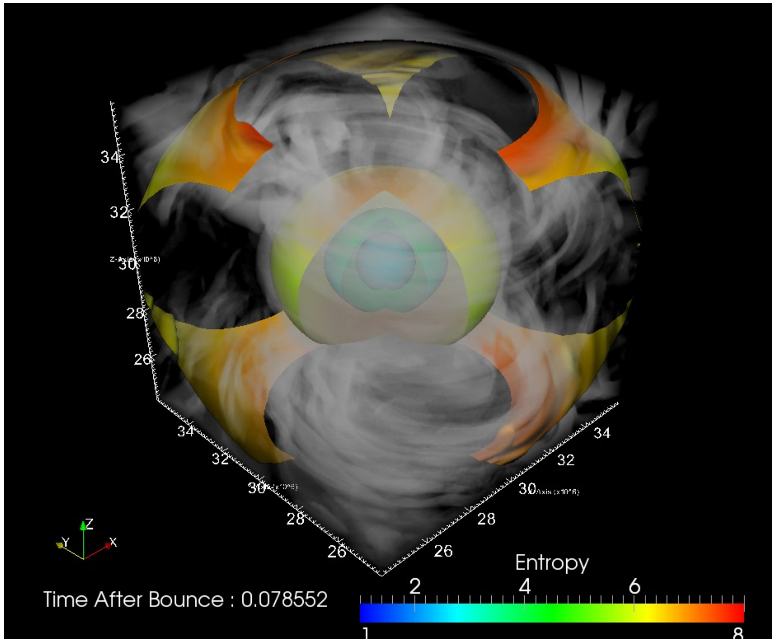
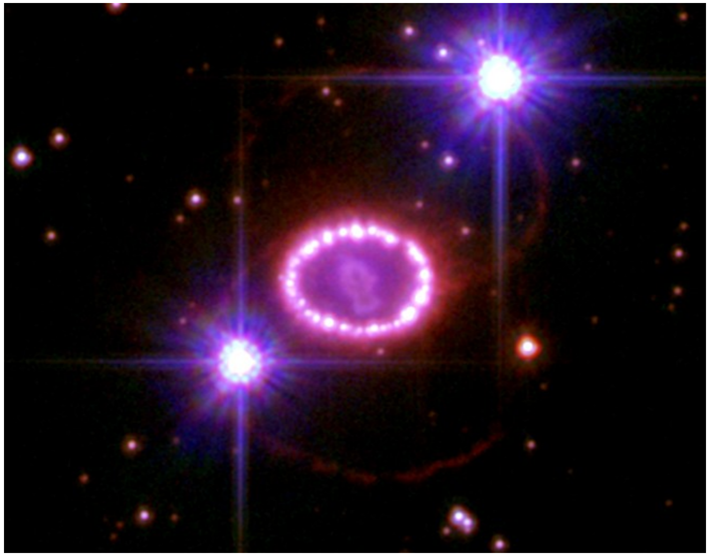
Simulation
of
Catastrophic Stellar Events

Roger Käppeli

Seminar for
Applied
Mathematics
SAM

ETH





Outline

- **Introduction**
  - A brief introduction of the problem
- **Physical model, numerical & computational aspects**
  - Physics ingredients & mathematical model
- **Simulation of magneto-rotational core-collapse**
  - MHD CCSN mechanism
  - Magnetic field amplification, formation and driving mechanism of bipolar outflow
  - Explosion energy, ejected mass and its composition
- **More on numerical & computational aspects**
  - Well-balanced methods for hydrostatic equilibrium
- **Conclusion**

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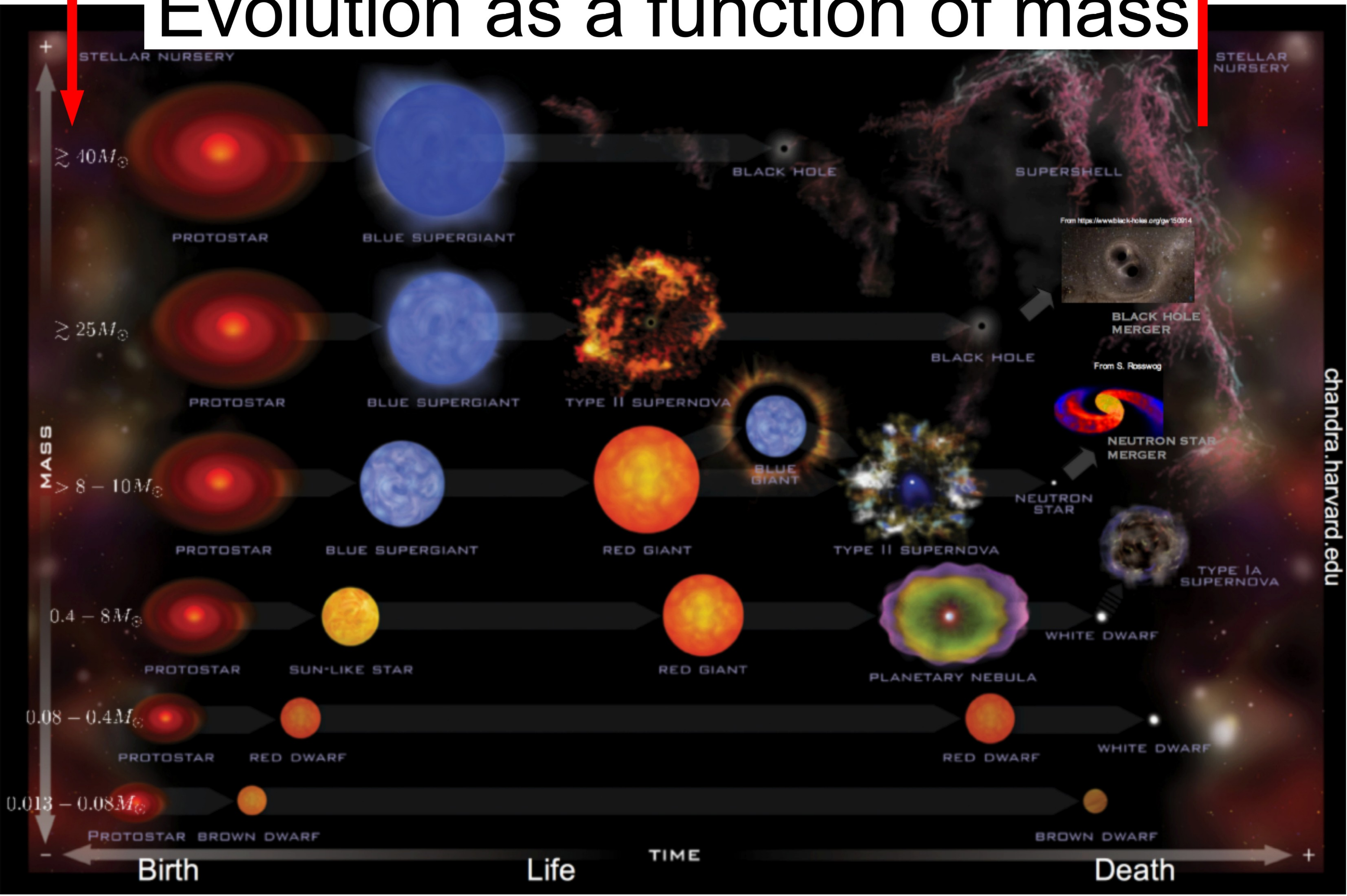
Stellar life cycle

