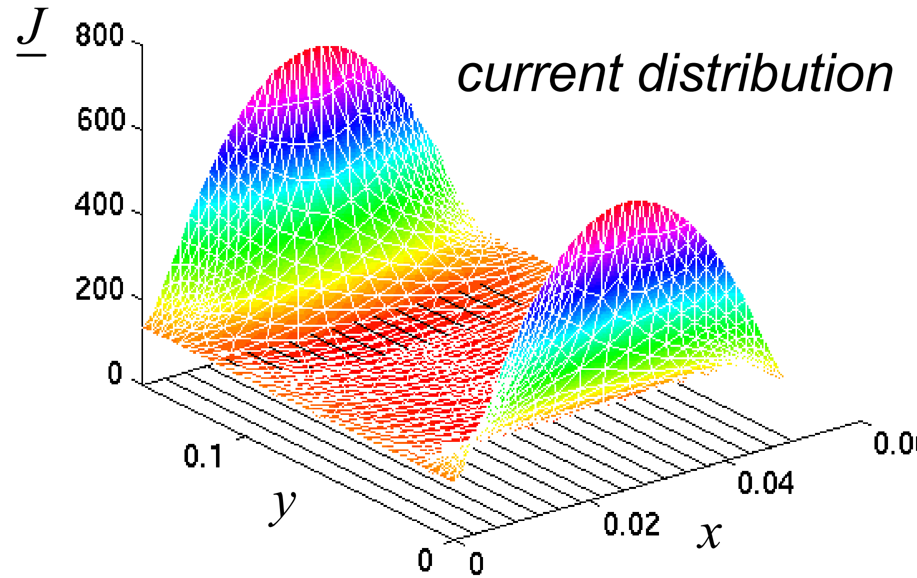
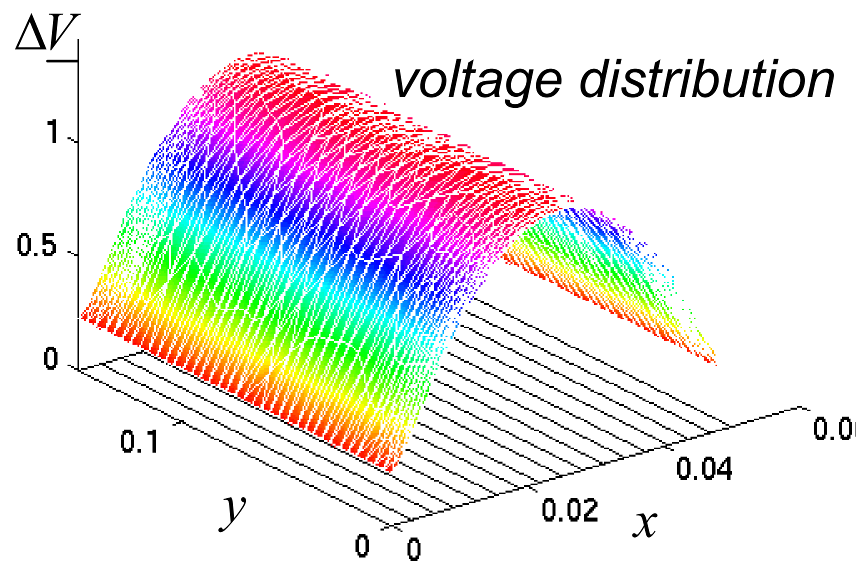
Foil windings

