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A Simple Multidimensional Euler-Scheme [1]

Michael Fey and Rolf Jeltsch

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Abstract

The idea of the decomposition of the vector of conserved quantities of the multidimensional Euler equations into three multidimensional waves is briefly described. It is implemented in the so-called transport method. Starting from this idea, the necessary properties of these waves to prove the consistency of a numerical scheme are collected. These properties are then used to construct a new and very simple method which preserves all the properties of the transport method. Numerical results obtained with this simple scheme are shown.

Keywords: Euler equations, multidimensional waves, dimensional splitting

Subject Classification: 35Q35, 65M06, 76M25, 76N15

In multidimensional flow calculations most of the finite difference or finite volume methods use a one-dimensional Euler solver in multiple directions. Here the main propagation directions are perpendicular to the cell interfaces of the underlying grid. With this standard dimensional splitting approach the accuracy of the solution is in general first order. With some special modifications the accuracy can be increased to at most second order [7]. But there are flow properties which cannot be correctly described by these splittings, e.g. a shock diagonal to the grid.

There are several new investigations of multidimensional Euler solvers to circumvent the problem mentioned above. The basic idea of these methods is to determine the main propagation direction and to solve a one-dimensional problem in this direction [3], or to construct a set of elementary waves which transport the residuum of one cell to some neighboring nodes [5]. Then the propagation directions are independent of the underlying grid. But these directions have to be calculated from the data of the flowfield and especially from some of their gradients. This causes a loss of robustness of the resulting scheme.

Our method is a synthesis of the previous ones. The underlying concept is based on a decoupling of the multidimensional flux into a finite number of multidimensional elementary waves comparable to the flux vector splitting in one space dimension. These elementary waves have most of the properties of the Euler equations as there is the invariance under a reflection or a rotation of space. These ideas lead to a numerical scheme which allows infinitely many propagation directions in contrast to only two for the dimensional splitting. Moreover, the main part of these waves does not depend on gradients of the data in contrast to the approaches in [3] and [5]. The gradients only affect higher order terms.

In this paper we will briefly describe the idea of the multidimensional waves which defines the structure of these waves and the resulting fluxes precisely. The disadvantage of this scheme is the large amount of computational work to calculate the numerical fluxes. Since we can prove the consistency of the scheme under weaker assumptions, we introduce some simplifications. It is possible to set up multidimensional waves such that for one-dimensional initial conditions the resulting scheme is identical to the Steger-Warming splitting. Even differentiable fluxes can be constructed. For these schemes the computational work decreases drastically so that it is comparable to that of standard methods.

In the following investigations we will restrict ourself to the case of the Euler equations. Before we start with the description of the idea we introduce the notation used in this paper. The one-dimensional homogeneous Euler equations can be written in the form

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial}{\partial x}\mathbf{F}(\mathbf{U}) = 0.$$

The vector of the conserved quantities $\mathbf{U}$ and the physical flux function $\mathbf{F}(\mathbf{U})$ are

$$\mathbf{U} = \begin{pmatrix} \rho \\ m \\ E \end{pmatrix}, \quad \mathbf{F}(\mathbf{U}) = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ u(E+p) \end{pmatrix}. \tag{1}$$

In this notation ρ is the density, m is the momentum, E is the total energy, $u = m/\rho$ is the velocity and p is the pressure. Using the equation of state for an ideal gas, we obtain

$$p = (\gamma - 1)\left(E - \rho\frac{u^2}{2}\right) \tag{2}$$

for the pressure. In our case γ is a constant with $\gamma = 1.4$, the value for air.

For simplicity, we consider the Euler equations in two space dimensions, although the ideas carry over to three dimensions. The differential equations then have the form

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial}{\partial x}\mathbf{F}(\mathbf{U}) + \frac{\partial}{\partial x}\mathbf{G}(\mathbf{U}) = 0.$$

The conservation equation for n, the y-component of the momentum, is added to the system. The vectors have the form

$$\mathbf{U} = \begin{pmatrix} \rho \\ m \\ n \\ E \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ u(E+p) \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} \rho u \\ \rho uv \\ \rho v^2 + p \\ v(E+p) \end{pmatrix}$$

where $v = n/\rho$ is the velocity in y-direction. Equation (2) becomes

$$p = (\gamma - 1)\left(E - \rho\frac{u^2 + v^2}{2}\right).$$

To complete this collection of formulas we add the eigenvalues and eigenvectors of the Jacobian matrix of $\mathbf{F}(\mathbf{U})$ in (1). The eigenvalues are $\lambda_1 = u - c$, $\lambda_2 = u$ and $\lambda_3 = u + c$ and

$$\mathbf{R} = \begin{pmatrix} 1 & 1 & 1 \\ u - c & u & u + c \\ H - uc & u^2/2 & H + uc \end{pmatrix} \tag{3}$$

defined by $c^2 = \gamma p/\rho$. The vector $\mathbf{R}^{-1}\mathbf{U}$ appearing in the one-dimensional flux vector splitting has the simple form

$$\mathbf{R}^{-1}\mathbf{U} = \rho\left(\frac{1}{2\gamma}, \frac{\gamma-1}{\gamma}, \frac{1}{2\gamma}\right)^T.$$

$\mathbf{I}$ denotes the unit matrix and $\mathbf{0}$ the corresponding vector of zeros. $\mathbf{x}$ is the coordinate vector and $\mathbf{u}$ the velocity vector in several space dimensions.