Evolution as a function of mass



From https://www.black-holes.org/gw150914

From S. Rosswog

Stellar life cycle

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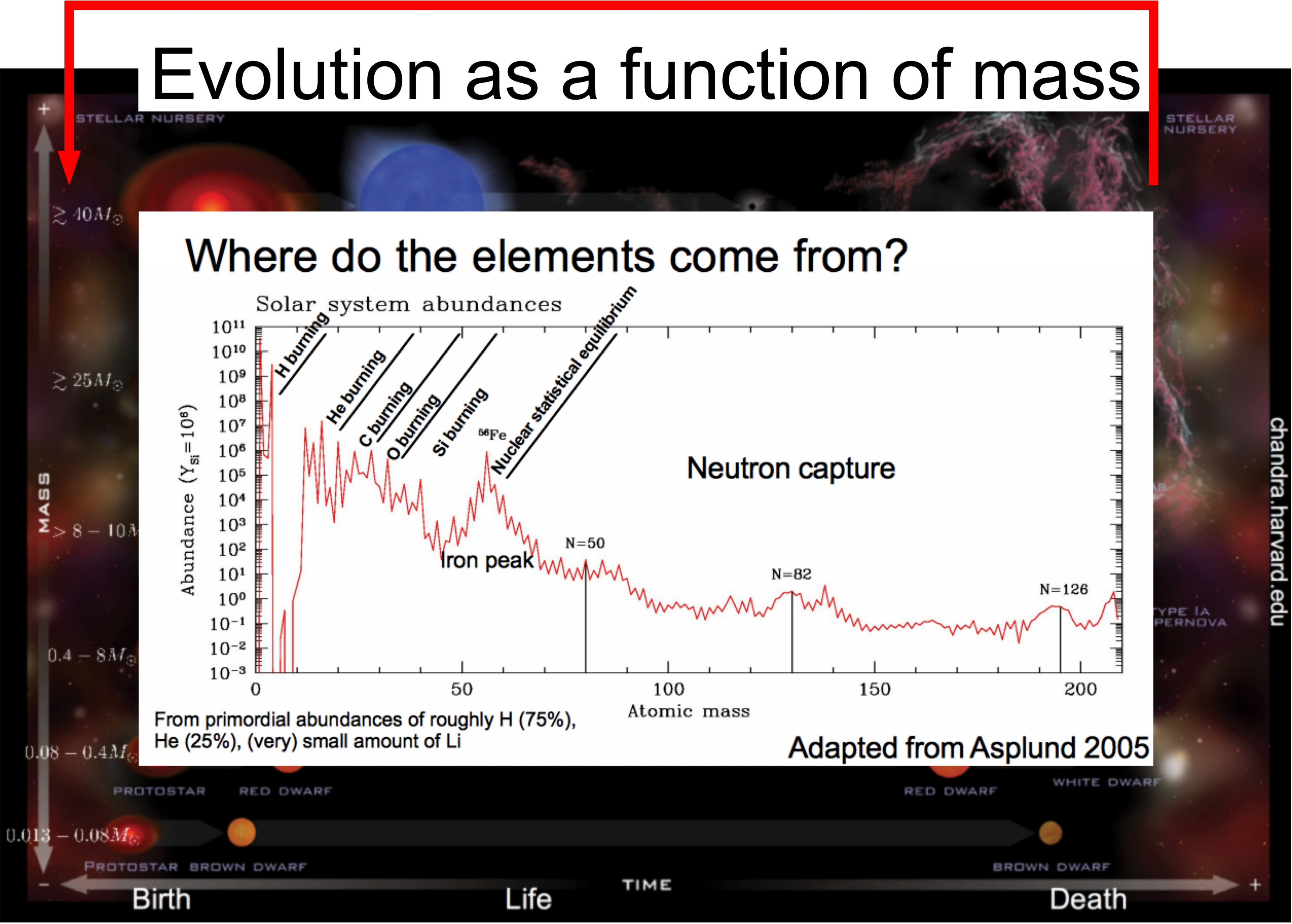

Where do the elements come from?

From primordial abundances of roughly H (75%), He (25%), (very) small amount of Li

