

# Theory and Numerics of Model Reduction

Web page:

<http://www.math.ethz.ch/~kressner/modred.php>

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# Examples for Model Reduction

## ▶ Oberwolfach Benchmark Collection.

- ▶ <http://www.imtek.de/simulation/benchmark/>
- ▶  $\approx 15$  examples from real applications, mostly linear but also some nonlinear systems
- ▶ Many examples have the form

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t).$$

where  $E$  is symmetric positive definite (mass matrix of a FE discretization).

## ▶ SLICOT Benchmark Collection.

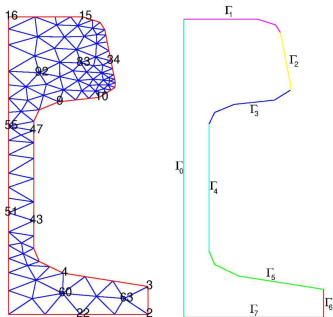
- ▶ See <http://www.inma.ucl.ac.be/~vdooren/publications/ChahlaouiV02a.pdf>.
- ▶ Matrices not online available at the moment.

# Example 1: Steel cooling (Oberwolfach BC)



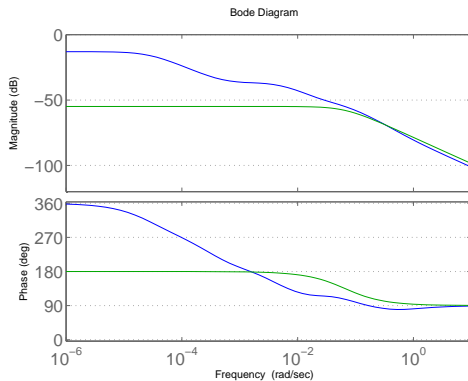
- ▶ Goal: Optimal cooling of glowing steel profile with water sprinklers.
- ▶ Too fast: Temperature differences too large  $\rightsquigarrow$  cracks.
- ▶ Too slow: expensive.
- ▶ Assumption: Profile homogeneous and infinite in one direction  $\rightsquigarrow$  2D heat equation.

- ▶ 7 Inputs: Piecewise constant Neumann boundary control.
- ▶ 6 Outputs: Temperature “measured” at control points in the inner domain.
- ▶ Discretization with linear FEs.



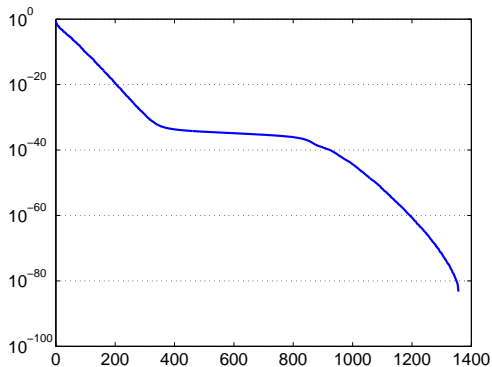
# Example 1: Model reduction with the naive approach

Bode plots for 1st input  $\rightarrow$  5th output



- ▶ Original model
- ▶ Reduced model when truncating all but first 100 states.

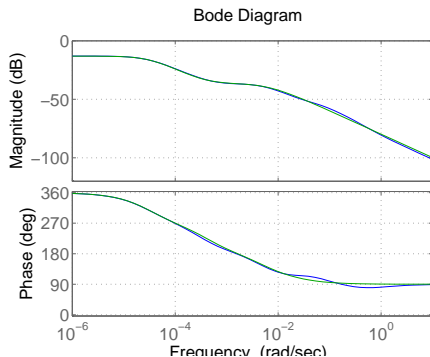
## Example 1: Hankel singular values



The Hankel singular values  $\sigma_i$  for  $i > 300$  are heavily corrupted by roundoff noise and should not be trusted.

# Example 1: Balanced truncation $r = 10$

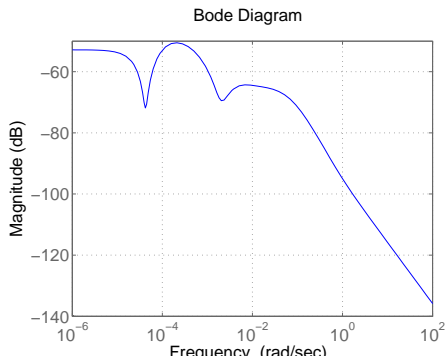
Bode plots for 1st input  $\rightarrow$  5th output



- ▶ Original model
- ▶ Reduced model when truncating all but first 10 states of the balanced system
- ▶  $H_\infty$  norm of error = 0.0105

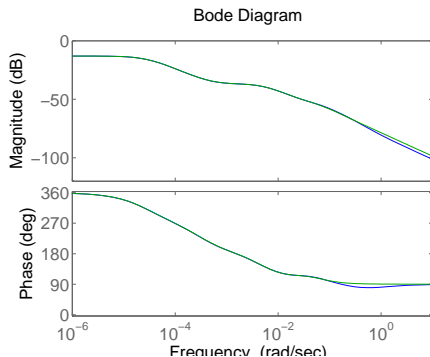
## Example 1: Balanced truncation $r = 10$

Bode magnitude plot for error in 1st input  $\rightarrow$  5th output



# Example 1: Balanced truncation $r = 20$

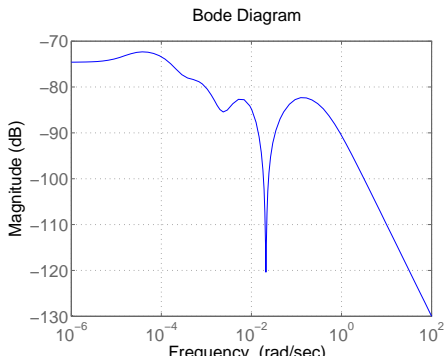
Bode plots for 1st input  $\rightarrow$  5th output



- ▶ Original model
- ▶ Reduced model when truncating all but first 20 states of the balanced system
- ▶  $H_{\infty}$  norm of error = 0.0012

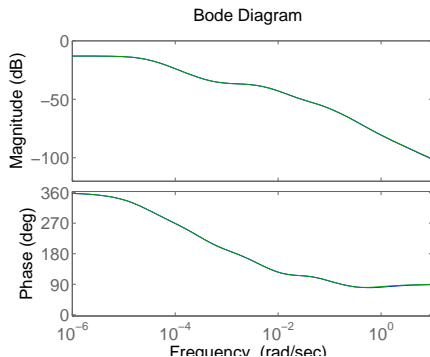
## Example 1: Balanced truncation $r = 20$

Bode magnitude plot for error in 1st input  $\rightarrow$  5th output



# Example 1: Balanced truncation $r = 40$

Bode plots for 1st input  $\rightarrow$  5th output



- ▶ Original model
- ▶ Reduced model when truncating all but first 20 states of the balanced system
- ▶  $H_\infty$  norm of error =  $3 \times 10^{-5}$

## Example 1: Balanced truncation $r = 40$

Bode magnitude plot for error in 1st input  $\rightarrow$  5th output