Method of transport

The main idea of this numerical scheme is to use the characteristic propagation directions in each point of the flowfield and to propagate special quantities along these directions. We will briefly describe this idea now. The main problem of extending the one-dimensional scheme is that the finite three propagation directions become infinitely many. The convection with velocity u is simple to adapt to more space dimensions since we have only one propagation direction $\mathbf{u}$. In this case the quantities are only advected with this velocity. The results in [2] obtained for a scalar equation can be used for this wave.

The extension is more difficult for the sonic waves propagating with speed $u \pm c$. These two directions change into the complete Mach cone with infinitely many directions. Let us use the behavior of waves in water as an example. Small disturbances move in all directions relative to the motion. For the scheme we will only allow one wave front and we will distribute the whole information, not only the difference between some steady states.

With these considerations we can define how the information is distributed by such a wave. We find that the physical quantities propagated by this wave are a general form of the linear combination of the two eigenvalues corresponding to the eigenvalues $u \pm c$ of the Jacobian matrix of $\mathbf{F}(\mathbf{U})$ in (1).

A new feature of our approach is a kind of momentum wave. This wave also propagates along the Mach cone and describes the interaction of pressure and momentum. In several space dimensions this wave distributes vectors, but in the one-dimensional case it is only the difference of the two eigenvectors corresponding to the eigenvalues $u \pm c$ times a constant.

We now use a finite volume discretization of the space on a Cartesian grid. As in most first order schemes we use the mean value of the conserved quantities as the function values in this cell. Then the Mach cones within the same cell are all the same. To

1. We calculate the wave front of each point $\mathbf{x}$ in the cell according to the Mach cone of this point. Since all values are the same, all the cones are the same. Then we sum over the wave fronts of all points $\mathbf{x}$ in a cell. The resulting distribution of the entire cell is called wave. This wave must be known analytically as a function of the velocity and the speed of sound because it cannot be computed numerically. To get an efficient numerical scheme we will simplify the construction of this wave.

2. We integrate this wave over the domain of each neighboring cell. We get the mass of information moving out from the central cell to its neighbors during the time Δt.

3. We repeat step 1 for all the neighboring cells. Then we integrate over the center cell and get the mass of information moving in from the neighbors during time Δt.

4. We update the mean value of the center cell by adding and subtracting the mass of information moving in and out.

This algorithm must be applied to all three waves mentioned above. The special form of the waves, their influence on the fluxes and the final form of this method of transport is described more in detail in [1]. There are also given some numerical results obtained with this scheme.

Definition of the waves

The waves of our method of transport derived from the ideas mentioned above are determined so that simplifying modifications are not possible. However, to show the consistency of the transport method we do not need all of the structure. In the algorithm explained above we used the behavior of each point within the cell to update the mean values of the cell. For the numerical method and to proof consistency it is only necessary to know the global behavior of a complete cell. We will put this behavior of the cell in a more general context and collect all the necessary requirements of these waves in four definitions.

Definition 1 *(Waves)*

For each rectangular domain $\Omega_0 \subset I\!R^N$ and for the vector of conserved quantities $\mathbf{U}$ we define a

$$
\begin{aligned}
\mathcal{U}_{\Omega_0}(\mathbf{x},\Delta t) &= \mathbf{R}_2(\mathbf{U}) f^u_{\Omega_0}(\mathbf{x},\Delta t)\\
\mathcal{C}_{\Omega_0}(\mathbf{x},\Delta t) &= \mathbf{R}_1(\mathbf{U}) f^c_{\Omega_0}(\mathbf{x},\Delta t)\\
\mathcal{C}^-_{\Omega_0}(\mathbf{x},\Delta t) &= \mathbf{L}_3(\mathbf{U}) \cdot f^{c-}_{\Omega_0}(\mathbf{x},\Delta t)
\end{aligned}
$$

with the functions

$$
\mathbf{R}_2(\mathbf{U}) := \frac{\gamma-1}{\gamma}\begin{pmatrix}\rho\\ \rho\mathbf{u}\\ \rho\mathbf{u}^2/2\end{pmatrix},
$$

$$
\mathbf{R}_1(\mathbf{U}) := \frac{1}{\gamma}\begin{pmatrix}\rho\\ \rho\mathbf{u}\\ \rho H\end{pmatrix},
$$

$$
\mathbf{L}_3(\mathbf{U}) := \frac{\rho c}{\gamma}\begin{pmatrix}\mathbf{0}^T\\ \mathbf{I}\\ \mathbf{u}^T\end{pmatrix}.
$$