Adapted from Asplund 2005

Stellar life cycle

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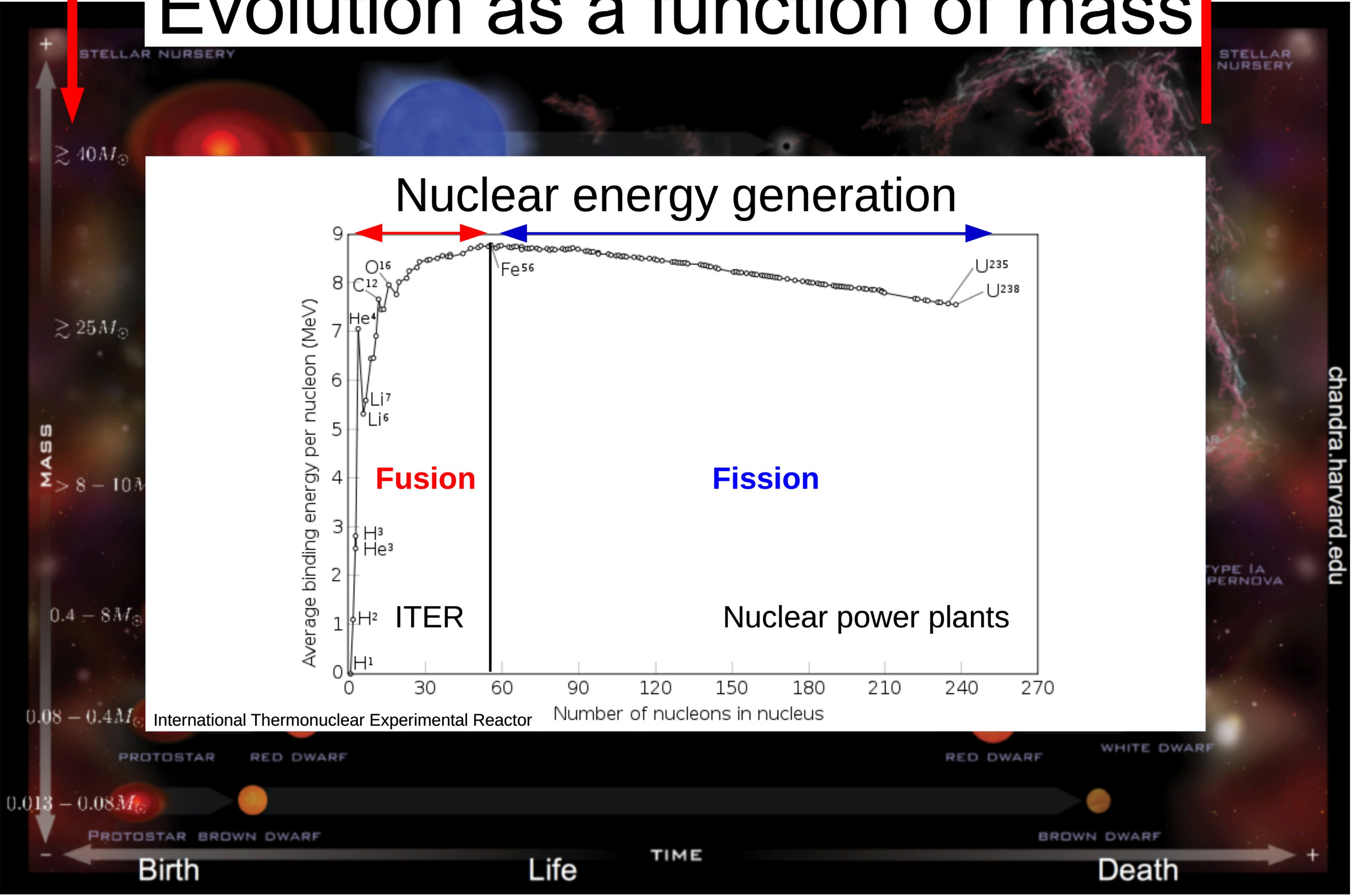


