LANCZOS-TYPE SOLVERS FOR NON-HERMITIAN LINEAR SYSTEMS

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Abstract

In this overview we discuss iterative methods for solving large linear systems with sparse (or, possibly, structured) nonsymmetric (or, non-Hermitian) matrix that are based on the Lanczos process. They feature short recurrences for the generation of the Krylov space and for the sequence of approximations to the solution. This means low cost and low memory requirement. For very large sparse non-Hermitian linear systems some of these (preconditioned) Lanczos-type Krylov space methods have proven to be the most effective. The basic approach, realized in the biconjugate gradient (BiCG) method of Lanczos (1952), is classical, but in the last twelve years much progress has been made regarding, for example,

1. avoiding breakdowns of the Lanczos process by look-ahead procedures,
2. avoiding the transposed matrix by using Lanczos-type product methods,
3. smoothing the convergence behavior by applying a local minimal residual or a global quasi-minimal residual process,
4. reducing the effects of roundoff errors,

The Lanczos approach also provides eigenvalue approximations. Moreover, the underlying theory is closely linked to formal orthogonal polynomials and Padé approximation.

Detailed coverage of all the above topics requires 3–4 lectures, but for shorter presentations a selection can be made. In particular, a short form covering the essentials of 1, 2, and 4 fits into a one-hour lecture.

References

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