High order semi-implicit schemes for evolutionary non linear partial differential equations

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Several systems of evolutionary partial differential equations may contain stiff terms, which require an implicit treatment. Typical examples are hyperbolic systems with stiff hyperbolic or parabolic relaxation and kinetic equations in regimes close to fluid dynamic limit.

In many cases, the stiff terms are clearly identified. For example, in hyperbolic systems with hyperbolic relaxation, the hyperbolic term is usually non stiff, while the relaxation term is stiff. A natural way to treat such systems is to adopt implicit-explicit schemes, in which the relaxation is treated by an implicit scheme, while the hyperbolic part is treated explicitly.

In several cases, however, such a distinction is not so clear. For example, in the case of hyperbolic systems with diffusive relaxation, a standard approach would lead to schemes, which in the stiff limit suffer from classical parabolic CFL restriction. Such systems can be treated by a penalization method, consisting in adding and subtracting the same term, so that the system appears as the limit relaxed system plus a small perturbation.

There are cases, however, in which stiff terns are not just additive, and the penaliztion method is not particularly effective, since the limit system itself is not easily solvable by standard techniques.

For many such systems, we present a new approach, which consists in identifying the stiff (linearly) dependence of the system on the unknown variable. Only this linear dependence will be treated implicitly, while the rest of the system is treated implicitly. This approach generalizes classical IMEX schemes based on additive or partitioned Runke-Kutta methods, and allows the construction of high order linearly implicit schemes, which are much simpler to use than fully implicit ones.

Several examples, including non linear diffusion and Fokker-Planck equation will be presented.