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# Multiple traces boundary integral formulation for Helmholtz transmission problems

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# Erratum for "*Multiple Traces formulation for Helmholtz transmission problems*" *Adv. Appl. Math.*, 37(1):39–91, 2012

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**Abstract** A flawed proof led to a wrong power of a logarithmic factor in the estimate of Lemma 11 of [R. Hiptmair and C. Jerez-Hanckes. Multiple traces boundary integral formulation for Helmholtz transmission problems. *Adv. Appl. Math.*, 37(1):39–91, 2012]. This error does not compromise the main results of this paper. A corrected version of that Lemma 11 is provided here.

## 1 Erroneous Estimate

As an auxiliary result required for the proof of convergence of the MTF, we introduced Lemma 11:

**Lemma 1 (Spurious Lemma 11 in [2])** *Given a quasi-uniform family of meshes* $\{\Gamma^h\}_{h>0}$ *for a polygon* $\Gamma$*, the following inverse estimate*

$$\|\varphi_h\|_{\widetilde{H}^{-1/2}(\Gamma)} \;\leq\; C_I(1+|\log h|)\,\|\varphi_h\|_{H^{-1/2}(\Gamma)} \tag{1}$$

*holds true for all piecewise constants* $\varphi_h \in \mathcal{S}^{-1,0}(\Gamma^h)$ *and with* $C_I$ *independent of the meshwidth* $h > 0$.

## 2 Corrected Estimate

The statement of Lemma 1 needs to be altered as the presented proof is flawed.

The new version of Lemma 1 is as follows:

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**Lemma 2 (Corrected version of Lemma 11 in [2])** *Given a quasi-uniform family of meshes $\{\Gamma^h\}_{h>0}$ for a polygon $\Gamma$, the following inverse estimate*

$$\|\varphi_h\|_{\widetilde{H}^{-1/2}(\Gamma)} \leq C_I(1+|\log h|)^{3/2}\,\|\varphi_h\|_{H^{-1/2}(\Gamma)} \tag{2}$$

*holds true for all piecewise constants $\varphi_h \in \mathcal{S}^{-1,0}(\Gamma^h)$ and with $C_I$ independent of the meshwidth $h > 0$.*

Compared to the original version there is an extra $\frac{1}{2}$ exponent in the logarithmic growth on $h$.

*Proof* Again we rely on the *dual mesh* $\hat{\Gamma}^h$ of the mesh $\Gamma^h$ of the curve $\Gamma$, see [2, Beginning of Sect. 4.1.1]. In detail, let $\mu : [0,1] \mapsto \Gamma$ denote a parametrization of the curve $\Gamma$, and denote by $\{x_i\}_{i=0}^{M}$ the $M+1$ nodes of the given partition $\Gamma^h$ that satisfy $x_i = \mu(i/M)$. The nodes $\xi_i$, $i = 0,\ldots,M$, of the dual mesh $\hat{\Gamma}^h$ are defined according to

$$\xi_0 = x_0\;,\quad \xi_{M+1} = x_M\;,\quad \xi_i = \mu\left(\tfrac{1}{M}\left(i-\tfrac{1}{2}\right)\right)\;,\quad i = 1,\ldots,M\;. \tag{3}$$

We write $\mathcal{S}^{0,1}(\hat{\Gamma}^h)$ for the space of piecewise linear functions on the dual mesh $\hat{\Gamma}^h$, and $\mathcal{S}_0^{0,1}(\hat{\Gamma}^h)$ for its subspace of functions vanishing in the endpoints $x_0 = \xi_0$ and $x_M = \xi_{M+1}$. The customary "tent function basis" of $\mathcal{S}^{0,1}(\hat{\Gamma}^h)$ comprises the functions $b_h^i$, $i = 0,\ldots,M+1$.