The functions $f^u, f^c : I\!R^N \times I\!R^+ \to I\!R$ *and* $f^{c-} : I\!R^N \times I\!R^+ \to I\!R^N$ *have the property that there exists a point* $\overline{\mathbf{x}}$ *so that*

$$
\begin{aligned}
f^u(\mathbf{x},\Delta t) &= f^u(\mathbf{x} - 2\mathbf{e}_i(\mathbf{x}-\overline{\mathbf{x}})\mathbf{e}_i,\Delta t)\\
f^c(\mathbf{x},\Delta t) &= f^c(\mathbf{x} - 2\mathbf{e}_i(\mathbf{x}-\overline{\mathbf{x}})\mathbf{e}_i,\Delta t)\\
f^{c-}_j(\mathbf{x},\Delta t) &= f^{c-}_j(\mathbf{x} - 2\mathbf{e}_i(\mathbf{x}-\overline{\mathbf{x}})\mathbf{e}_i,\Delta t) \qquad i \neq j,\\
f^{c-}_j(\mathbf{x},\Delta t) &= -f^{c-}_j(\mathbf{x} - 2\mathbf{e}_i(\mathbf{x}-\overline{\mathbf{x}})\mathbf{e}_i,\Delta t) \quad i = j
\end{aligned}
$$

holds, where $\mathbf{e}_i$ *is the unit vector in direction* x_i, $i = 1,\ldots,N$ *and* $\mathbf{x} \in I\!R^N$. *The support of the functions* f *is bounded for any fixed* Δt *and for* $\Delta t = 0$ *it is*

$$
f^u_{\Omega_0}(\mathbf{x},0) = f^c_{\Omega_0}(\mathbf{x},0) = \begin{cases} 1 & \text{if } \mathbf{x} \in \Omega_0 \\ 0 & \text{otherwise}\end{cases}
$$

$$
f^{c-}_{\Omega_0}(\mathbf{x},0) = \mathbf{0}.
$$

This defines the basic structure of the waves. The functions $\mathbf{R}_2$, $\mathbf{R}_1$ and $\mathbf{L}_3$ contain the physical quantities propagating with time. One can easily see the connection between the eigenvectors in (3) and these functions. In the one-dimensional case $\mathbf{R}_2$, $\mathbf{R}_1$ and $\mathbf{L}_3$ are only a linear combination of the columns of $\mathbf{R}$. The last equations of f^u and f^c in Def. 1 represent the deformation of the shape of the cell with increasing time Δt. In contrast to this, the function f^{c-} acts as a source term of momentum influenced by the pressure.

The next three definitions describe additional properties. First we have to require that the sum of two waves corresponding to two disjoint domains are equal to the wave of the union of these two domains.

Definition 2 *(Compatibility condition)*

$$
f_{\Omega_k}(\mathbf{x},\Delta t) + f_{\Omega_l}(\mathbf{x},\Delta t) = f_{\Omega_k \cup \Omega_l}(\mathbf{x},\Delta t)
$$

holds.

We will also assume a kind of conservation property. The numerical scheme is written in conservation form and therefore we obtain conservation after each step, independent of the behavior of the waves. At time t, i.e. for $\Delta t = 0$, Def. 1 implies $\mathcal{U}(\mathbf{x},0) + \mathcal{C}(\mathbf{x},0) + \mathcal{C}^-(\mathbf{x},0) = \mathbf{U}$. We will require this condition for a constant function $\mathbf{U}$ for all $\Delta t > 0$.

Definition 3 *(Completeness)*

A wave f *from Definition 1 is called complete if for all* $\mathbf{x} \in I\!R^N$ *and a constant vector* $\mathbf{U}$

$$
\begin{aligned}
\sum_i f^u_{\Omega_i}(\mathbf{x},\Delta t) &= 1\\
\sum_i f^c_{\Omega_i}(\mathbf{x},\Delta t) &= 1\\
\sum_i f^{c-}_{\Omega_i}(\mathbf{x},\Delta t) &= \mathbf{0}
\end{aligned}
$$

holds, summing over all Ω_i *of the grid.*

Finally we demand that the waves have something to do with the problem, i. e., the Euler equations.

Definition 4 *(Consistency)*

A set of waves from Definition 1 is called consistent if

(a) the point $\overline{\mathbf{x}}$ *moves with velocity* $\mathbf{u}$, *that is* $\overline{\mathbf{x}}(\Delta t) = \overline{\mathbf{x}}(0) + \Delta t\mathbf{u}$ *and*

(b) $\int_0^\infty \left[f^{c-}_{\Omega_0}(\overline{\mathbf{x}} + h\mathbf{e}_i,\Delta t)\right]_j dh = \Delta t c \delta_{i,j}$
holds for $i,j = 1,\ldots,N$. *Here* $\delta_{i,j}$ *denotes the Kronecker symbol.*