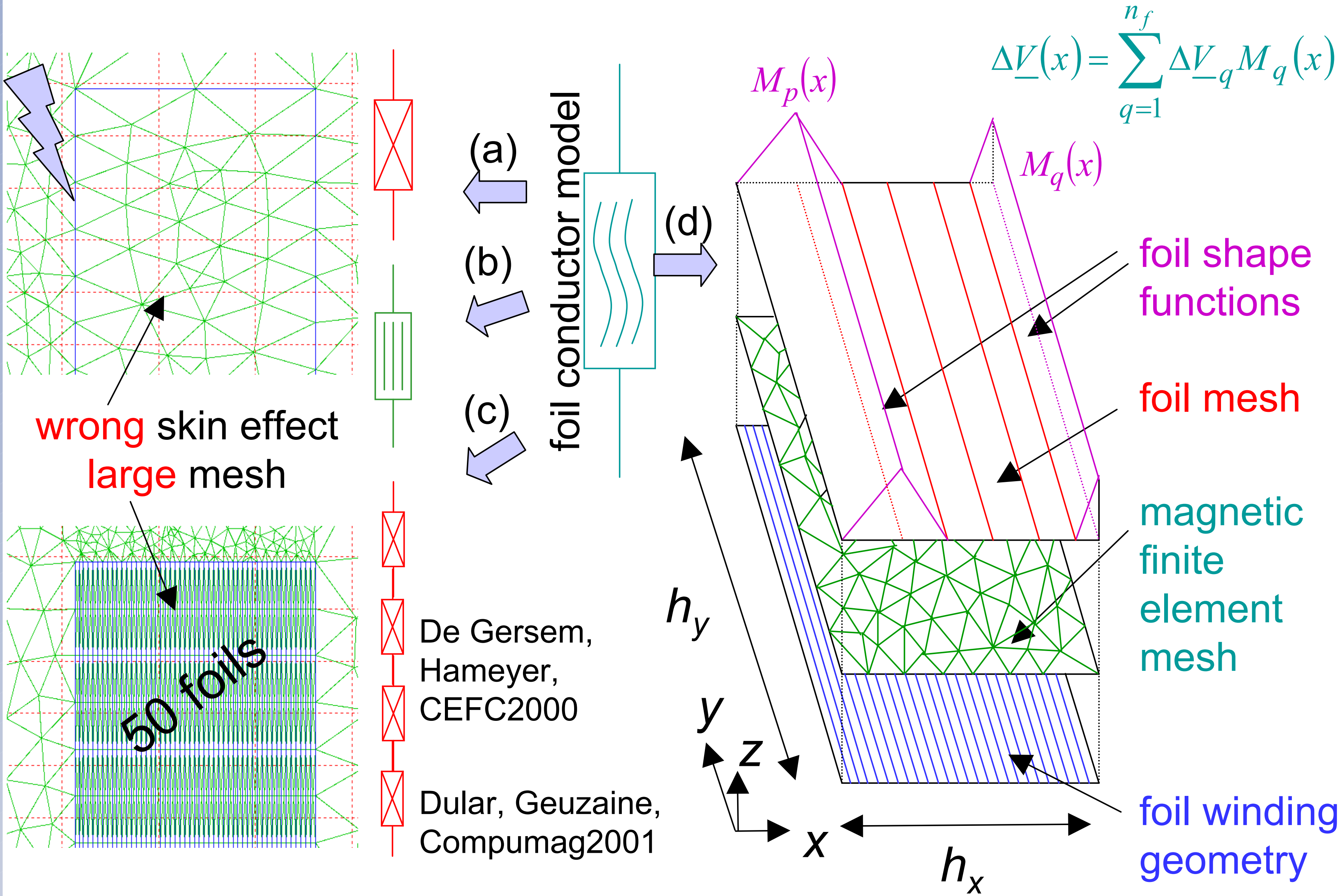
foil winding transformer







Foil conductor model



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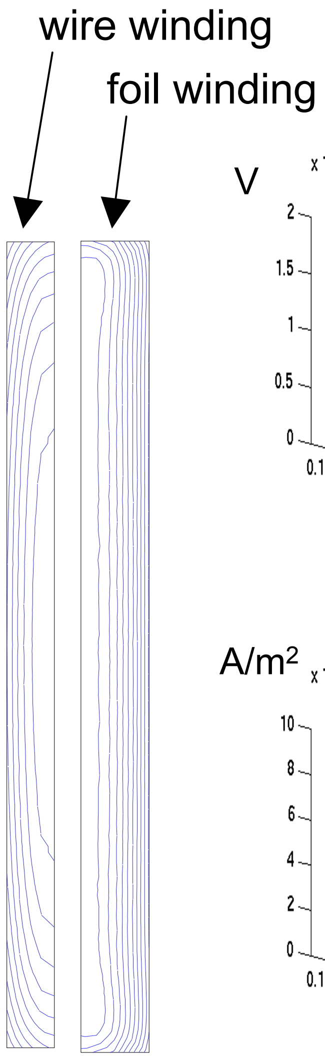
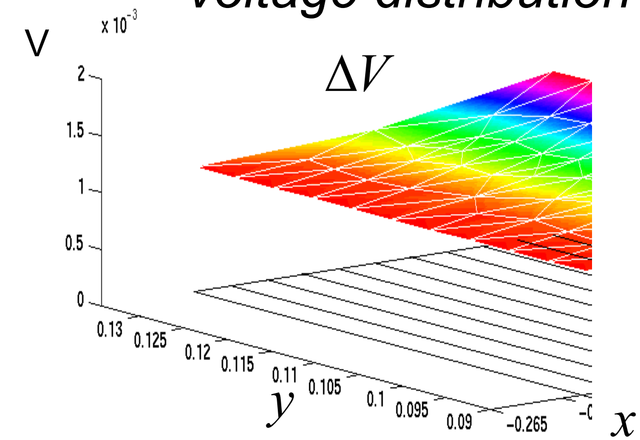
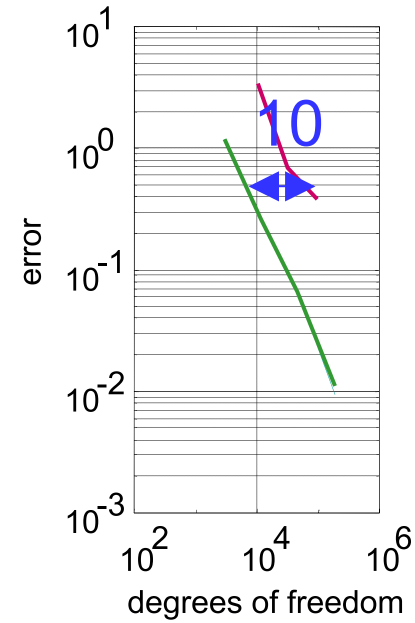
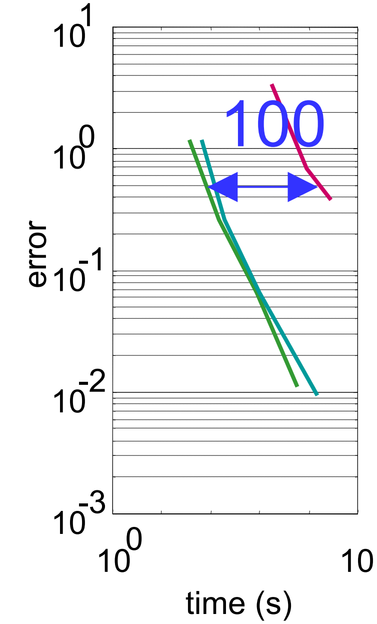
Foil winding transformer



voltage distribution

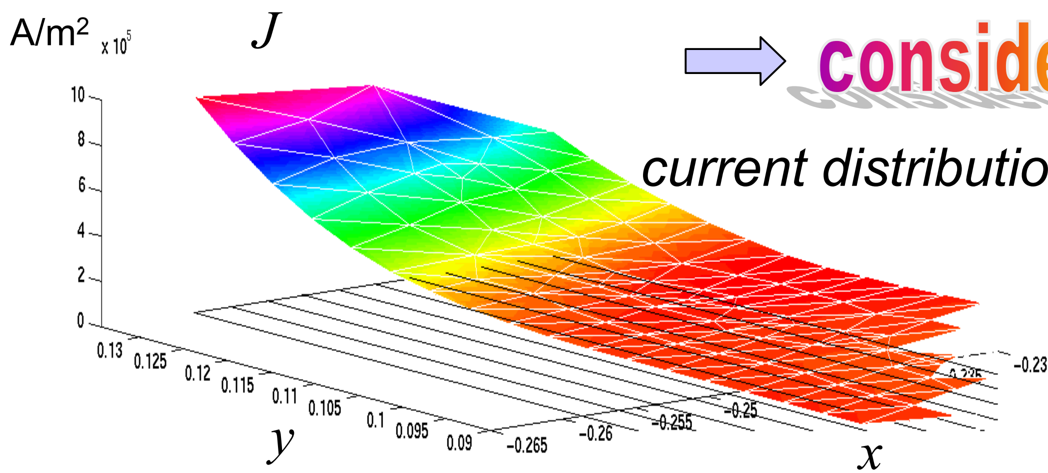


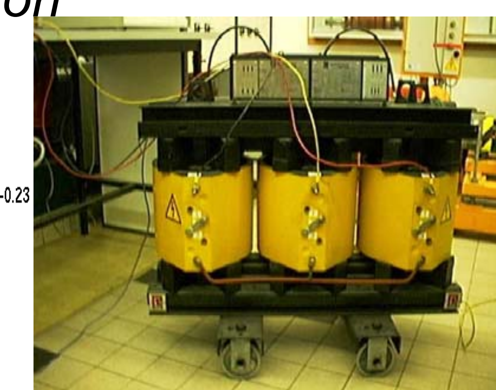




considerable saving

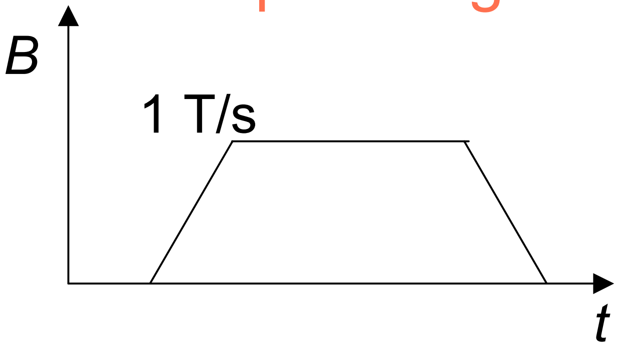
current distribution



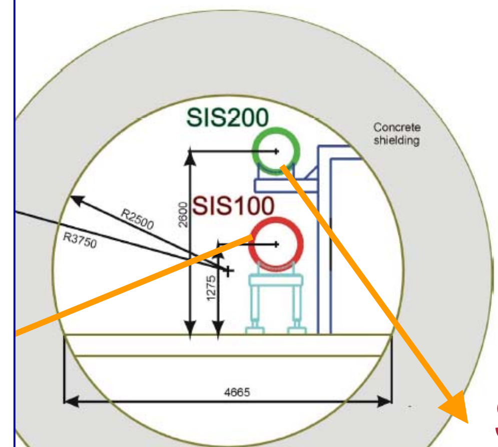


Superconductive magnets (1)