Nuclear energy generation

Fusion

Fission

ITER

Nuclear power plants

International Thermonuclear Experimental Reactor

Birth

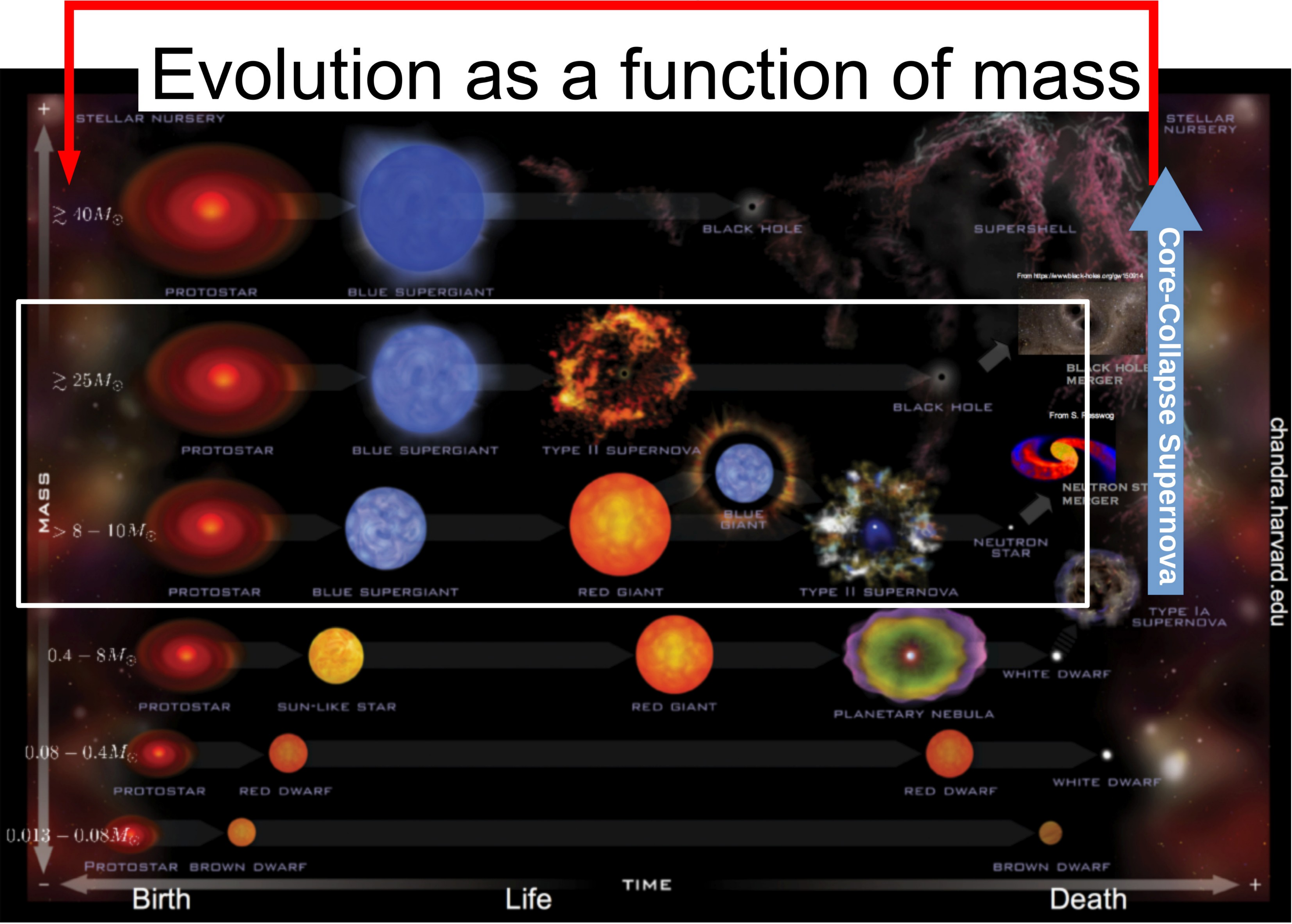
Life

Death

chandra.harvard.edu

Stellar life cycle

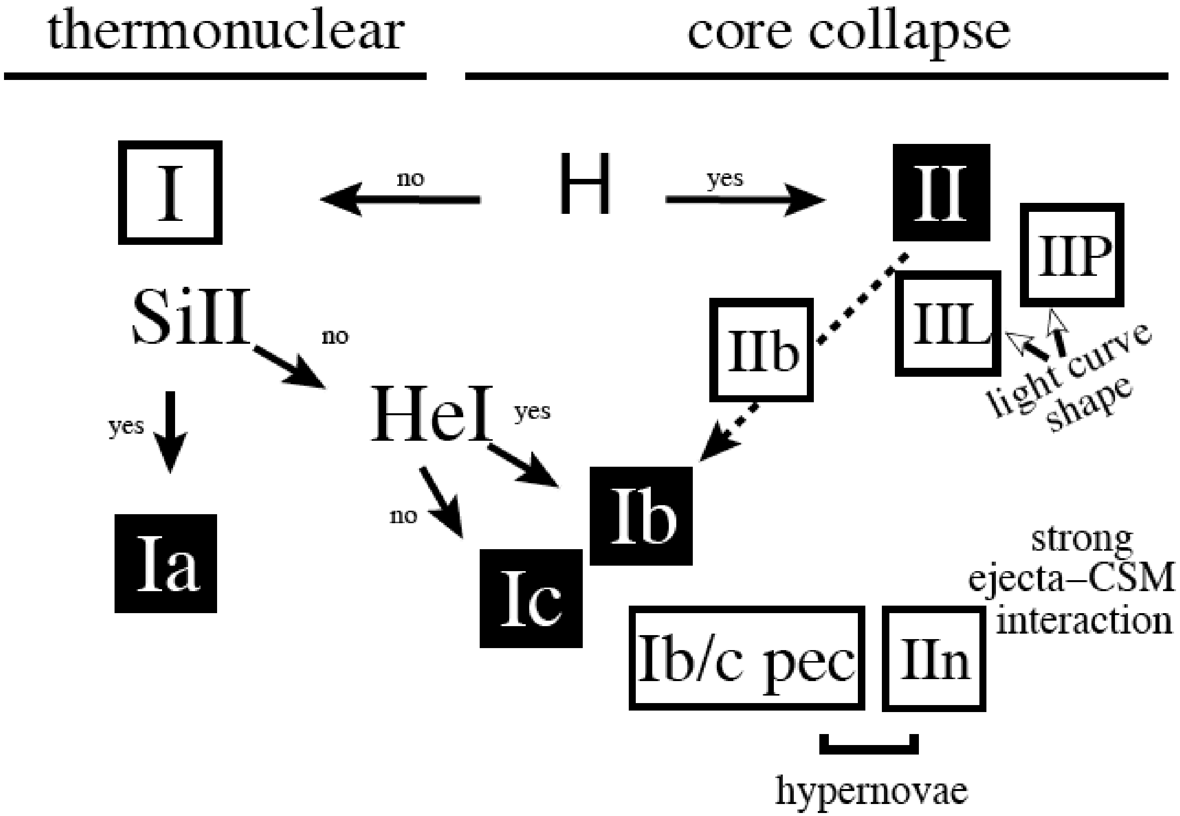
Evolution as a function of mass



Supernovae classification

- Taxonomical/Morphological approach

Like botanists and zoologists, find observable characteristics that eventually provide a deeper physical understanding. However, not all necessarily meaning full...



Tarutto 2003

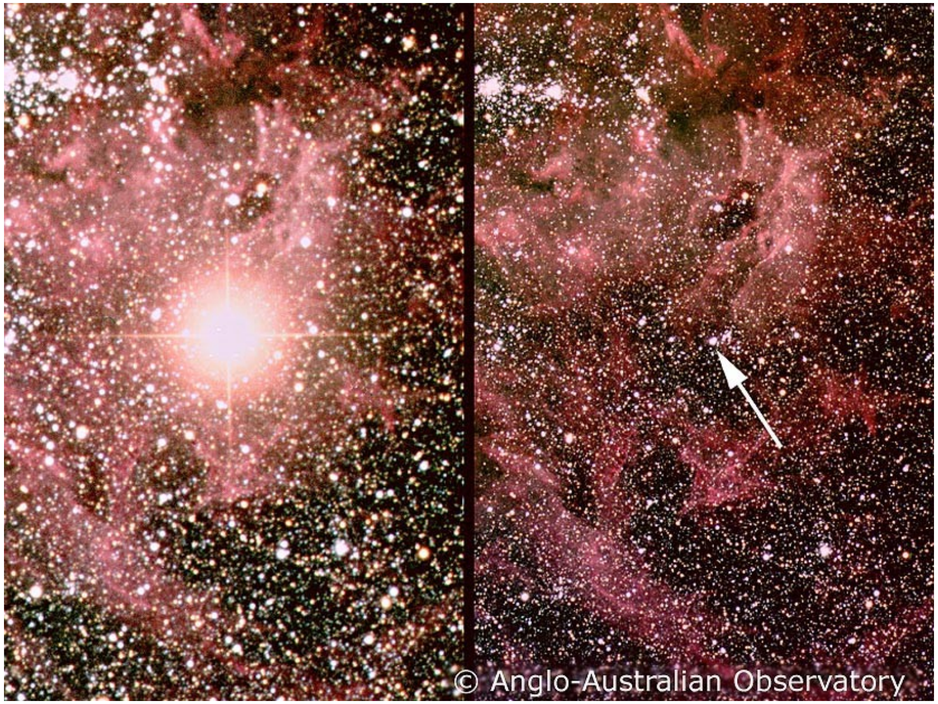
Core-collapse supernova

- Huge energy scales
  - ~1e+53 erg neutrinos
  - ~1e+51 erg mechanical
  - ~1e+48 erg elm
  - ~1e+41 erg visible elm
- Observables
  - Elm
  - Neutrinos
  - Gravitational waves

$1\ \mathrm{erg} = 10^{-7}\mathrm{J}$

World Energy Consumption ~ 1e+27 erg/yr
Sun ~ 1e+41 erg/yr

SN1987A



Australian Astronomical Observatory

After

Before

$\}$ Neutrino & GW astronomy!!!

SN1987A in the Large Magellanic Cloud (a nearby galaxy) ~ 163'000 light-years away!
~1.5e+21 km!

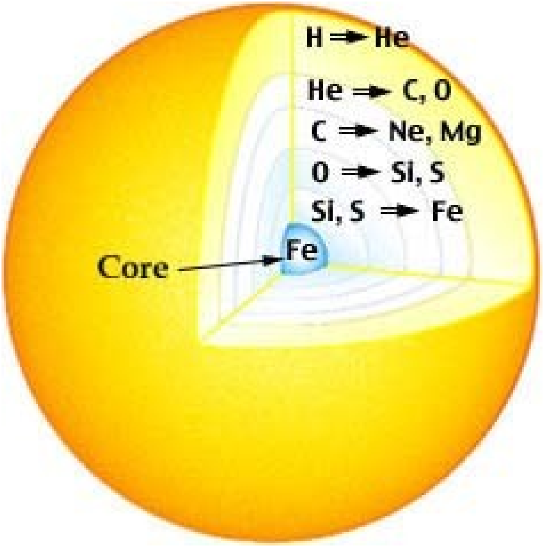
Core-collapse supernova

- General idea:
  - Implosion of iron core of massive star $M \gtrsim 8 M_\odot$ at the end of thermonuclear evolution
  - Explosion powered by gravitational binding energy of forming compact remnant:

$$E_b \approx 3 \times 10^{53} \left(\frac{M}{M_\odot} \right)^2 \left(\frac{R}{10\mathrm{km}} \right)^{-1} \mathrm{erg}$$

M Mass of remnant

R Radius of remnan



GRAVITY BOMB!