The fluxes from a domain Ω_0 to another domain Ω_1 are given by

$$
\mathbf{F}_{\Omega_0\Omega_1} = \mathbf{F}^u_{\Omega_0\Omega_1} + \mathbf{F}^c_{\Omega_0\Omega_1} + \mathbf{F}^{c-}_{\Omega_0\Omega_1}
$$

with

$$
\begin{aligned}
\mathbf{F}^u_{\Omega_0\Omega_1} &= \mathbf{R}_2(\mathbf{U}) \int_{\Omega_1} f^u_{\Omega_0}(\mathbf{y},\Delta t)d\mathbf{y}\\
\mathbf{F}^c_{\Omega_0\Omega_1} &= \mathbf{R}_1(\mathbf{U}) \int_{\Omega_1} f^c_{\Omega_0}(\mathbf{y},\Delta t)d\mathbf{y}\\
\mathbf{F}^{c-}_{\Omega_0\Omega_1} &= \mathbf{L}_1(\mathbf{U}) \cdot \int_{\Omega_1} f^{c-}_{\Omega_0}(\mathbf{y},\Delta t)d\mathbf{y}
\end{aligned}
$$

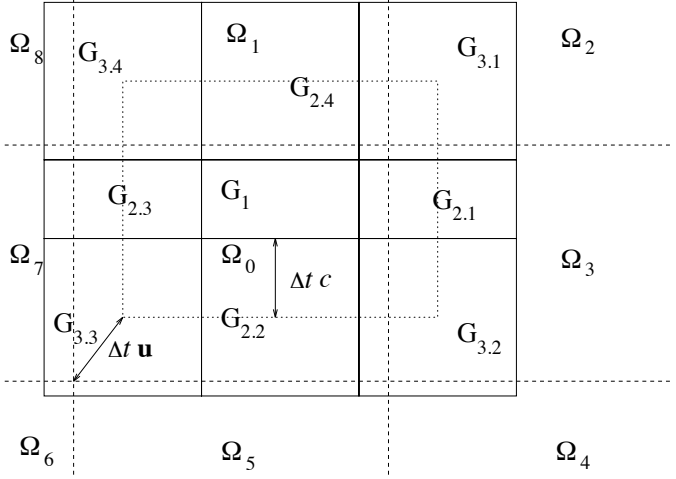

Figure 1: Partition of the support G of the functions f^c and f^{c-} at time $t = \Delta t$ in sub-domains.

and the values at time $t + \Delta t$ are obtained by

$$\mathbf{U}^{n+1}_{\Omega_0} = \mathbf{U}^n_{\Omega_0} - \frac{1}{V_{\Omega_0}} \left(\sum_{l \neq 0} \mathbf{F}_{\Omega_0 \Omega_l} - \mathbf{F}_{\Omega_l \Omega_0} \right)$$

where $\mathbf{U}^n_{\Omega_0}$ denotes the numerical values at the cell Ω_0 and V_{Ω_0} is the volume of the domain Ω_0. The sum runs over all cells except Ω_0. In practice one will restrict the step size Δt such that $\mathbf{F}_{\Omega_0 \Omega_l} = \mathbf{F}_{\Omega_l \Omega_0} = 0$ for all Ω_l with $\overline{\Omega_0} \cap \overline{\Omega_l} = \emptyset$, i. e., the sum runs only over the $3^N - 1$ neighboring cells of Ω_0.

Numerical method

With Definitions 1 – 4 we will construct a simple numerical method in two space dimensions. For the integration part (step 2) of the calculation it is preferable that the support of the waves can be divided in rectangular sub-domains where the functions are constant. This is the disadvantage of the method of transport. There the boundaries of some sub-domains are circles and so the integration procedure was difficult because of the complex geometry.

For the function f^u we simply use the same function as in the method of transport. We get

$$f^u_{\Omega_0}(\mathbf{x}, \Delta t) = \begin{cases} 1 & \mathbf{x} - \Delta t \mathbf{u} \in \Omega_0 \\ 0 & \text{elsewhere} \end{cases}.$$

For the functions f^c and f^{c-} we choose a partitioning of the support as shown in Figures 1. The dashed lines denote the Cartesian grid and separate the domains Ω_j. The dotted line forms the boundary of the center cell advected with velocity $\mathbf{u}$ and the solid line domains $\mathrm{G}_{k.l}$. It can easily be seen that this kind of wave satisfies most of the assumptions in Defs. 1 and 4. We obtain the desired symmetry from the symmetric partitioning and the shift with $\Delta t \mathbf{u}$ guarantees the part (a) of the consistency. The conditions of completeness and compatibility force the following definition for the piecewise constant functions f^c and f^{c-}:

$$f^c_{\Omega_0}(\mathbf{x} - \Delta t \mathbf{u}, \Delta t) = \begin{cases} 1 & \mathbf{x} \in \mathrm{G}_1 \\ \frac{1}{2} & \mathbf{x} \in \mathrm{G}_2 \\ \frac{1}{4} & \mathbf{x} \in \mathrm{G}_3 \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