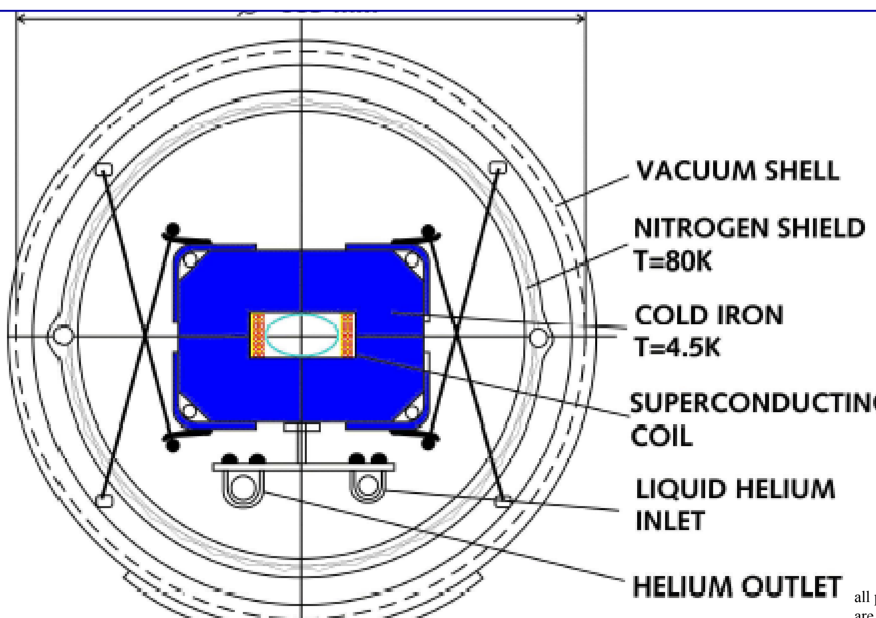
fast ramped magnets

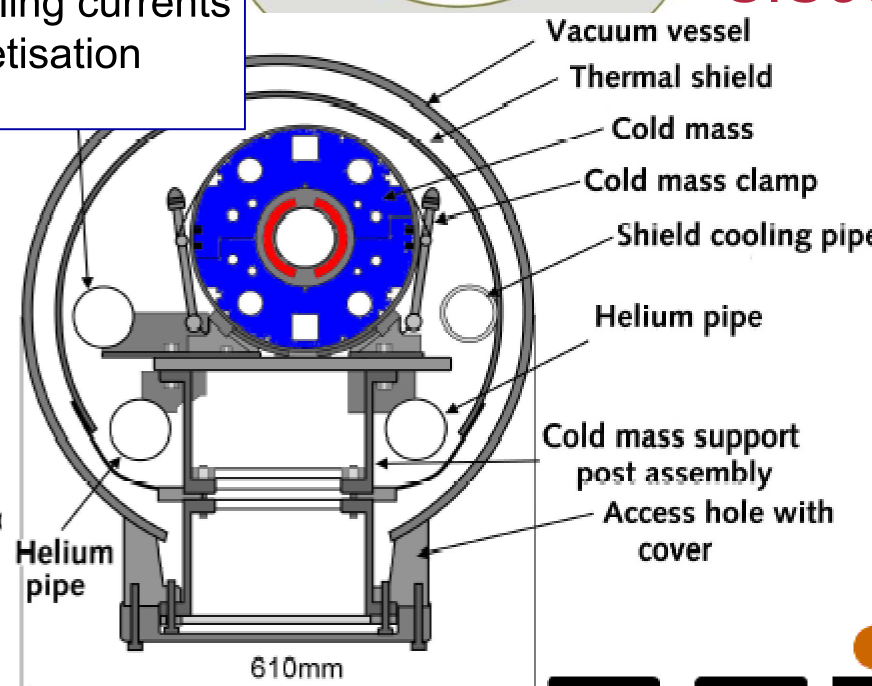


yoke: eddy currents & hysteresis
superconductive filaments: persistent & coupling currents
Rutherford cable: cable eddy currents/magnetisation

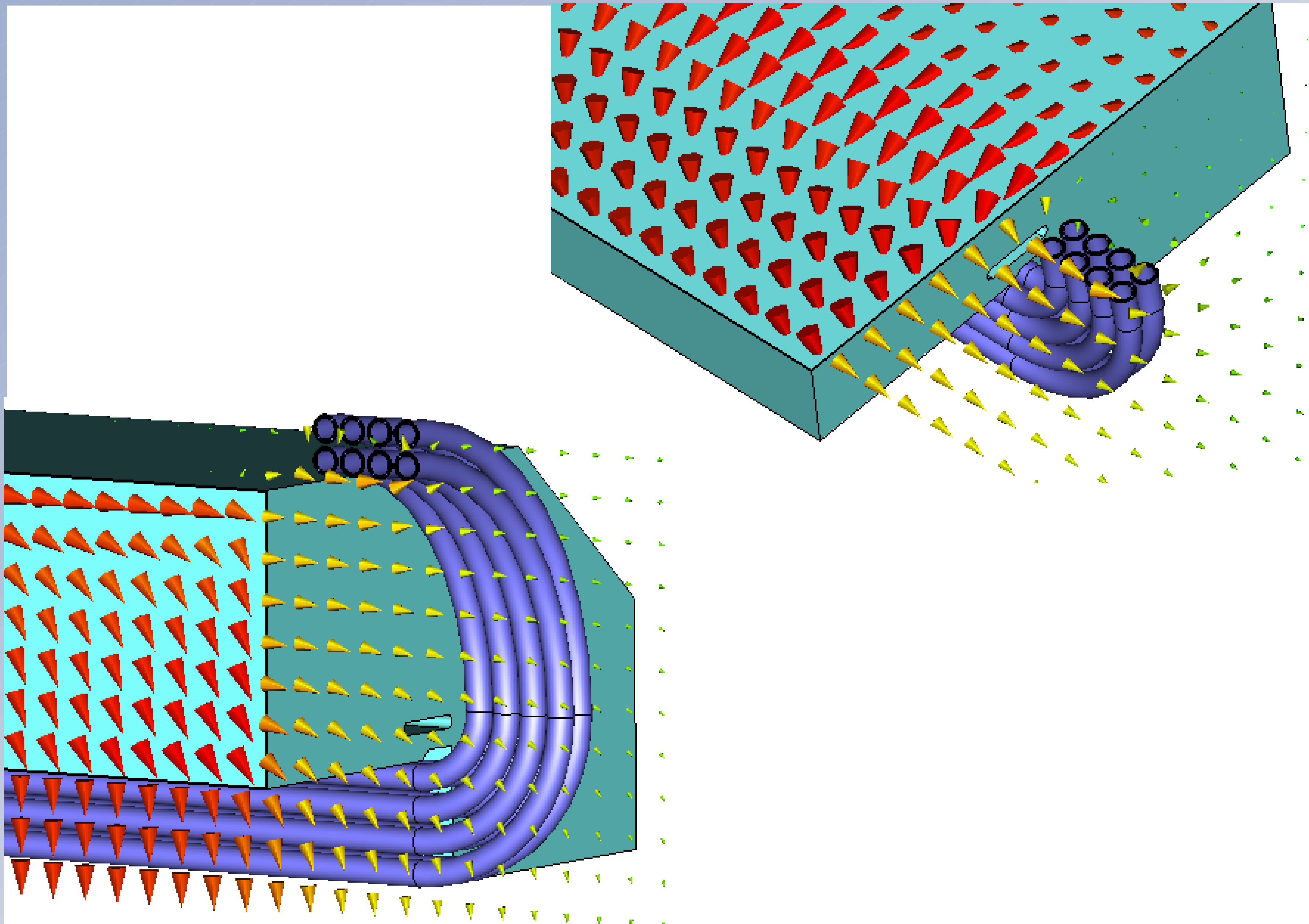


SIS300





Superconductive magnets (2)