Let us introduce the space $\tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}_h) \subset \mathcal{S}^{0,1}(\hat{\Gamma}^h)$ defined as

$$\tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}_h) := \left\{ v_h \in \mathcal{S}^{0,1}(\hat{\Gamma}_h) : v_h|_{[\xi_0,\xi_2]}, v_h|_{[\xi_{M-1},\xi_{M+1}]} \in \mathbb{P}_1 \right\} \tag{4}$$

In words, $\tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}_h)$ is the space of p.w-linear functions over the dual mesh with restrictions at the endpoints, thus leaving $M$ degrees of freedom which is equal to the number of dofs for $\mathcal{S}^{-1,0}(\Gamma^h)$. As basis of $\tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}_h)$ is given by $\{\tilde{b}_h^1, b_h^2, \ldots, b_h^{M-1}, \tilde{b}_h^M\}$, where $\tilde{b}_h^1$ and $\tilde{b}_h^M$ are special linear combinations of $b_h^0, b_h^1$ and $b_h^M, b_h^{M+1}$, respectively, see Figure 1.

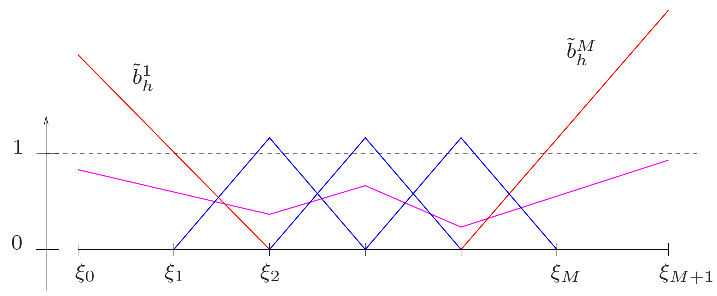


**Fig. 1** A function in $\tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}_h)$ (magenta) and nodal basis functions (blue and red).

The rationale for choosing $\tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}_h)$ was to make the stability property

$$\sup_{\psi_h \in \mathcal{S}^{-1,0}(\Gamma^h)} \frac{|\langle \psi_h\,,\, v_h \rangle|}{\|\psi_h\|_{L^2(\Gamma)}} \geq C_{\mathrm{ST}}\,\|v_h\|_{L^2(\Gamma)} \quad \forall\, v_h \in \tilde{\mathcal{S}}^{0,1}(\Gamma), \quad \forall h > 0, \tag{5}$$

hold. This can be proved by computing the Galerkin matrix of $\langle \psi_h \,,\, v_h \rangle$ with respect to a suitably scaled local basis of $\mathcal{S}^{-1,0}(\Gamma^h)$ and $\tilde{\mathcal{S}}^{0,1}(\Gamma)$, respectively. This matrix turns out to be strictly diagonally dominant.

Owing to (5) we can define the linear projector $\mathsf{J}_h^1 : L^2(\Gamma) \to \tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}_h)$ according to

$$\left\langle \varphi_h \,,\, \mathsf{J}_h^1 \, u \right\rangle \;=\; \langle \varphi_h \,,\, u \rangle \,, \quad \forall\, \varphi_h \in \mathcal{S}^{-1,0}(\Gamma^h) \,, \tag{6}$$

and conclude from (5)

$$\left\| \mathsf{J}_h^1 \, u \right\|_{L^2(\Gamma)} \;\leq\; C_{\mathrm{ST}}^{-1} \, \|u\|_{L^2(\Gamma)} \,, \quad \forall\, h > 0 \,. \tag{7}$$

Also, instead of the projection operator $\mathsf{P}_h$, we use the nodal interpolation operator $\mathsf{I}_h : H^1(\Gamma) \to \tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}_h)$. Thanks to the continuous embedding of $\mathcal{C}^0(\Gamma)$ in $H^1(\Gamma)$ we have

$$\|\mathsf{I}_h f\|_{H^1(\Gamma)} \leq C_{\mathsf{I}} \, \|f\|_{H^1(\Gamma)} \,, \quad h > 0, \tag{8}$$

and one can prove

$$\|f - \mathsf{I}_h f\|_{L^2(\Gamma)} \leq C_2 h \, \|f\|_{H^1(\Gamma)} \,, \quad h > 0. \tag{9}$$

Note that for dimensions higher than one these results are no longer true.