$$f^{c-}_{\Omega_0}(\mathbf{x} - \Delta t \mathbf{u}, \Delta t) = \begin{cases} \left(\frac{1}{2}, 0\right)^T & \mathbf{x} \in \mathrm{G}_{2.1} \\ \left(0, -\frac{1}{2}\right)^T & \mathbf{x} \in \mathrm{G}_{2.2} \\ \left(-\frac{1}{2}, 0\right)^T & \mathbf{x} \in \mathrm{G}_{2.3} \\ \left(0, \frac{1}{2}\right)^T & \mathbf{x} \in \mathrm{G}_{2.4} \\ \left(\frac{1}{4}, \frac{1}{4}\right)^T & \mathbf{x} \in \mathrm{G}_{3.1} \\ \left(\frac{1}{4}, -\frac{1}{4}\right)^T & \mathbf{x} \in \mathrm{G}_{3.2} \\ \left(-\frac{1}{4}, -\frac{1}{4}\right)^T & \mathbf{x} \in \mathrm{G}_{3.3} \\ \left(-\frac{1}{4}, \frac{1}{4}\right)^T & \mathbf{x} \in \mathrm{G}_{3.4} \\ (0,0)^T & \text{elsewhere} \end{cases} \tag{5}$$

The corresponding fluxes are obtained by integration of the waves. This can be done easily and the result is the sum of the areas of some rectangles times some factors depending on the sub-domain. With the auxiliary functions

$$\begin{aligned} h_1(x) &= \max(0, x+1) \\ h_2(x) &= \max(0, x-1) \\ h_3(x) &= \min(2, h_1(x)) \\ h_4(x,y) &= y - 1 + \min(0, 1 - |x|) \\ h_5(x) &= \min(1, \max(-1, -x)) \end{aligned}$$

we obtain

$$\begin{aligned} \mathbf{F}^u_{\Omega_0 \Omega_1} &= \mathbf{R}_2(\mathbf{U})(\Delta t c)^2 \max(0, M_v) h_4\left(M_u, \frac{\Delta y}{\Delta t c}\right) \\ \mathbf{F}^u_{\Omega_0 \Omega_2} &= \mathbf{R}_2(\mathbf{U})(\Delta t c)^2 \max(0, M_v) \max(0, M_u) \end{aligned}$$

for the flux of wave $\mathcal{U}$. The integration of the function f^c yields

$$\begin{aligned} \mathbf{F}^u_{\Omega_0 \Omega_1} &= \mathbf{R}_1(\mathbf{U}) \frac{(\Delta t c)^2}{2} h_4\left(M_u, \frac{\Delta y}{\Delta t c}\right) \\ &\qquad (h_1(M_v) + h_2(M_v)) \\ \mathbf{F}^u_{\Omega_0 \Omega_2} &= \mathbf{R}_1(\mathbf{U}) \left(\frac{\Delta t c}{2}\right)^2 \\ &\qquad (h_1(M_u) + h_2(M_u))(h_1(M_v) + h_2(M_v)) \end{aligned}$$

$$\mathbf{z}_1(x,y,z) = \frac{1}{2}\begin{pmatrix} h_5(x)(h_1(y)+h_2(y)) \\ h_4(x,z)\,h_3(y) \end{pmatrix}$$

$$\mathbf{z}_2(x,y) = \frac{1}{4}\begin{pmatrix} h_3(x)(h_1(y)+h_2(y)) \\ h_3(y)(h_1(x)+h_2(x)) \end{pmatrix}$$

the fluxes $\mathbf{F}^{c-}$ have the form

$$\mathbf{F}^{c-}_{\Omega_0\Omega_1} = \mathbf{L}_3(\mathbf{U})(\Delta tc)^2\mathbf{z}_1\left(M_u, M_v, \frac{\Delta y}{\Delta tc}\right)$$

$$\mathbf{F}^{c-}_{\Omega_0\Omega_2} = \mathbf{L}_3(\mathbf{U})(\Delta tc)^2\mathbf{z}_2\left(M_u, M_v\right)$$

where $M_u = u/c$ and $M_v = v/c$ denote the Mach number in x- and y-direction.

The fluxes into the remaining domains can be obtained by changing the Mach numbers (e.g. $M_u \rightarrow -M_u$ in $\mathbf{F}_{\Omega_0\Omega_2}$ to get $\mathbf{F}_{\Omega_0\Omega_8}$ etc.). Now all the necessary fluxes are known to update the quantities in the flowfield.

Numerical results

Figures 2 and 3 show the solution of a two-dimensional Riemann problem. The four quadrants are connected by a simple wave. These kind of problems are discussed in detail in [6] and they are good tests because of the simple boundary conditions. Figure 2 shows density contours obtained with the unsplit method. For comparison, Figure 3 shows the same quantities obtained by a van Leer flux vector splitting method and dimensional splitting [4]. In this case the results are nearly the same. The van Leer scheme is 20% faster because it only computes four fluxes per cell instead of eight in our unsplit method.