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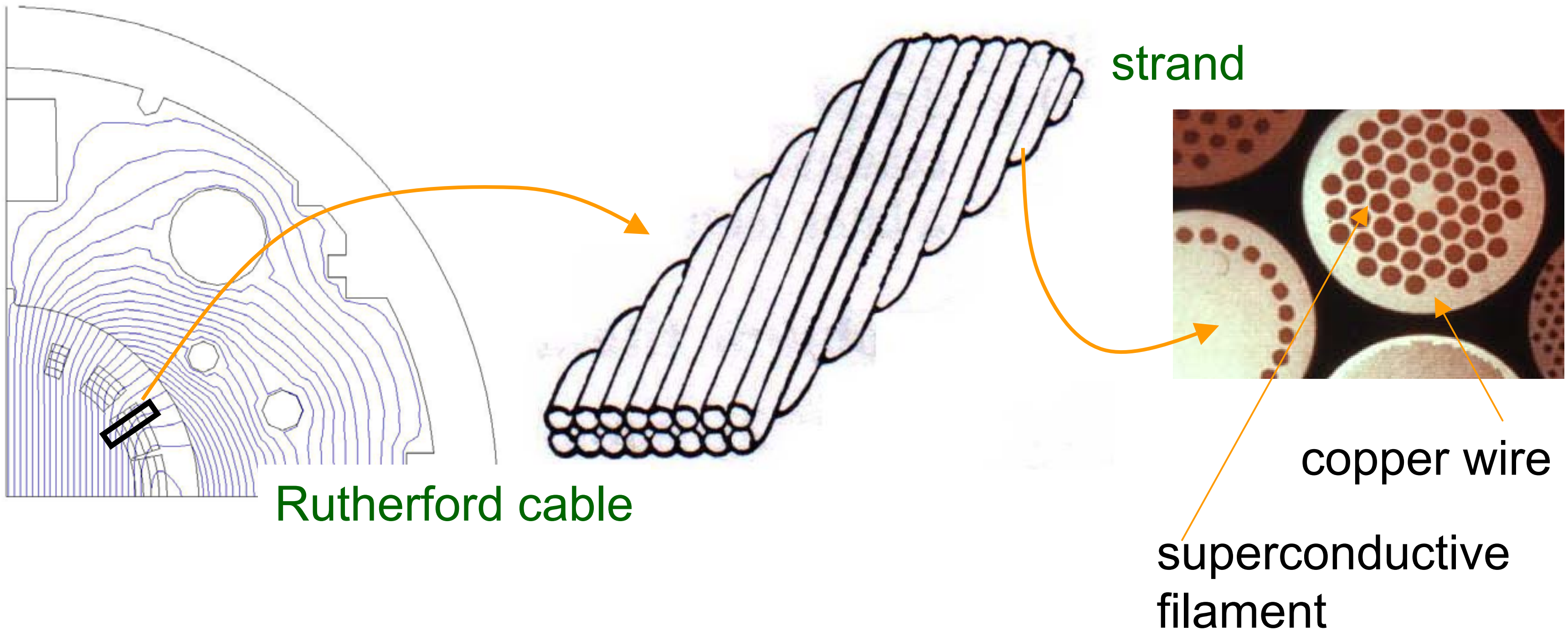
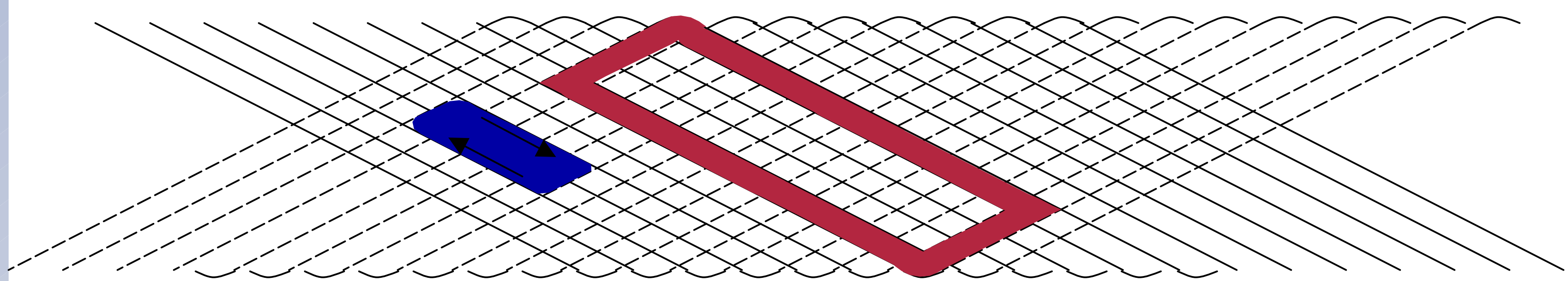
Rutherford Cable (1)





Rutherford Cable (2)

