PRECONDITIONING BY SIMILARITY TRANSFORMATIONS: ANOTHER VALID OPTION?

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Abstract

Preconditioning a linear system $\mathbf{Ax} = \mathbf{b}$ consists in replacing the matrix $\mathbf{A}$ by some other matrix for which the iterative solver that is to be applied is likely to converge faster. Typically, the goal is to find a preconditioned matrix that is better conditioned and whose spectrum is mostly clustered around 1. Here, we consider another modification, which fits formally still into preconditioning, but does not change the spectrum of the matrix, only its departure from normality, and thus its pseudospectrum. This formal preconditioning can be applied independently of another, standard preconditioner.

One proposal is to determine it by the initial residual, in which case it can be considered as "residual-based preconditioning". A natural way of applying this seems the flexible one, where, for example, in restarted FOM and GMRES the formal preconditioner is modified at each restart. We show that for these two methods this approach is equivalent to applying restarted FOM and GMRES with a varying inner product depending on the residual, as recently proposed by A. Essai (*Numer. Algor.* **18** *(1998), pp. 277–292*). We see in examples that sometimes convergence can be accelerated dramatically, while in other cases there is virtually no or even an adverse effect. We have made efforts to analyze this behavior by computing the eigen-components of the residuals and by monitoring the departure of normality of the preconditioned matrix. Amazingly, many results turned out to be counter-intuitive.

Other ways to apply the general approach may turn out to be rewarding. For example, balancing fits into this framework too, but our tests with it were also quite negative.

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