Numerical solution of matrix eigenvalue problems Part 0: Introduction

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Typology of matrix eigenvalue problems

- Standard eigenvalue problem

$$Ax = \lambda x, \quad x \neq 0, \quad A \in \{\mathbb{R}, \mathbb{C}\}^{n \times n}.$$

- Generalized eigenvalue problem

$$Ax = \lambda Bx, \quad x \neq 0, \quad A, B \in \{\mathbb{R}, \mathbb{C}\}^{n \times n}.$$

- Polynomial eigenvalue problem

$$(A_0 + \lambda A_1 + \cdots + \lambda^l A_l)x = 0, \quad x \neq 0, \quad A_j \in \{\mathbb{R}, \mathbb{C}\}^{n \times n}.$$

- Nonlinear eigenvalue problem

$$A(\lambda)x = 0, \quad x \neq 0, \quad A : \mathbb{C} \mapsto \mathbb{C}^{n \times n}.$$

Typically, coefficients are highly structured (sparse, symmetric, low rank structures, . . .).

Some typical tasks

- Find $k \ll n$ largest eigenvalues/eigenvectors (PageRank, stationary vector of Markov chain, PCA, spectral clustering, Floquet multipliers, ...).
- Find $k \ll n$ smallest eigenvalues/eigenvectors (structural dynamics, ground state compt, spectral clustering, spectral bundle methods, ...).
- Find $k \ll n$ eigenvalues/eigenvectors closest to imaginary axis (stability analysis, model reduction, ...).
- Find all eigenvalues/eigenvectors on real/imaginary axis ($\mathcal{H}_\infty$ control, gyroscopic systems, robust stability analysis, ...).
- Find many or all eigenvalues/eigenvectors (optimal control, DFT, stable/unstable separation, ...).

Typology of algorithms

- Dense methods: QR algorithm, Jacobi, Divide-and-conquer, MRRR.
- Power method and friends: power method, inverse iteration, subspace iteration.
- Krylov subspace methods: Arnoldi, Lanczos.
- Preconditioned methods: preconditioned inverse iteration, LOPCG.
- Newton-like methods: Newton, Jacobi-Davidson, Rayleigh quotient iteration (RQI), Grassmann RQI.
- Matrix function methods: sign function iteration, Newton-Schultz iteration, Cauchy integral.

Overview of lectures

Part 1 Power method and friends.

Part 2 Krylov subspace methods.

Part 3 Preconditioned eigensolvers.

Omissions:

- Dense eigenvalue solvers.
- Details.
- Implementation/parallelization aspects.
- Generalized/structured/nonlinear eigenvalue problems.
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