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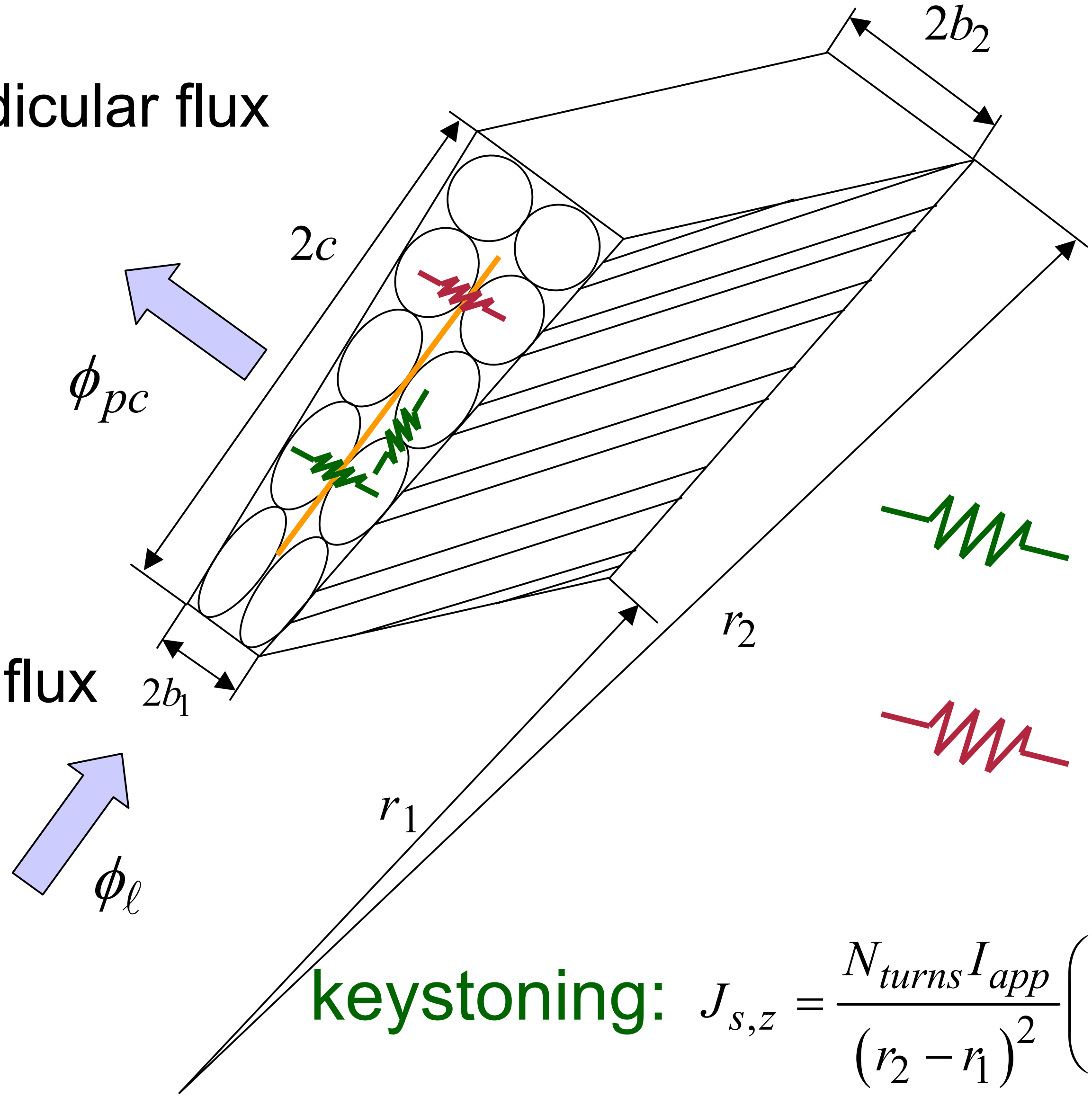
perpendicular flux

$B_p(r,\theta)$

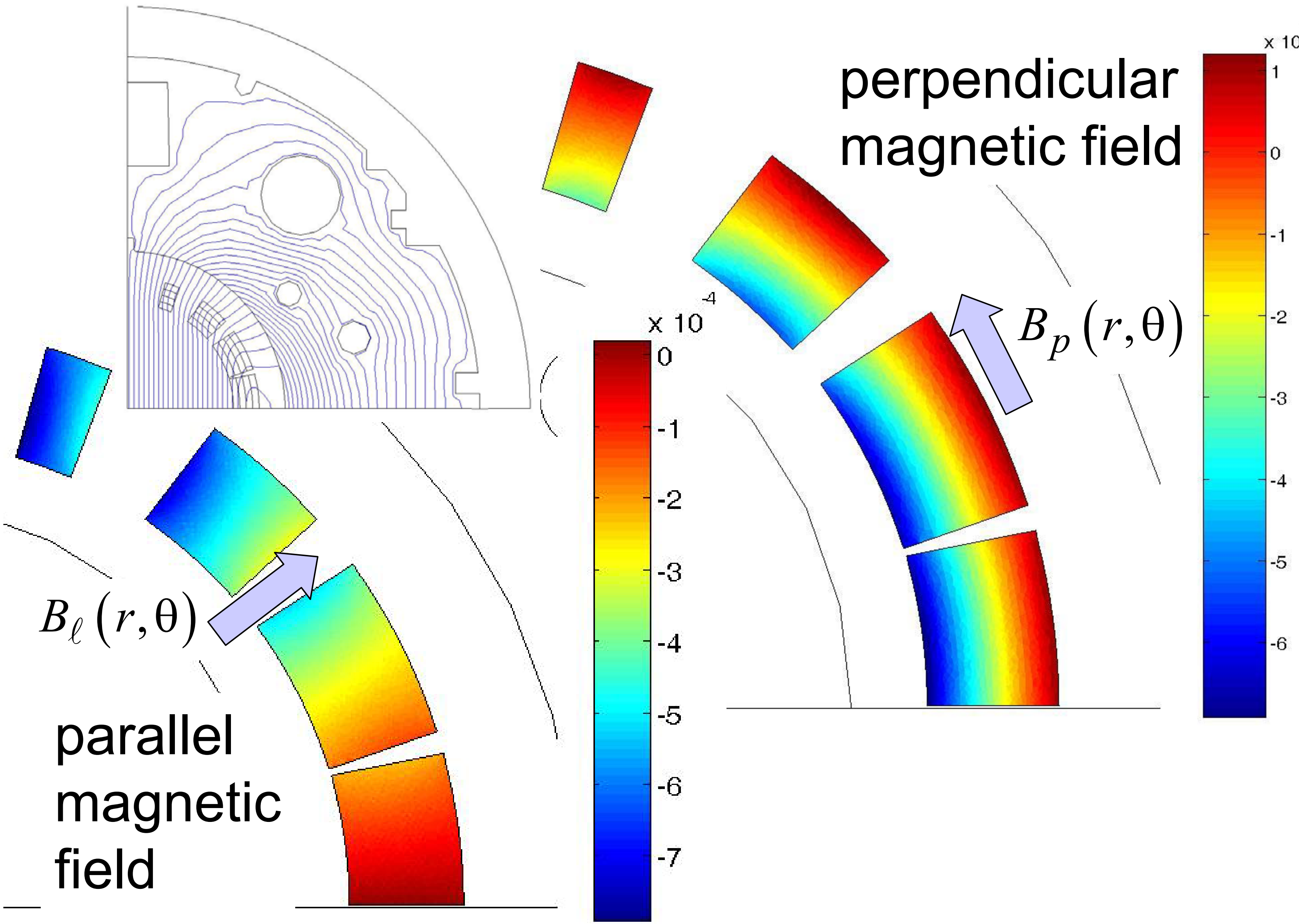
parallel flux

$B_\ell(r,\theta)$



keystoning: $J_{s,z} = \frac{N_{turns} I_{app}}{(r_2 - r_1)^2} \left(\frac{r_2 - r}{2b_1} + \frac{r - r_1}{2b_2} \right)$

Magnetic flux density



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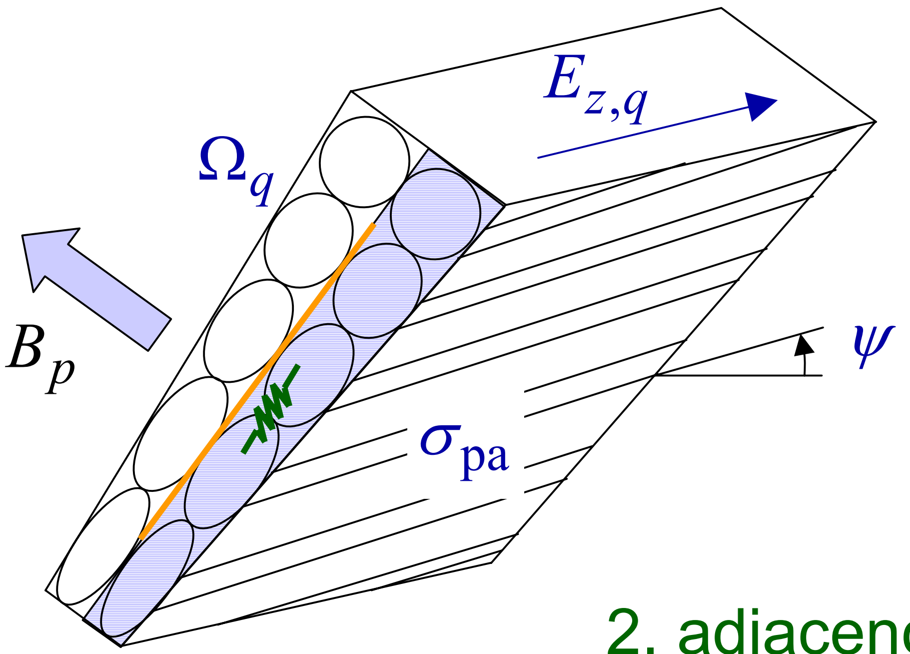
Adjacency eddy current (1)



