Theory and Numerics of Model Reduction

Web page:
`http://www.math.ethz.ch/~kressner/modred.php`

Martin Gutknecht and Daniel Kressner

The basic ADI method

$X_0 = 0$

for $i = 1, \ldots, \ell$

Solve $(A + p_i)X_{i-1/2} = -BB^T - X_{i-1}(A^T - p_i I)$.

Solve $(A + p_i)X_i^T = -BB^T - X_{i-1/2}^T(A^T - p_i I)$.

end

If A is symmetric, $\Lambda(A) \subset [\alpha, \beta]$, suboptimal parameters

$$\min_{\{p_1,\ldots,p_\ell\}\subset\mathbb{C}} \max_{t\in[\alpha,\beta]} \frac{|r_\ell(t)|}{|r_\ell(-t)|}$$

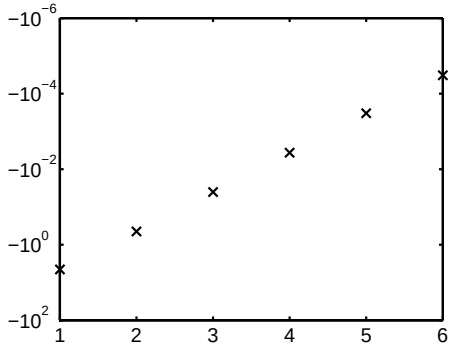
with $r_\ell(t) = (t - p_1)\cdots(t - p_\ell)$ can be determined explicitly using elliptic functions (see [Wachspress'95]).

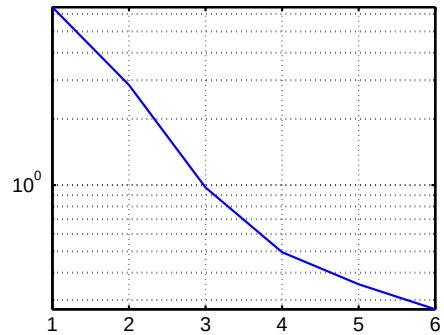
Supoptimal ADI parameters for steel cooling example

Parameters determined with MATLAB functions from `http://www-user.tu-chemnitz.de/~saak/Software/adipars.php`.
$\Lambda(A) \subset [-8.18, -1.8 \times 10^{-5}]$.

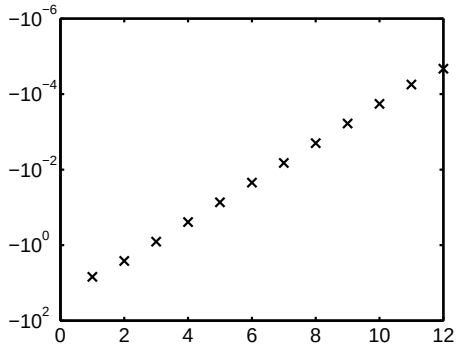
ADI parameters $\qquad \|AX_i + X_iA^T + BB^T\|_F$

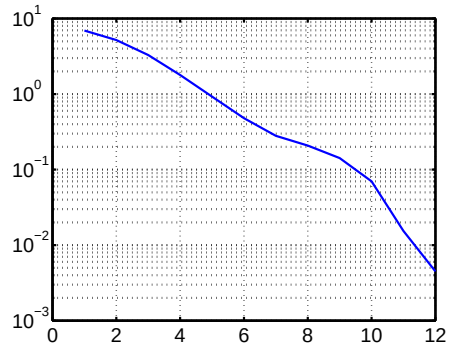
- $\ell = 6$:





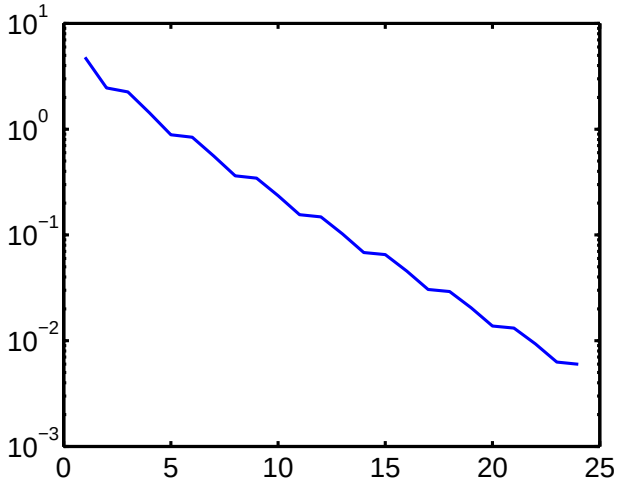
- $\ell = 12$:



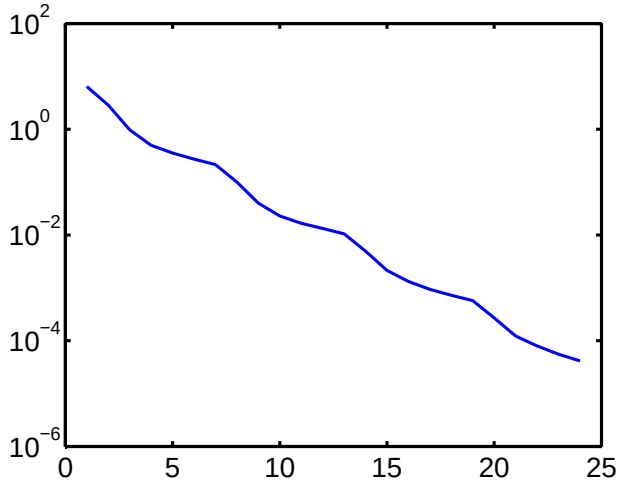


Cyclic ADI

$\ell = 3$, 8 cycles



$\ell = 6$, 4 cycles



$\ell = 12$, 2 cycles