As a second test case we took a Mach 25 flow. The free stream conditions are those in 75 km altitude. Because of the loss of a boundary condition for a solid wall we inserted a source into the flowfield which is strong enough to build a strong moving shock wave. Figures 4 – 9 show the numerical results on different meshes and at different times. In this case the dimensional splitting is not suitable for large time steps. In contrast to this, the unsplit method runs also in this case with the maximum time step. There are also small differences in the position and the shape of the shocks, and these differences become larger for coarser meshes.

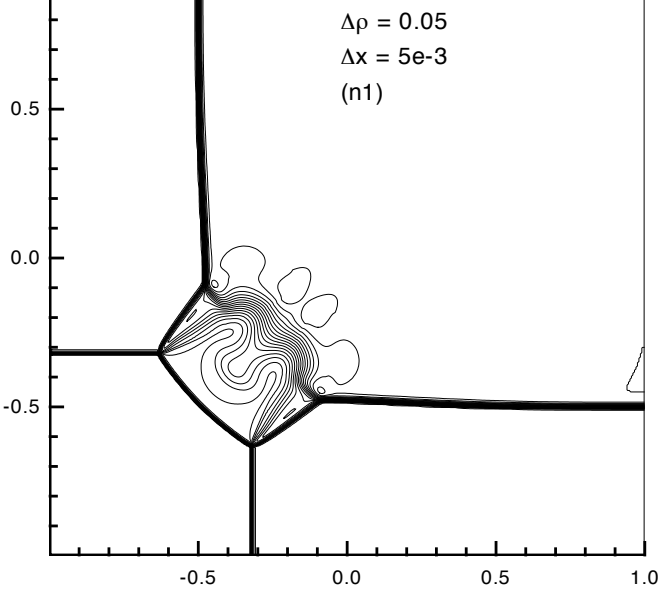

Figure 2: Solution of a Riemann problem with our unsplit method.

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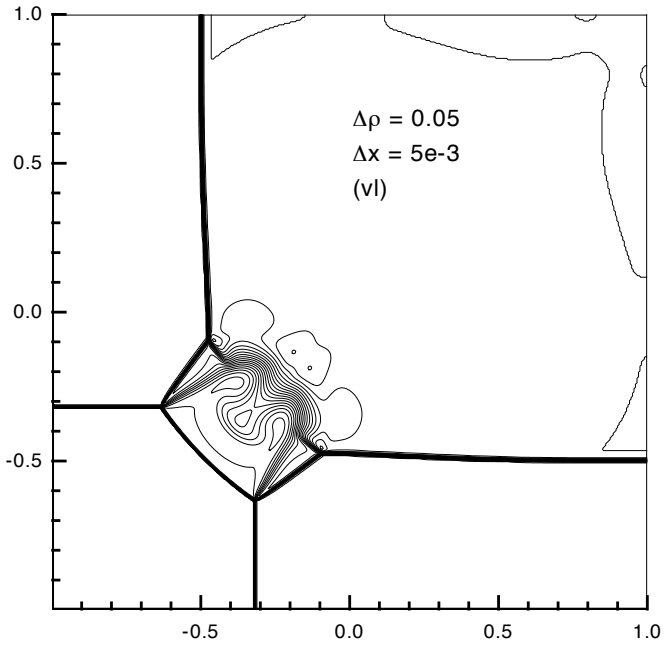

Figure 3: Solution of a Riemann problem with van Leer flux vector splitting.

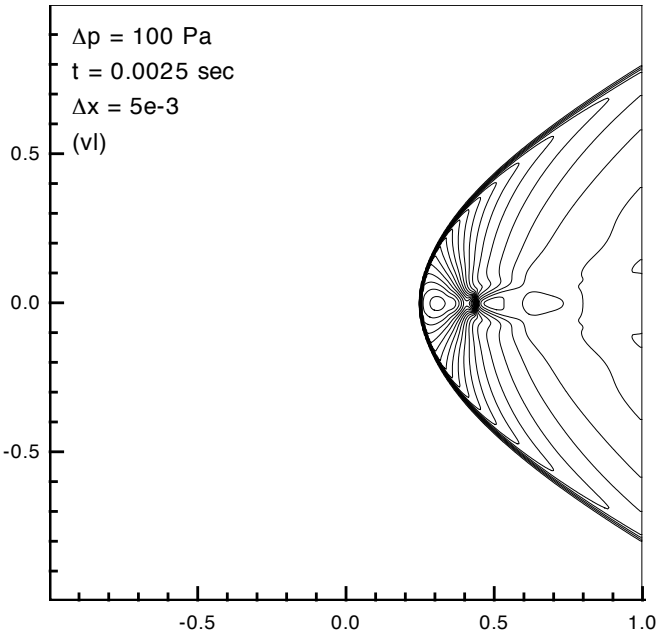

Figure 5: Solution of a Mach 25 flow on the fine mesh with van Leer flux vector splitting.