due to perpendicular magnetic field

1. additional discretisation for unknown electric field $E_{z,q}$:

$$E_z(x,y) = \sum_q E_{z,q} M_q(x,y)$$

2. adjacency eddy current density :

$$J_{\text{pa},z}(r,\theta) = \sigma_{\text{pa}} E_{z,q} - \sigma_{\text{pa}} \frac{\partial A_z}{\partial t}$$

3. netto current through Ω_q = 0

$$I_{z,q} = \int_{\Omega_q} J_{\text{pa},z}(r,\theta)\, d\Omega = 0$$

} *additional constraint !*

Adjacency eddy current (2)

additional load term for magnetic FE model

$$g_{\text{pa}} = M_{\text{pa}} \frac{\partial u}{\partial t} - Z_{\text{pa}} e_{\text{pa}}$$

additional constraint

$$-Z_{\text{pa}}^{\text{T}} \frac{\partial u}{\partial t} + G_{\text{pa}} e_{\text{pa}} = 0$$

degrees of freedom for $E_{z,q}$

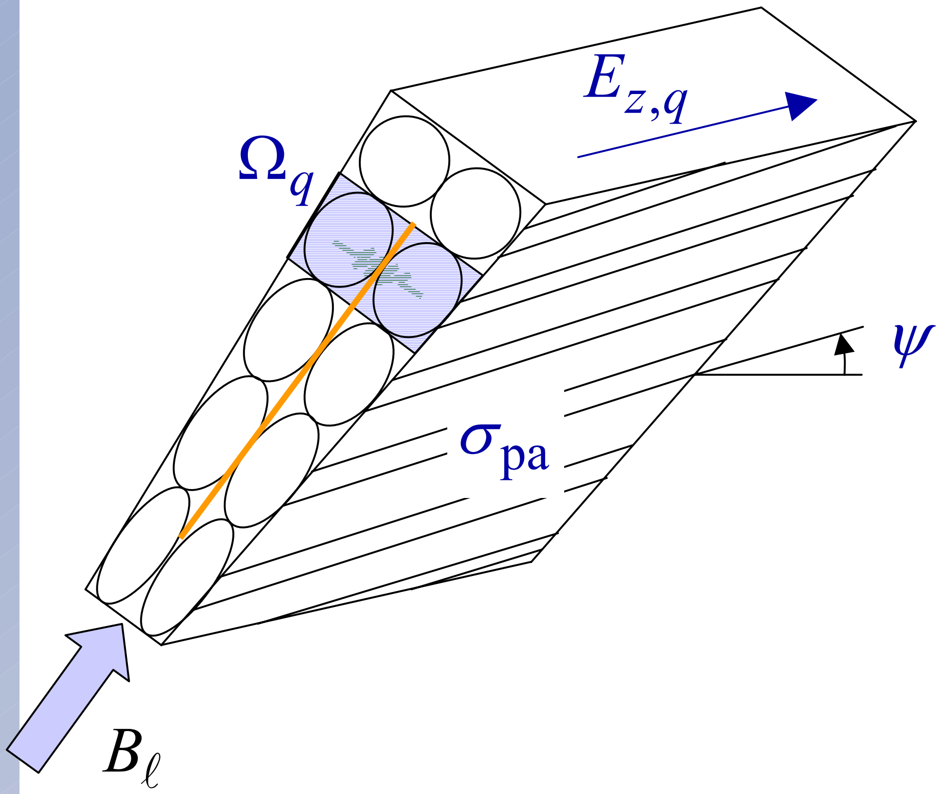
$$\begin{bmatrix} M_{\text{pa}} & 0 \\ Z_{\text{pa}}^{\text{T}} & 0 \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} u \\ e_{\text{pa}} \end{bmatrix} + \begin{bmatrix} K & Z_{\text{pa}} \\ 0 & G_{\text{pa}} \end{bmatrix} \begin{bmatrix} u \\ e_{\text{pa}} \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

$$M_{\text{pa},ij} = \int_{\Omega} \sigma_{\text{pa}} N_i(x,y) N_j(x,y)\, \mathrm{d}\Omega$$

$$Z_{\text{pa},iq} = \int_{\Omega} \sigma_{\text{pa}} N_i(x,y) M_q(x,y)\, \mathrm{d}\Omega$$

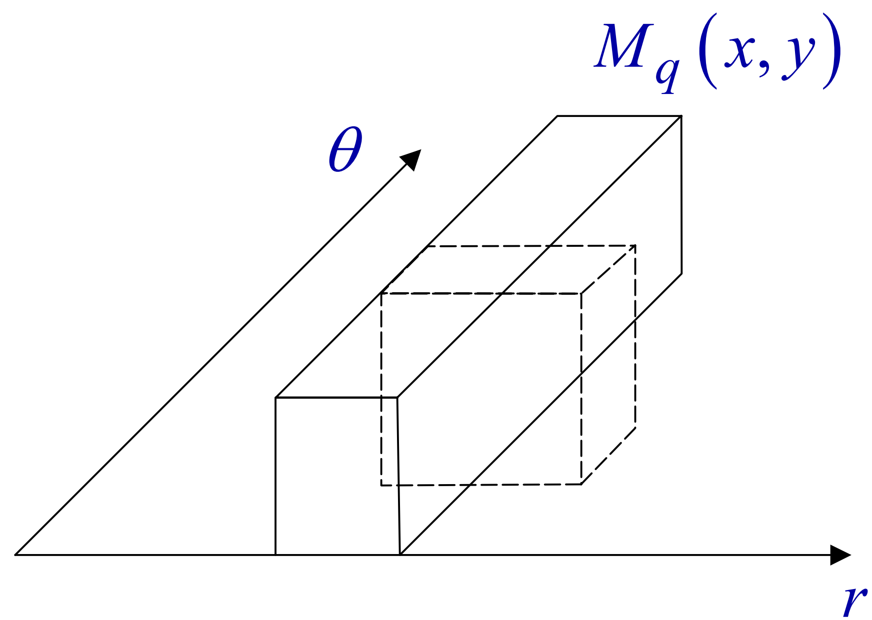
$$G_{\text{pa},pq} = \int_{\Omega} \sigma_{\text{pa}} M_p(x,y) M_q(x,y)\, \mathrm{d}\Omega$$

Adjacency eddy current (3)



due to parallel magnetic field

shape functions $M_q(x, y)$ related (but not necessarily equal) to the zones of current redistribution



