LOOK-AHEAD LANCZOS REVISITED

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Abstract

The construction of the iterates and residuals of Lanczos-type solvers is based on either a three-term recurrence or a pair of coupled two-term recurrences for the Lanczos vectors that generate two Krylov spaces. However, the Lanczos process for non-Hermitian matrices is well-known to be endangered by breakdowns. These breakdowns and the corresponding near-breakdowns can be circumvented by look-ahead steps in all but very exceptional situations (the so-called incurable breakdowns). Look-ahead means to replace ill-defined or ill-conditioned Lanczos vectors by other, so-called inner vectors, which still expand the Krylov space, but do not satisfy all the biorthogonality conditions. There are various legitimate ways to define these inner vectors. We discuss the pros and cons of several choices and show that the introduction of certain auxiliary vectors reduces the memory overhead in the case of successive look-ahead steps.

This talk is based on joint work with Marlis Hochbruck and Klaus Ressel