Conditions at onset of collapse

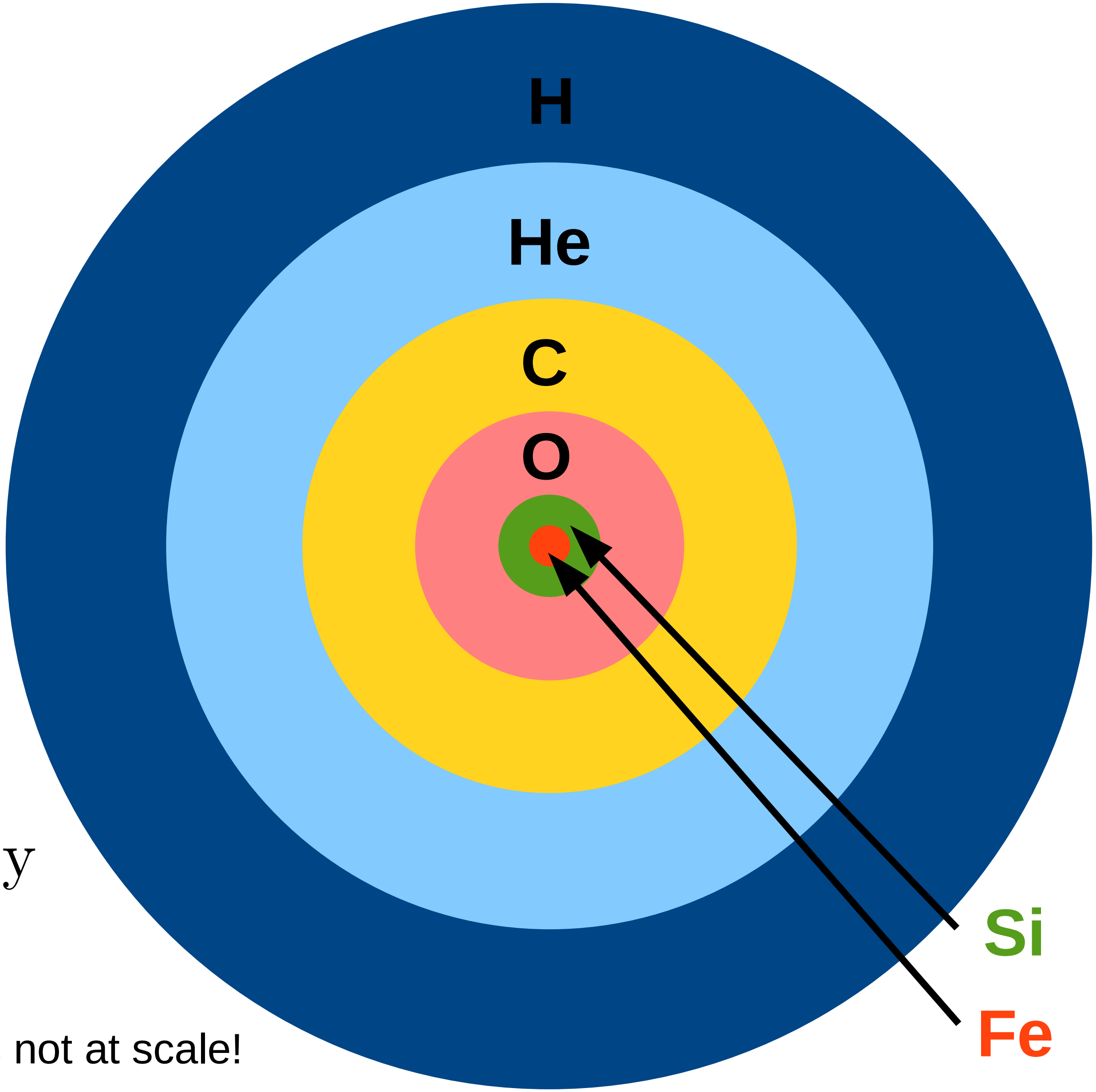
Evolved massive star
prior to collapse

Onion-like structure
due to nuclear burning
stages

E.g.: Mass ~ $15M_\odot$

Age ~ 2×10^6 y

Size ~ 10^8 km



Shells not at scale!