Consequently, one can derive the following estimate:

$$\begin{aligned}
\left\| \mathsf{J}_h^1 \, u \right\|_{H^1(\Gamma)} &\leq \left\| \mathsf{J}_h^1 \, u - \mathsf{I}_h u \right\|_{H^1(\Gamma)} + \|\mathsf{I}_h u\|_{H^1(\Gamma)} \\
\text{inverse inequality and (8)} \qquad &\leq C_{\mathrm{inv}} h^{-1} \left\| \mathsf{J}_h^1 \, u - \mathsf{I}_h u \right\|_{L^2(\Gamma)} + C_{\mathsf{I}} \, \|u\|_{H^1(\Gamma)} \\
&= C_{\mathrm{inv}} h^{-1} \left\| \mathsf{J}_h^1 (\mathsf{Id} - \mathsf{I}_h) u \right\|_{L^2(\Gamma)} + C_{\mathsf{I}} \, \|u\|_{H^1(\Gamma)} \\
\text{(7)} \qquad &\leq C_{\mathrm{inv}} h^{-1} C_{\mathrm{ST}}^{-1} \, \|(\mathsf{Id} - \mathsf{I}_h) u\|_{L^2(\Gamma)} + C_{\mathsf{I}} \, \|u\|_{H^1(\Gamma)} \\
\text{(9)} \qquad &\leq C_{\mathrm{inv}} h^{-1} C_{\mathrm{ST}}^{-1} C_2 h \, \|u\|_{H^1(\Gamma)} + C_{\mathsf{I}} \, \|u\|_{H^1(\Gamma)} \\
&\leq \widetilde{C}_1 \, \|u\|_{H^1(\Gamma)}
\end{aligned} \tag{10}$$

with $\widetilde{C}_1 := C_{\mathrm{inv}} C_{\mathrm{ST}}^{-1} C_2 + C_{\mathsf{I}}$.
Interpolation between $L^2(\Gamma)$ and $H^1(\Gamma)$ using (7) and (10) yields

$$\left\| \mathsf{J}_h^1 \, u \right\|_{H^{1/2}(\Gamma)} \;\leq\; C_3 \, \|u\|_{H^{1/2}(\Gamma)} \quad \forall\, u \in H^{1/2}(\Gamma), \quad \forall h > 0. \tag{11}$$

where $C_3 := C_{\mathrm{ST}}^{-1/2} \widetilde{C}_1^{1/2}$.

Let us introduce a chop-off operator $\mathring{\mathsf{I}}_h$: $\mathcal{S}^{0,1}(\hat{\Gamma}_h) \to \mathcal{S}_0^{0,1}(\hat{\Gamma}_h)$, which sets $v_h(\xi_0) = v_h(\xi_{M+1}) = 0$. This is equivalent to dropping the contribution of basis functions of $\mathcal{S}^{0,1}(\hat{\Gamma}^h)$ associated with the endpoints. The impact of chopping off can be controlled thanks to the following lemma.

**Lemma 3** *For all $v_h$ in $\mathcal{S}^{0,1}(\Gamma_h)$ we have*

$$\left\| \mathring{\mathsf{I}}_h \, v_h \right\|_{\widetilde{H}^{1/2}(\Gamma)} \;\leq\; C_4 (1 + |\log h|)^{3/2} \, \|v_h\|_{H^{1/2}(\Gamma)} \,. \tag{12}$$

To prove this, proceed as follows. Write

$$v_h = \mathring{\mathsf{I}}_h\, v_h + v_h(\xi_0) b_h^0 + v_h(\xi_{M+1}) b_h^{M+1} \tag{13}$$

Based on the inverse inequality [1, Lemma 1]

$$\|v_h\|_{L^\infty(\Gamma)} \leq C_{\mathrm{D}}(1 + |\log h|)^{1/2} \|v_h\|_{H^{1/2}(\Gamma)} \qquad \forall\, v_h \in \mathcal{S}^{0,1}(\hat{\Gamma}^h), \tag{14}$$

we get

$$|v_h(\xi_0)|\,, |v_h(\xi_{M+1})| \leq C(1 + |\log h|)^{1/2} \|v_h\|_{H^{1/2}(\Gamma)} \tag{15}$$