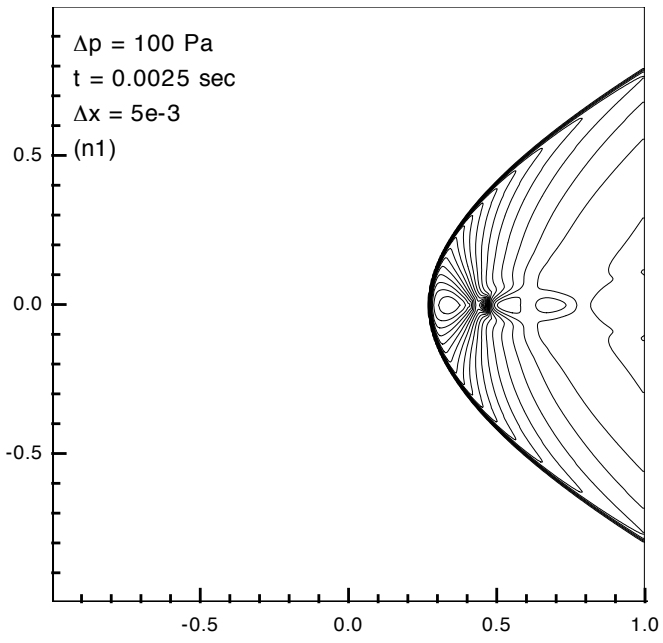

Figure 4: Solution of a Mach 25 flow on the fine mesh with our unsplit method.

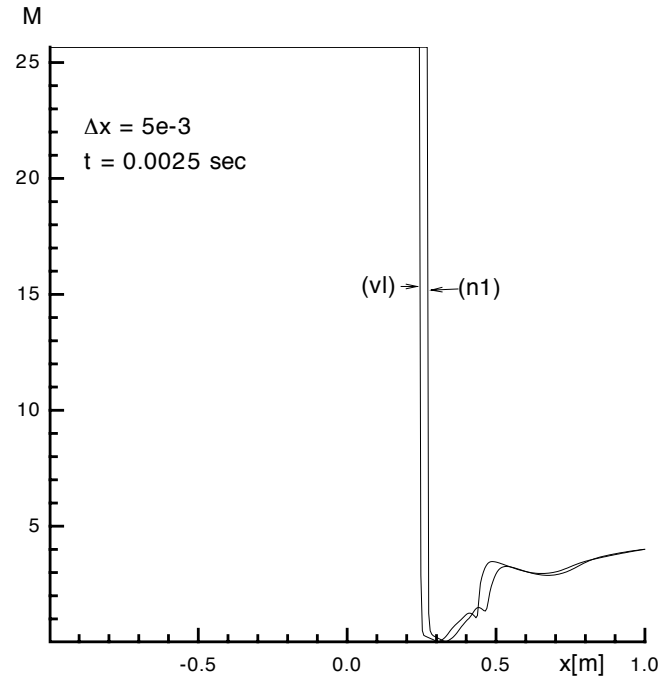

Figure 6: Mach number distribution at the symmetry line of Figures 4 and 5.

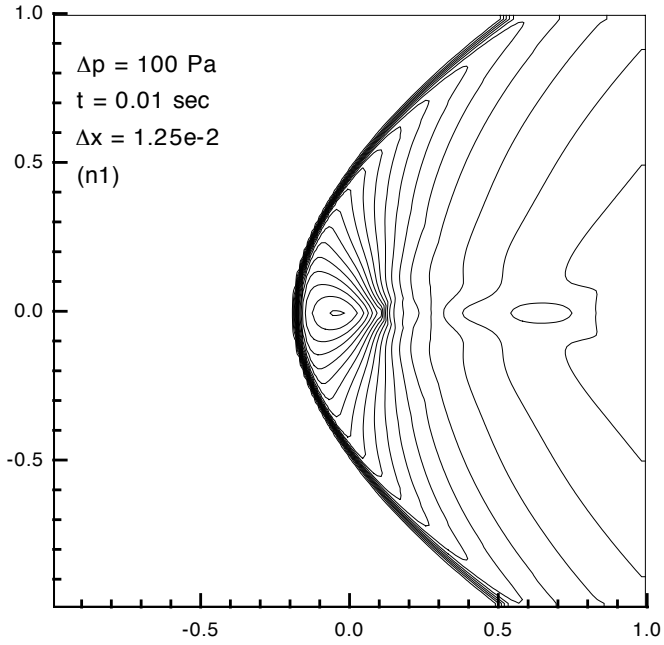

Figure 7: Solution of a Mach 25 flow on the coarse mesh with our unsplit method.