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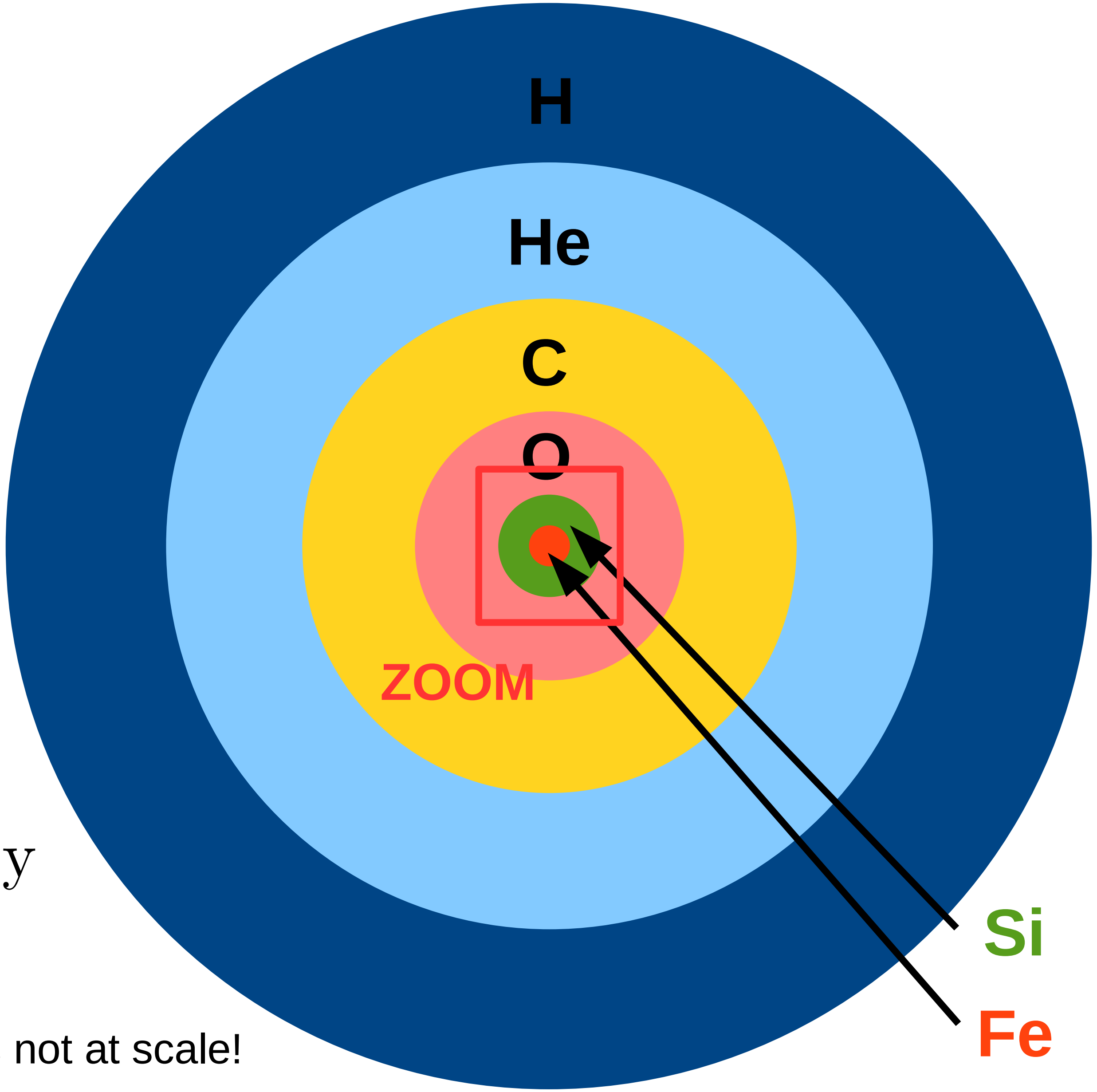


Shells not at scale!

Conditions at onset of collapse

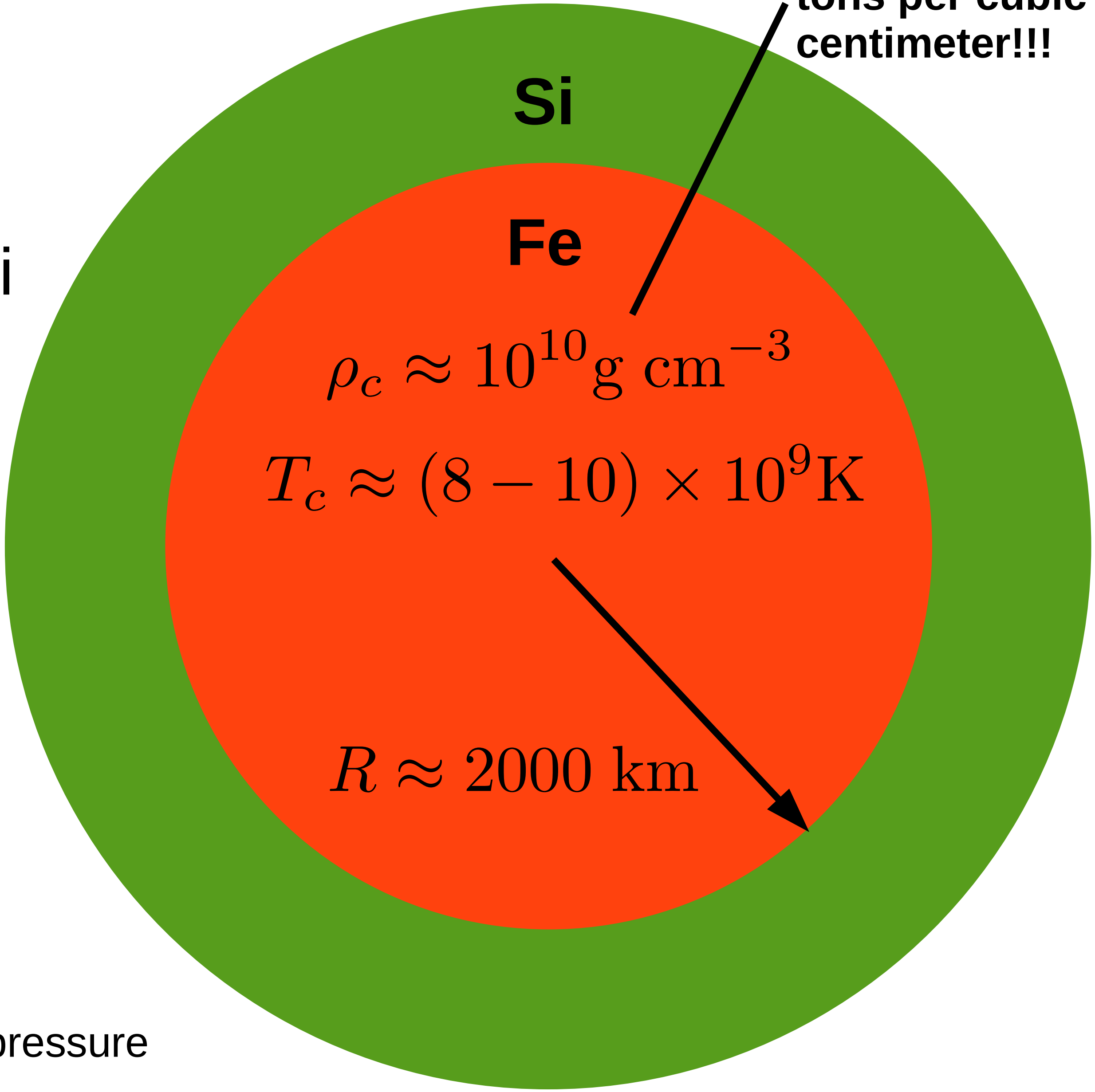
Core made of ashes from Silicone burning...
Mainly iron group nuclei

➡ **IRON CORE**

Iron core stabilized against gravity by relativistic and degenerate electrons

$$P_e \gg P_{ion} \gg P_{rad}$$

Electron Ion Photon pressure



$$\rho_c \approx 10^{10}\,\mathrm{g\,cm^{-3}}$$

$$T_c \approx (8-10)\times 10^9\,\mathrm{K}$$

$$R \approx 2000\,\mathrm{km}$$

Ten thousand tons per cubic centimeter!!!