The above together with the knowledge that

$$\left\|b_h^0\right\|_{H^{1/2}(\Gamma)}, \left\|b_h^{M+1}\right\|_{H^{1/2}(\Gamma)} \leq C$$

gives

$$\left\|v_h - \mathring{\mathsf{I}}_h\, v_h\right\|_{H^{1/2}(\Gamma)} \leq \tilde{C}(1 + |\log h|)^{1/2} \|v_h\|_{H^{1/2}(\Gamma)} \tag{16}$$

from where

$$\left\|\mathring{\mathsf{I}}_h\, v_h\right\|_{H^{1/2}(\Gamma)} \leq \hat{C}(1 + |\log h|)^{1/2} \|v_h\|_{H^{1/2}(\Gamma)}\,. \tag{17}$$

Finally, we use a result by McLean and Steinbach [4], [2, Eq. (166)]

$$\|u_h\|_{\widetilde{H}^{1/2}(\Gamma)} \leq C_{\mathrm{MS}}(1 + |\log h|) \|u_h\|_{H^{1/2}(\Gamma)} \quad , \quad \forall u_h \in \mathcal{S}_0^{0,1}(\hat{\Gamma}^h)\,. \tag{18}$$

Thus, we arrive at the estimate

$$\left\|\mathring{\mathsf{I}}_h\, v_h\right\|_{\widetilde{H}^{1/2}(\Gamma)} \leq C_{\mathrm{MS}}\hat{C}(1 + |\log h|)^{3/2} \|v_h\|_{H^{1/2}(\Gamma)} \tag{19}$$

since $\mathring{\mathsf{I}}_h\, v_h \in \mathcal{S}_0^{0,1}(\hat{\Gamma}^h) \subset \widetilde{H}^{1/2}(\Gamma)$.

The estimates (17) and (19) permit us to finish the proof of Lemma 2 as follows. Let $\varphi_h \in \mathcal{S}^{-1,0}(\Gamma^h)$, then by definition of the dual norm:

$$\begin{aligned}
\|\varphi_h\|_{\widetilde{H}^{-1/2}(\Gamma)} &= \sup_{0\neq v \in H^{1/2}(\Gamma)} \frac{|\langle \varphi_h\,,\, v\rangle|}{\|v\|_{H^{1/2}(\Gamma)}} \\
\text{(11), (6)} \quad &\leq C_3 \sup_{0\neq v \in H^{1/2}(\Gamma)} \frac{\left|\left\langle \varphi_h\,,\, \mathsf{J}_h^1\, v\right\rangle\right|}{\left\|\mathsf{J}_h^1\, v\right\|_{H^{1/2}(\Gamma)}} \\
&\leq C_3 \sup_{0\neq v_h \in \tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}^h)} \frac{\left|\left\langle \varphi_h\,,\, \mathring{\mathsf{I}}_h\, v_h\right\rangle\right|}{\|v_h\|_{H^{1/2}(\Gamma)}} \\
&\quad + C_3 \sup_{0\neq v \in \tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}^h)} \frac{\left|\left\langle \varphi_h\,,\, (\mathsf{Id} - \mathring{\mathsf{I}}_h) v_h\right\rangle\right|}{\|v_h\|_{H^{1/2}(\Gamma)}}
\end{aligned} \tag{20}$$

