

(18) Verbesserter Euler

$$k_1 = f(t_j, y(t_j))$$

$$k_2 = f(t_j + h/2, y(t_j) + h/2 k_1)$$

$$y_{j+1} = y(t_j) + h \cdot k_2$$

Entwickeln der Verfahrens-Funktion

$$\phi(t_j, y(t_j), h) = f\left(t_j + \frac{h}{2}, y(t_j) + \frac{h}{2} k_1\right)$$

$$= f(t_j, y(t_j)) \leftarrow \phi(t_j, y(t_j), 0)$$

$$\left. \dot{\phi}(t_j, y(t_j), 0) \cdot h \right\} + \frac{\partial f}{\partial t}(t_j, y(t_j)) \cdot \frac{h}{2} + \frac{\partial f}{\partial y}(t_j, y(t_j)) \cdot \frac{h}{2} k_1$$

$$\left. \ddot{\phi}(t_j, y(t_j), 0) \cdot \frac{h^2}{2} \right\} \left\{ \begin{array}{l} + \frac{1}{2} \frac{\partial^2 f}{\partial t^2}(t_j, y(t_j)) \cdot \left(\frac{h}{2}\right)^2 + \frac{\partial^2 f}{\partial t \partial y}(t_j, y(t_j)) \cdot \left(\frac{h}{2}\right)^2 \cdot k_1 \\ + \frac{1}{2} \frac{\partial^2 f}{\partial y^2}(t_j, y(t_j)) \cdot \left(\frac{h}{2} k_1\right)^2 \end{array} \right.$$

+ ...

Durch Vergleich

$$\phi(t_j, y(t_j), 0) = f(t_j, y(t_j)) = \dot{y}(t_j) \quad \checkmark$$

$$2. \quad \dot{\phi}(t_j, y(t_j), 0) = \left( \frac{\partial f}{\partial t}(t_j, y(t_j)) + \frac{\partial f}{\partial y}(t_j, y(t_j)) \cdot f(t_j, y(t_j)) \right)$$

$$= \ddot{y}(t_j) \quad \checkmark$$

$$3. \quad \ddot{\phi}(t_j, y(t_j), 0) = \dots \quad \times$$