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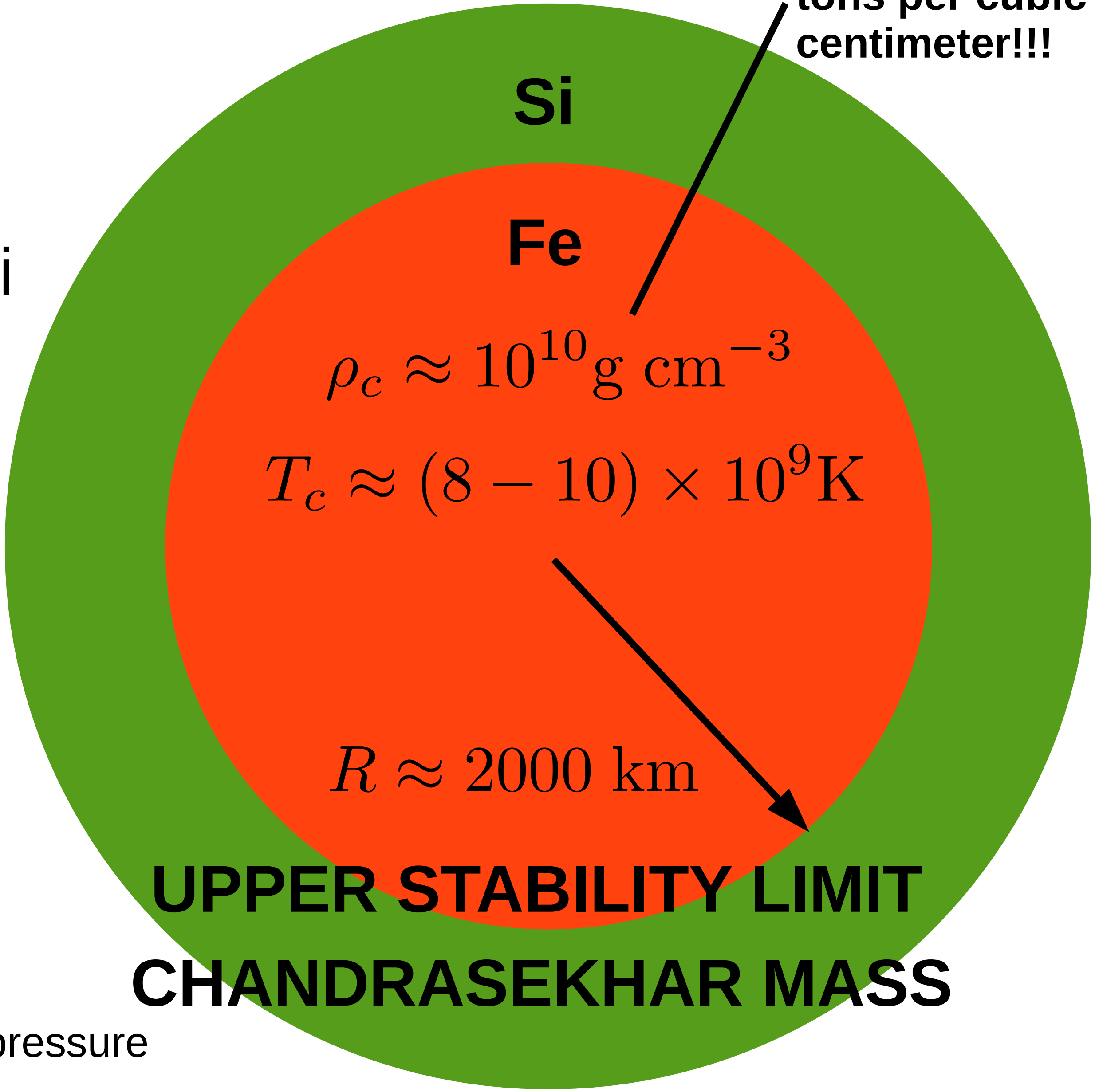


Ten thousand tons per cubic centimeter!!!

Si

Fe

$$\rho_c \approx 10^{10}\,\mathrm{g\ cm^{-3}}$$

$$T_c \approx (8-10) \times 10^{9}\,\mathrm{K}$$

$$R \approx 2000\ \mathrm{km}$$

UPPER STABILITY LIMIT

CHANDRASEKHAR MASS

Stability of self-gravitating hydrostatic equilibrium

polytropic exponent

Polytropic EoS: $P \propto \rho^{\gamma}$

Average grav. force: $F_G \propto \bar{\rho}\dfrac{GM}{R^2} \propto \dfrac{M}{R^3}\dfrac{GM}{R^2} \propto R^{-5}$

Av. pressure gradient: $\dfrac{\mathrm{d}\bar{P}}{\mathrm{d}r} \propto \dfrac{\bar{P}}{R} \propto \dfrac{\bar{\rho}^{\gamma}}{R} \propto R^{-3\gamma-1}$