Due to (12), the first term sup on the right-hand side satisfies

$$\begin{aligned}
\sup_{0\neq v_h\in\tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}^h)} \frac{\left|\left\langle \varphi_h\,,\,\mathring{\mathsf{I}}_h\,v_h\right\rangle\right|}{\|v_h\|_{H^{1/2}(\Gamma)}} &\le C_4(1+|\log h|)^{3/2} \sup_{0\neq v_h\in\tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}^h)} \frac{\left|\left\langle \varphi_h\,,\,\mathring{\mathsf{I}}_h\,v_h\right\rangle\right|}{\left\|\mathring{\mathsf{I}}_h\,v_h\right\|_{\widetilde{H}^{1/2}(\Gamma)}}\\
&= C_4(1+|\log h|)^{3/2} \sup_{0\neq w_h\in\mathcal{S}_0^{0,1}(\hat{\Gamma}^h)} \frac{|\langle\varphi_h\,,\,w_h\rangle|}{\|w_h\|_{\widetilde{H}^{1/2}(\Gamma)}}\\
&\le C_4(1+|\log h|)^{3/2} \sup_{0\neq w\in\widetilde{H}^{1/2}(\Gamma)} \frac{|\langle\varphi_h\,,\,w\rangle|}{\|w\|_{\widetilde{H}^{1/2}(\Gamma)}}\\
&= C_4(1+|\log h|)^{3/2}\,\|\varphi_h\|_{H^{-1/2}(\Gamma)}\,.
\end{aligned} \tag{21}$$

The second term is tackled by a simple Cauchy-Schwarz inequality

$$\sup_{0\neq v_h\in\tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}^h)} \frac{\left|\left\langle \varphi_h\,,\,(\mathsf{Id}-\mathring{\mathsf{I}}_h)v_h\right\rangle\right|}{\|v_h\|_{H^{1/2}(\Gamma)}} \le \sup_{0\neq v_h\in\tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}^h)} \frac{\|\varphi_h\|_{L^2(\Gamma)}\left\|(\mathsf{Id}-\mathring{\mathsf{I}}_h)v_h\right\|_{L^2(\Gamma)}}{\|v_h\|_{H^{1/2}(\Gamma)}} \tag{22}$$

Using a scaling argument and (16), one can show that

$$\begin{aligned}
\left\|(\mathsf{Id}-\mathring{\mathsf{I}}_h)v_h\right\|_{L^2(\Gamma)} &\le C_5h^{1/2}\left\|(\mathsf{Id}-\mathring{\mathsf{I}}_h)v_h\right\|_{H^{1/2}(\Gamma)}\\
&\le C_5h^{1/2}\tilde{C}(1+|\log h|)^{1/2}\,\|v_h\|_{H^{1/2}(\Gamma)}\,.
\end{aligned} \tag{23}$$

On the other hand, following the reasoning in the proof of [3, Lemma 4.5] we obtain the inverse inequality

$$\|\varphi_h\|_{L^2(\Gamma)} \le C_6h^{-1/2}\,\|\varphi_h\|_{H^{-1/2}(\Gamma)} \tag{24}$$

so that combining (23) and (24) for (22) boils down to

$$\begin{aligned}
\sup_{0\neq v_h\in\tilde{\mathcal{S}}^{0,1}(\hat{\Gamma}^h)} \frac{\left|\left\langle \varphi_h\,,\,(\mathsf{Id}-\mathring{\mathsf{I}}_h)v_h\right\rangle\right|}{\|v_h\|_{H^{1/2}(\Gamma)}} &\le C_6h^{-1/2}C_5h^{1/2}\tilde{C}(1+|\log h|)^{1/2}\,\|\varphi_h\|_{H^{-1/2}(\Gamma)}\\
&\le C_7(1+|\log h|)^{1/2}\,\|\varphi_h\|_{H^{-1/2}(\Gamma)}\,,
\end{aligned} \tag{25}$$

where now $C_7 := C_6C_5\tilde{C}$. To conclude, using (21) and (25) we get

$$\begin{aligned}
\|\varphi_h\|_{\widetilde{H}^{-1/2}(\Gamma)} &\le C_3C_4(1+|\log h|)^{3/2}\,\|\varphi_h\|_{H^{-1/2}(\Gamma)}\\
&\quad + C_3C_7(1+|\log h|)^{1/2}\,\|\varphi_h\|_{H^{-1/2}(\Gamma)}\\
&\le C_I(1+|\log h|)^{3/2}\,\|\varphi_h\|_{H^{-1/2}(\Gamma)}\,,
\end{aligned} \tag{26}$$

where the constant $C_I$ is defined adequately.