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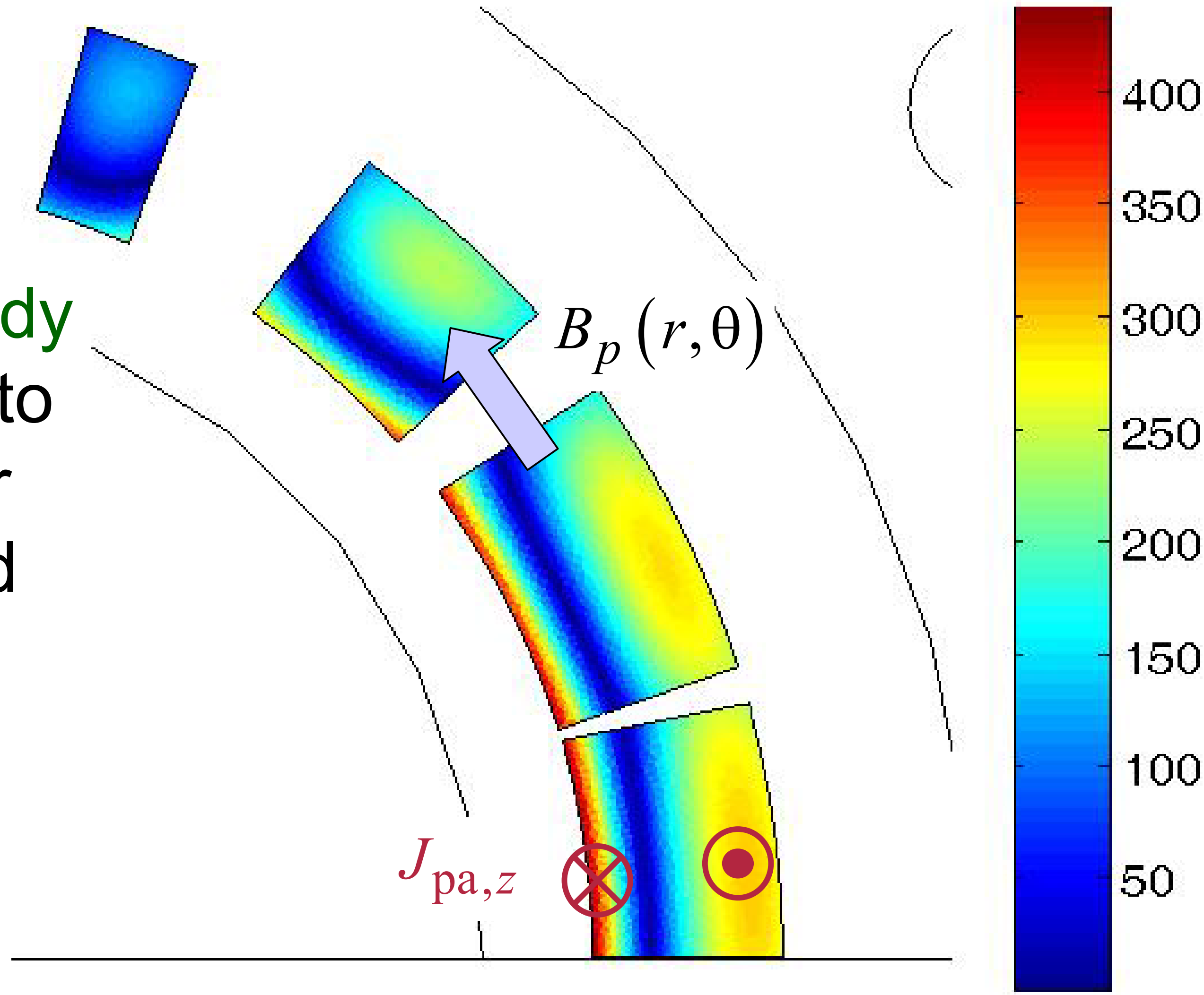
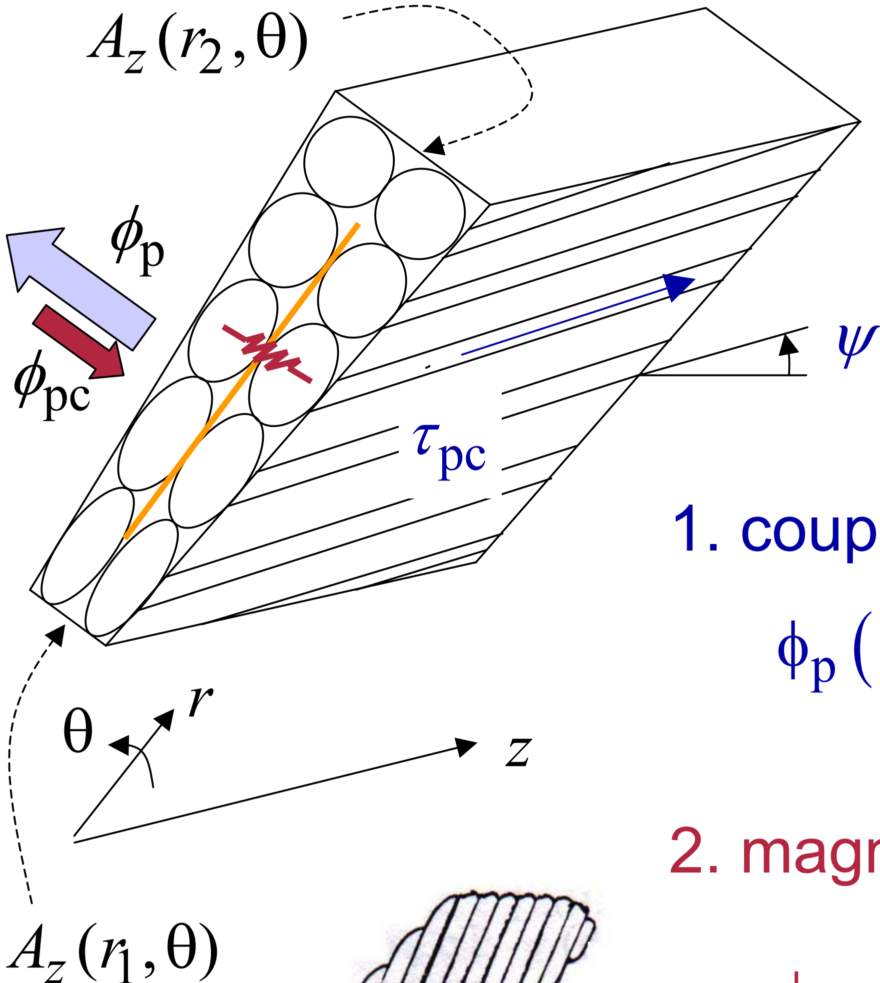
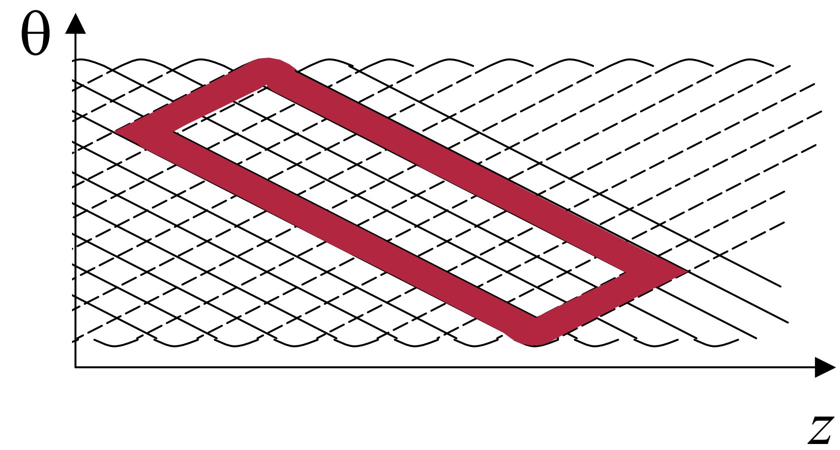
Adjacency eddy current

