THE CONJUGATE GRADIENT METHOD

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Abstract

This is an easy introduction into the conjugate gradient (CG) method of Hestens and Stiefel (1952), presented on a level suitable to undergraduate students with a good working knowledge of linear algebra.

We start by the rationale for using Krylov space methods and introduce CG as the one that builds up an orthogonal basis of the Krylov space. We then turn to the geometric interpretation of the CG method, which manifests the striking superiority of CG over the steepest descent method. While the first approach leads to the three-term formulation, the geometric approach leads directly to the classical coupled two-term formulation. We show how the two are related by LR decomposition of a tridiagonal matrix.

Time permitted we can also talk about some more recent results on roundoff and add a discussion of the standard convergence rate estimate and its weaknesses.