Recent advances in Residual distribution schemes Applications to compressible fluid problems

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Abstract

The residual distribution schemes (RD), or fluctuation splitting schemes, have been introduced by P.L. Roe in several papers, among which his 1981 JCP paper, and then in collaboration with VKI. Considering a transport equation, the idea is to represent the solution by its nodal values, the mesh being described in a non structured manner, and to evolve it using the most possible compact stencil with the highest possible accuracy that can be reached with the scheme stencil without sacrificing the non oscilalatory properties of the exact solutions. For example, using a triangular mesh, one can get second order accuracy only using the nearest neighbors of any given vertex. The discrete scheme formulation shares many similarities with stabilised finite element, though they are very different in that the schemes can be proved to be L^{∞} stable. They also share many similarities with standard finite volume schemes since any finite volume scheme can be reformulated as a RD scheme, at least in their first order version. Of course the classical MUSCL schemes violate the spencil compactness requirements of RD schemes, hence at least from a pure computational point of view, the second order version of the RD schemes are superior to standard second order FV scheme.

Since the early versions of RD scheme, many improvement have been realised. First, it is possible to construct very high order schemes on hybrid meshes in two and three dimensions. Efficient unsteady schemes exists now. Recently, it has been possible to design very high order non oscillatory schemes for advection diffusion equations.

In this lecture, we will describe the design principle of RD scheme and show how very high order can be reached. Efficient unsteady version will be presented as well as the very high order approximation of viscous problems. I will also present some open questions.