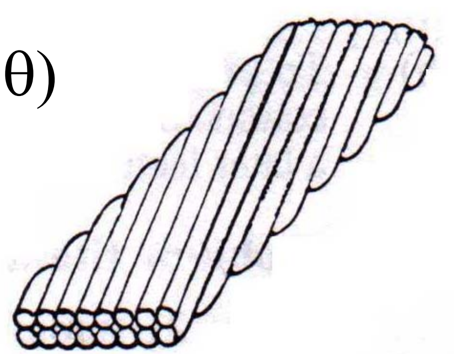
adjacency eddy currents due to perpendicular magnetic field



Cross-over eddy current (1)







1. coupled flux

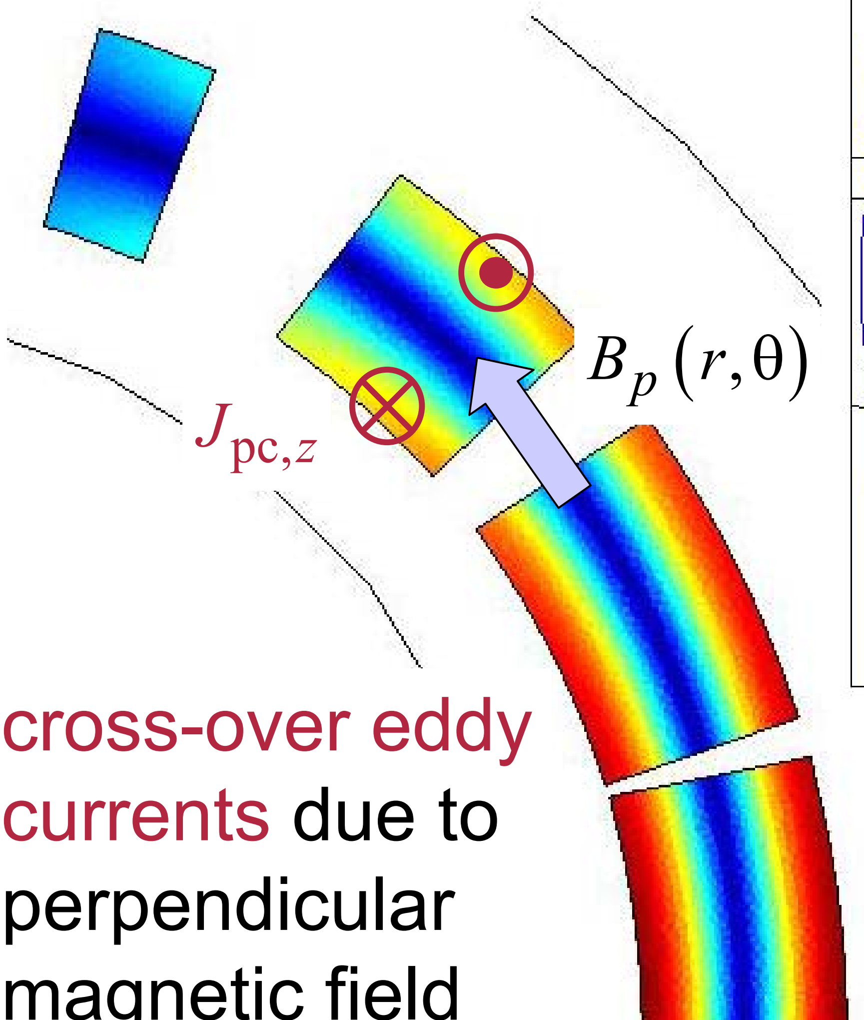
$$\phi_p(\theta) = \ell_z \left(A_z(r_2,\theta) - A_z(r_1,\theta) \right)$$

2. magnetisation

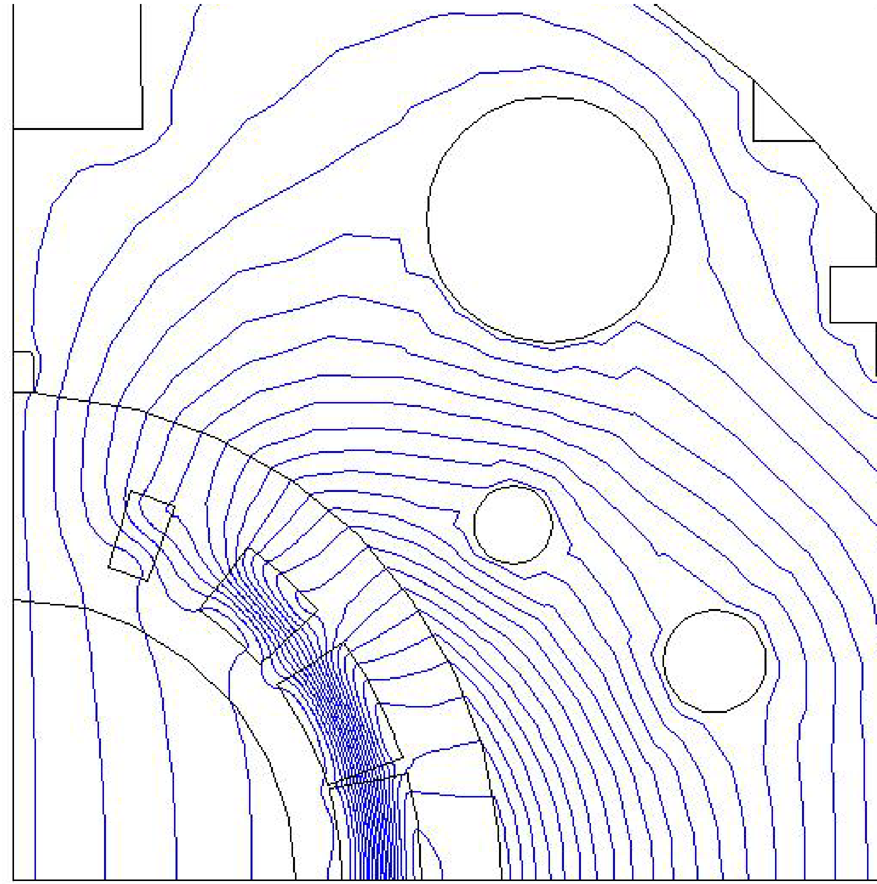
$$\phi_{pc}(\theta) = \tau_{pc} \frac{\partial \phi_p(\theta)}{\partial t}$$

time constant

Cross-over eddy current



cross-over eddy currents due to perpendicular magnetic field



cross-over magnetisation

Conclusions

- hybrid method for circuit analysis
  ⇨ *convenient field-circuit modelling*
- solution of the coupled algebraic systems of equations
- specialised conductor models
  ⇨ *complicated eddy current phenomena*
- applications to
  - electrical machine
  - foil winding transformer
  - superconductive magnets, Rutherford cable