

Numerical solution of matrix eigenvalue problems Part 0: Introduction

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Typology of matrix eigenvalue problems

▶ Standard eigenvalue problem

$$Ax = \lambda x, \quad x \neq 0, \quad A \in \{\mathbb{R}, \mathbb{C}\}^{n \times n}.$$

▶ Generalized eigenvalue problem

$$Ax = \lambda Bx, \quad x \neq 0, \quad A, B \in \{\mathbb{R}, \mathbb{C}\}^{n \times n}.$$

▶ Polynomial eigenvalue problem

$$(A_0 + \lambda A_1 + \dots + \lambda^l A_l)x = 0, \quad x \neq 0, \quad A_j \in \{\mathbb{R}, \mathbb{C}\}^{n \times n}.$$

▶ Nonlinear eigenvalue problem

$$A(\lambda)x = 0, \quad x \neq 0, \quad A: \mathbb{C} \mapsto \mathbb{C}^{n \times n}.$$

Typically, coefficients are highly structured (sparse, symmetric, low rank structures, ...).

- ▶ Find $k \ll n$ largest eigenvalues/eigenvectors (PageRank, stationary vector of Markov chain, PCA, spectral clustering, Floquet multipliers, ...).
- ▶ Find $k \ll n$ smallest eigenvalues/eigenvectors (structural dynamics, ground state compt, spectral clustering, spectral bundle methods, ...).
- ▶ Find $k \ll n$ eigenvalues/eigenvectors closest to imaginary axis (stability analysis, model reduction, ...).
- ▶ Find all eigenvalues/eigenvectors on real/imaginary axis (\mathcal{H}_∞ control, gyroscopic systems, robust stability analysis, ...).
- ▶ Find many or all eigenvalues/eigenvectors (optimal control, DFT, stable/unstable separation, ...).

Typology of algorithms

- ▶ **Dense methods**: QR algorithm, Jacobi, Divide-and-conquer, MRRR.
- ▶ **Power method and friends**: power method, inverse iteration, subspace iteration.
- ▶ **Krylov subspace methods**: Arnoldi, Lanczos.
- ▶ **Preconditioned methods**: preconditioned inverse iteration, LOPCG.
- ▶ **Newton-like methods**: Newton, Jacobi-Davidson, Rayleigh quotient iteration (RQI), Grassmann RQI.
- ▶ **Matrix function methods**: sign function iteration, Newton-Schultz iteration, Cauchy integral.

Part 1 Power method and friends.

Part 2 Krylov subspace methods.

Part 3 Preconditioned eigensolvers.

Omissions:

- ▶ Dense eigenvalue solvers.
- ▶ Details.
- ▶ Implementation/parallelization aspects.
- ▶ Generalized/structured/nonlinear eigenvalue problems.
- ▶ ...