

SOLUTION of (3.7.a):

Let $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. Then, for every $i = 1, \dots, n$:

$$\begin{bmatrix} p_1^i \\ p_2^i \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1^i \\ x_2^i \end{bmatrix} = \begin{bmatrix} a_{11}x_1^i + a_{12}x_2^i \\ a_{21}x_1^i + a_{22}x_2^i \end{bmatrix} \quad (3.0.43)$$

Since all entries of \mathbf{A} are unknown, we want to write the above system as $\mathbf{p}^i = \mathbf{X}^i \mathbf{v}$, where \mathbf{X}^i is given and:

$$\mathbf{v} = \begin{bmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \end{bmatrix} \quad (3.0.44)$$

Rewriting (3.0.43):

$$\begin{bmatrix} p_1^i \\ p_2^i \end{bmatrix} = \begin{bmatrix} x_1^i & x_2^i & 0 & 0 \\ 0 & 0 & x_1^i & x_2^i \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \end{bmatrix} \quad (3.0.45)$$

Vertically stacking all the equations, for $i = 1, \dots, n$:

$$\underbrace{\begin{bmatrix} p_1^1 \\ p_2^1 \\ p_1^2 \\ p_2^2 \\ \vdots \\ p_1^n \\ p_2^n \end{bmatrix}}_{\mathbf{w}} = \underbrace{\begin{bmatrix} x_1^1 & x_2^1 & 0 & 0 \\ 0 & 0 & x_1^1 & x_2^1 \\ x_1^2 & x_2^2 & 0 & 0 \\ 0 & 0 & x_1^2 & x_2^2 \\ \vdots & \vdots & \vdots & \vdots \\ x_1^n & x_2^n & 0 & 0 \\ 0 & 0 & x_1^n & x_2^n \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \end{bmatrix}}_{\mathbf{v}}.$$

The matrix \mathbf{B} is a $2n \times 4$ matrix, \mathbf{w} is a $2n \times 1$ vector and \mathbf{v} a 4×1 vector.