## 3 Implications

The power of the logarithmic factor $(1 + |\log h|)$ in the estimate of Corollary 3 of [2] has to be raised to $\frac{3}{2}$ as well. In turns, this corollary is used in the proof of Theorem 13 of [2]. However, the inverse estimate of Corollary 3 only enters in Equation (182). There it is crucial that $h^{1/2-\epsilon}(1+|\log h|)^{\beta} \to 0$ for $h \to 0$, which holds for any $\beta \in \mathbb{R}$. Hence, the slightly changed power of the logarithmic factor in Lemma 11 of [2] does not make any difference.

## References

1. M. Dryja. A method of domain decomposition for three-dimensional finite element elliptic problems. In *First International Symposium on Domain Decomposition Methods for Partial Differential Equations (Paris, 1987)*, pages 43–61. SIAM, Philadelphia, PA, 1988.
2. R. Hiptmair and C. Jerez-Hanckes. Multiple traces boundary integral formulation for Helmholtz transmission problems. *Adv. Appl. Math.*, 37(1):39–91, 2012.
3. R. Hiptmair and S.-P. Mao. Stable multilevel splittings of boundary edge element spaces. *BIT*, 2012. published electronically, DOI: 10.1007/s10543-012-0369-1.
4. W. McLean and O. Steinbach. Boundary element preconditioners for a hypersingular integral equations on an interval. *Adv. Comp. Math.*, 11(4):271–286, 1999.

# Multiple Traces Boundary Integral Formulation for Helmholtz Transmission Problems

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**Abstract**

We present a novel boundary integral formulation of the Helmholtz transmission problem for bounded composite scatterers (that is, piecewise constant material parameters in "subdomains") that directly lends itself to operator preconditioning via Calderón projectors. The method relies on local traces on subdomains and weak enforcement of transmission conditions. The variational formulation is set in Cartesian products of standard Dirichlet and special Neumann trace spaces for which restriction and extension by zero are well defined. In particular, the Neumann trace spaces over each subdomain boundary are built as piecewise $\widetilde{H}^{-1/2}$-distributions over each associated interface. Through the use of interior Calderón projectors, the problem is cast in variational Galerkin form with an operator matrix whose diagonal is composed of block boundary integral operators associated with the subdomains. We show existence and uniqueness of solutions based on an extension of Lions' projection lemma for non-closed subspaces. We also investigate asymptotic quasi-optimality of conforming boundary element Galerkin discretization. Numerical experiments in 2-D confirm the efficacy of the method and a performance matching that of another widely used boundary element discretization. They also demonstrate its amenability to different types of preconditioning.

**Keywords**: Acoustic scattering, boundary integral equations, trace spaces, Calderón projectors, boundary elements.

## 1 Introduction

We focus on the time-harmonic scattering of acoustic waves by a bounded penetrable object $\Omega \in \mathbb{R}^d$, $d = 2, 3$, composed of several subdomains $\Omega_i$, $i = 1, \dots, N$. Specifically, in each subdomain $\Omega_i$ the solution $u$ satisfies a Helmholtz equation with wave-number $\kappa_i$. This is generally referred to as *Helmholtz Transmission Problem* (HTP) and is a relevant model for applications ranging from ultrasound and electromagnetic biomedical imaging [53, 1] to blood cell scattering [15] and antenna design [42]. A solution of the HTP on a given subdomain is related to the surrounding ones via continuity or *transmission conditions* for Dirichlet and Neumann traces across interfaces. More precisely, if $u$ represents the total wave inside the scatterer $\Omega$ and the scattered field in the exterior $\Omega_0 := \mathbb{R}^d \setminus \bar{\Omega}$, the problem for $N$ subdomains can be stated as follows:

*Problem* 1.1 (Multiple Transmission Problem). Seek $u$ in a suitable functional space such that:

$$\begin{cases} -\Delta u - \kappa_i^2 u \ = \ 0 & \text{in } \Omega_i \qquad i = 0, \dots, N, \\ +\text{inhom. transmission conditions} & \text{on } \partial\Omega, \\ +\text{homogeneous transmission conditions} & \text{on all interfaces } \Omega_i \cap \Omega_j, \\ +\text{radiation conditions} & \text{for } |\mathbf{x}| \longrightarrow \infty. \end{cases} \tag{1}$$

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