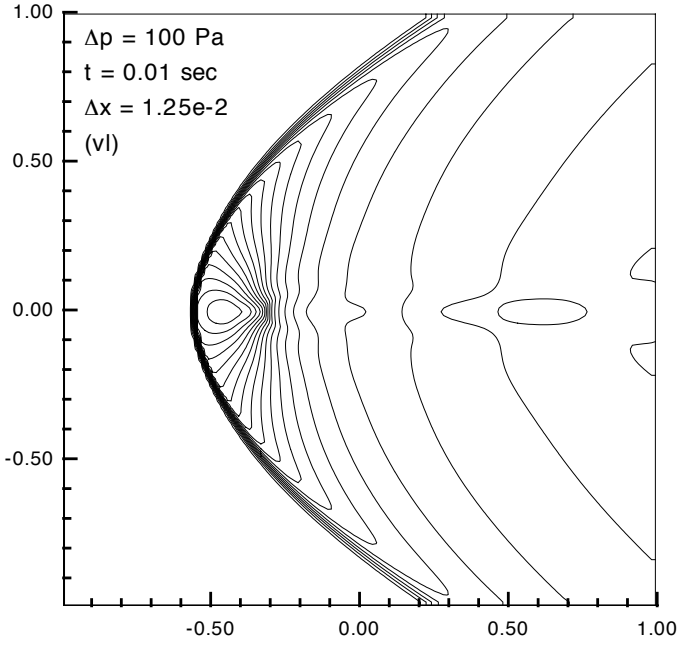

Figure 8: Solution of a Mach 25 flow with on the coarse mesh van Leer flux vector splitting.

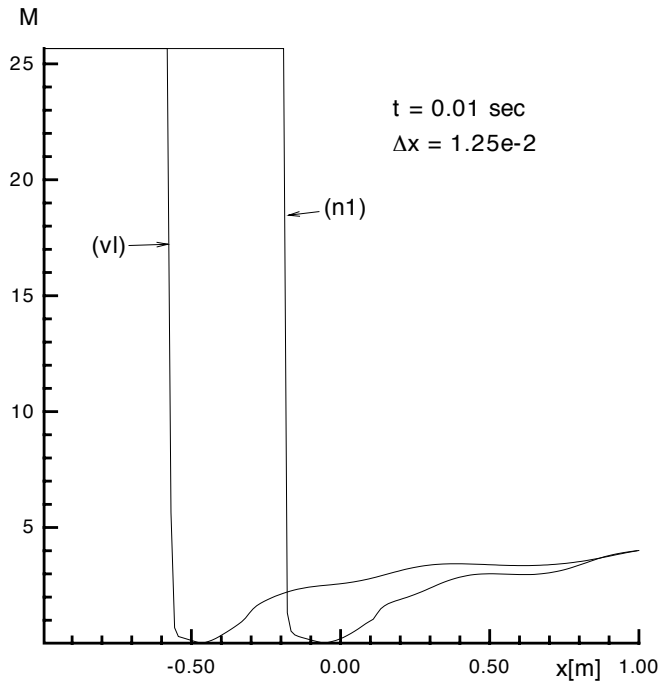

Figure 9: Mach number distribution at the symmetry line of Figures 7 and 8.

92-10	M. Fey, R. Jeltsch	A Simple Multidimensional Euler Scheme
92-09	M. Fey, R. Jeltsch	A New Multidimensional Euler Scheme
92-08	M. Fey, R. Jeltsch, P. Karmann	Numerical solution of a nozzle flow
92-07	M. Fey, R. Jeltsch, P. Karmann	Special aspects of reacting inviscid blunt body flow
92-06	M. Fey, R. Jeltsch, S. Müller	The influence of a source term, an example: chemically reacting hypersonic flow
92-05	N. Botta, J. Sesterhenn	Deficiencies in the numerical computation of nozzle flow
92-04	Ch. Lubich	Integration of stiff mechanical systems by Runge-Kutta methods
92-03	M. Fey, R. Jeltsch, S. Müller	Stagnation point analysis
92-02	C. W. Schulz-Rinne, J. P. Collins, H. M. Glaz	Numerical Solution of the Riemann Problem for Two-Dimensional Gas Dynamics
92-01	R. J. LeVeque, K. M. Shyue	Shock Tracking Based on High Resolution Wave Propagation Methods
91-10	M. Fey, R. Jeltsch	Influence of numerical diffusion in high temperature flow
91-09	R. J. LeVeque, R. Walder	Grid Alignment Effects and Rotated Methods for Computing Complex Flows in Astrophysics
91-08	Ch. Lubich, R. Schneider	Time discretization of parabolic boundary integral equations
91-07	M. Pirovino	On the Definition of Nonlinear Stability for Numerical Methods
91-06	Ch. Lubich, A. Ostermann	Runge-Kutta Methods for Parabolic Equations and Convolution Quadrature
91-05	C. W. Schulz-Rinne	Classification of the Riemann Problem for Two-Dimensional Gas Dynamics
91-04	R. Jeltsch, J. H. Smit	Accuracy Barriers of Three Time Level Difference Schemes for Hyperbolic Equations
91-03	I. Vecchi	Concentration-cancellation and Hardy spaces
91-02	R. Jeltsch, B. Pohl	Waveform Relaxation with Overlapping Splittings