R Radius

M Mass

$\bar{\rho} = \dfrac{M}{R^3}$ Average density

$\bar{p}$ Average pressure

➡ $$\frac{\mathrm{d}P/\mathrm{d}r}{F_G} \propto R^{-3(\gamma-4/3)}$$

Poly

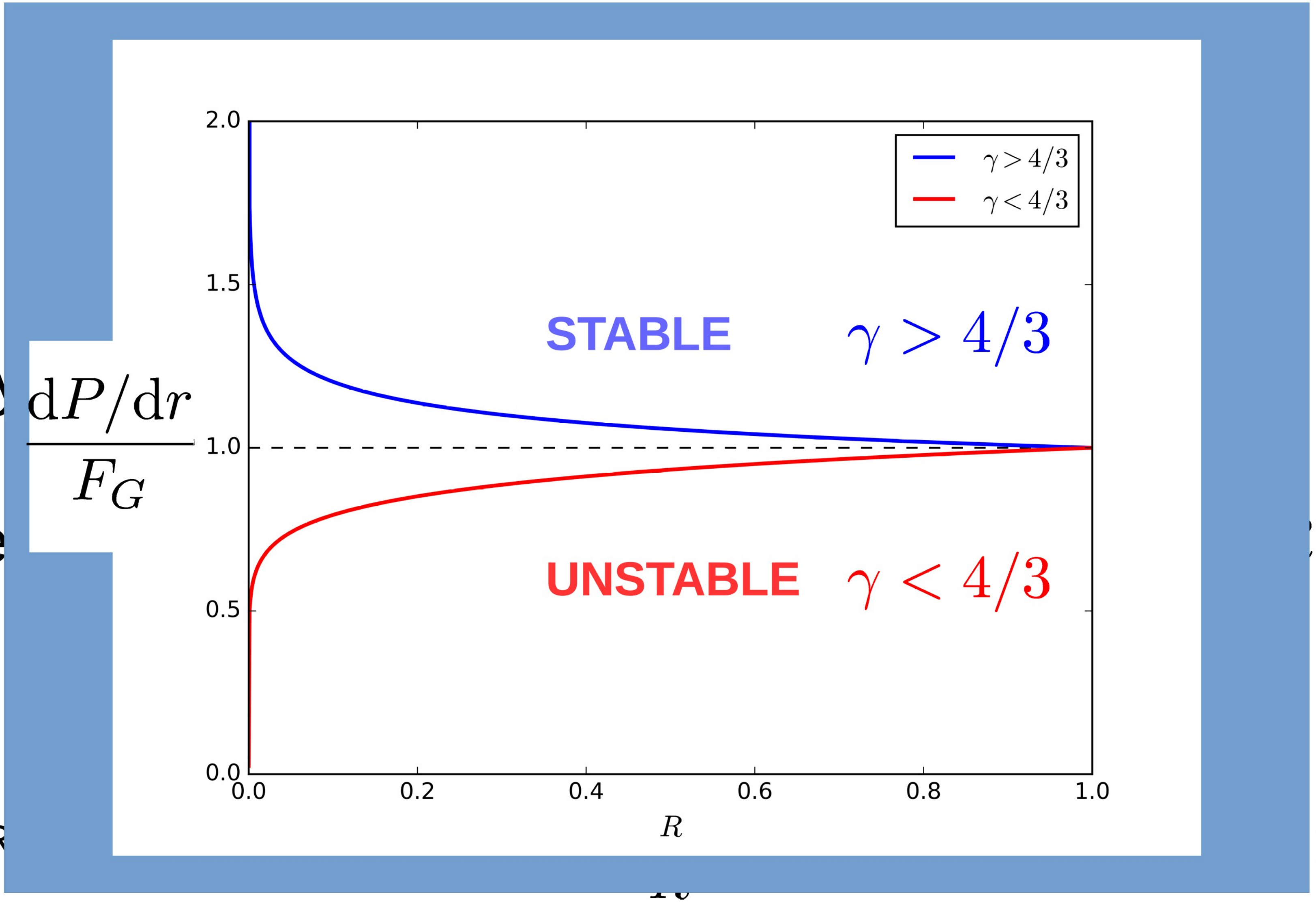
Ave

$^{-5}$

Av.

R R

pressure



R

$$\frac{\mathrm{d}P/\mathrm{d}r}{F_G} \propto R^{-3(\gamma-4/3)}$$

Collapse

Iron core collapse due:

① Mass grows due to accreting Si burning ashes ultimately reaching

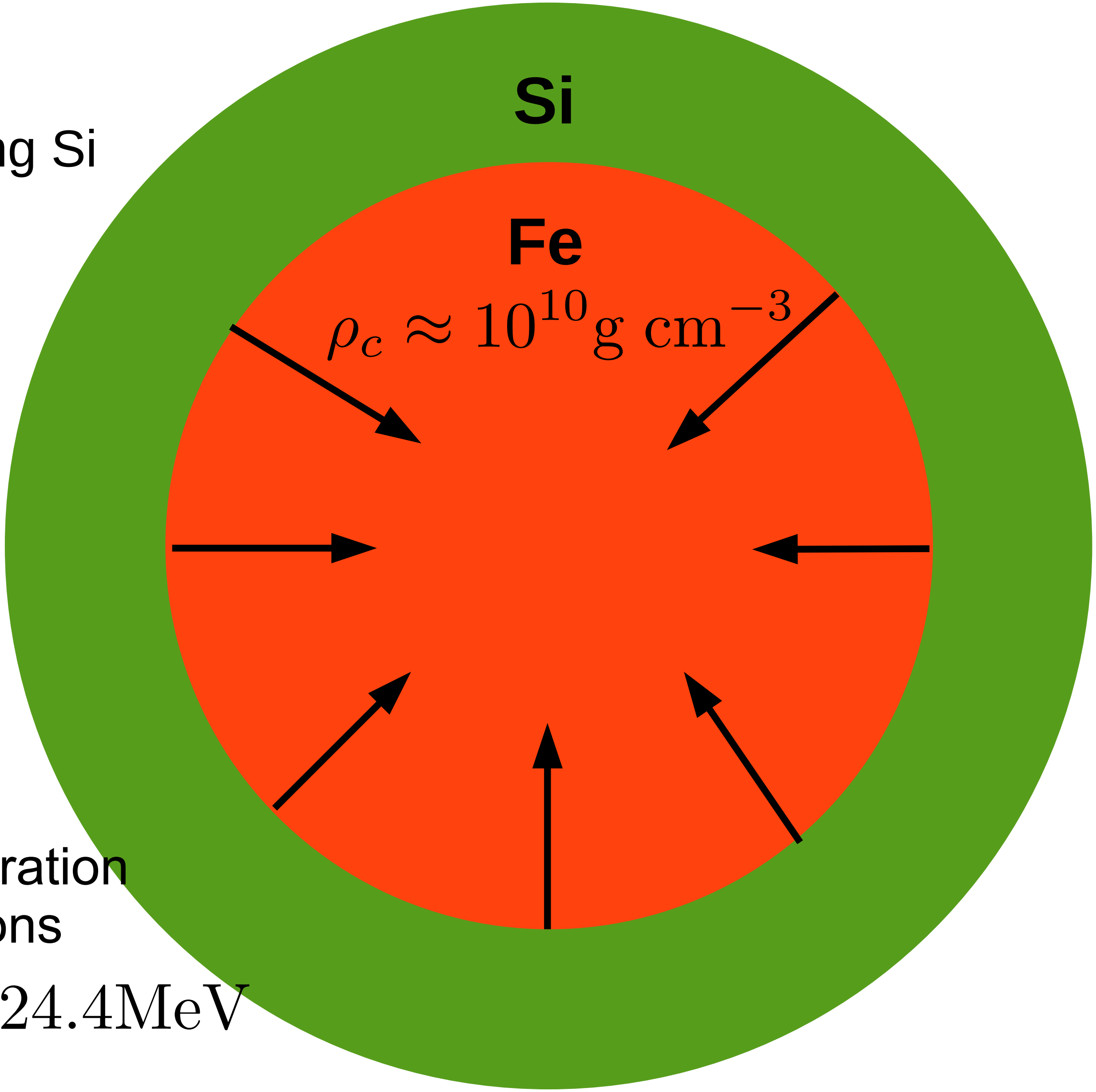
$$M_{Ch} = 1.457(2Y_e)^2 M_\odot$$

② Electron captures reduce lepton number and neutrinos escape freely

$$^{56}Fe + e^- \rightarrow {}^{56}Mn + \nu_e$$

③ Pressure reduced due to endothermic photo-disintegration of nuclei by energetic photons

$$\gamma + {}^{56}Fe \rightarrow 13\alpha + 4n \; - 124.4\mathrm{MeV}$$



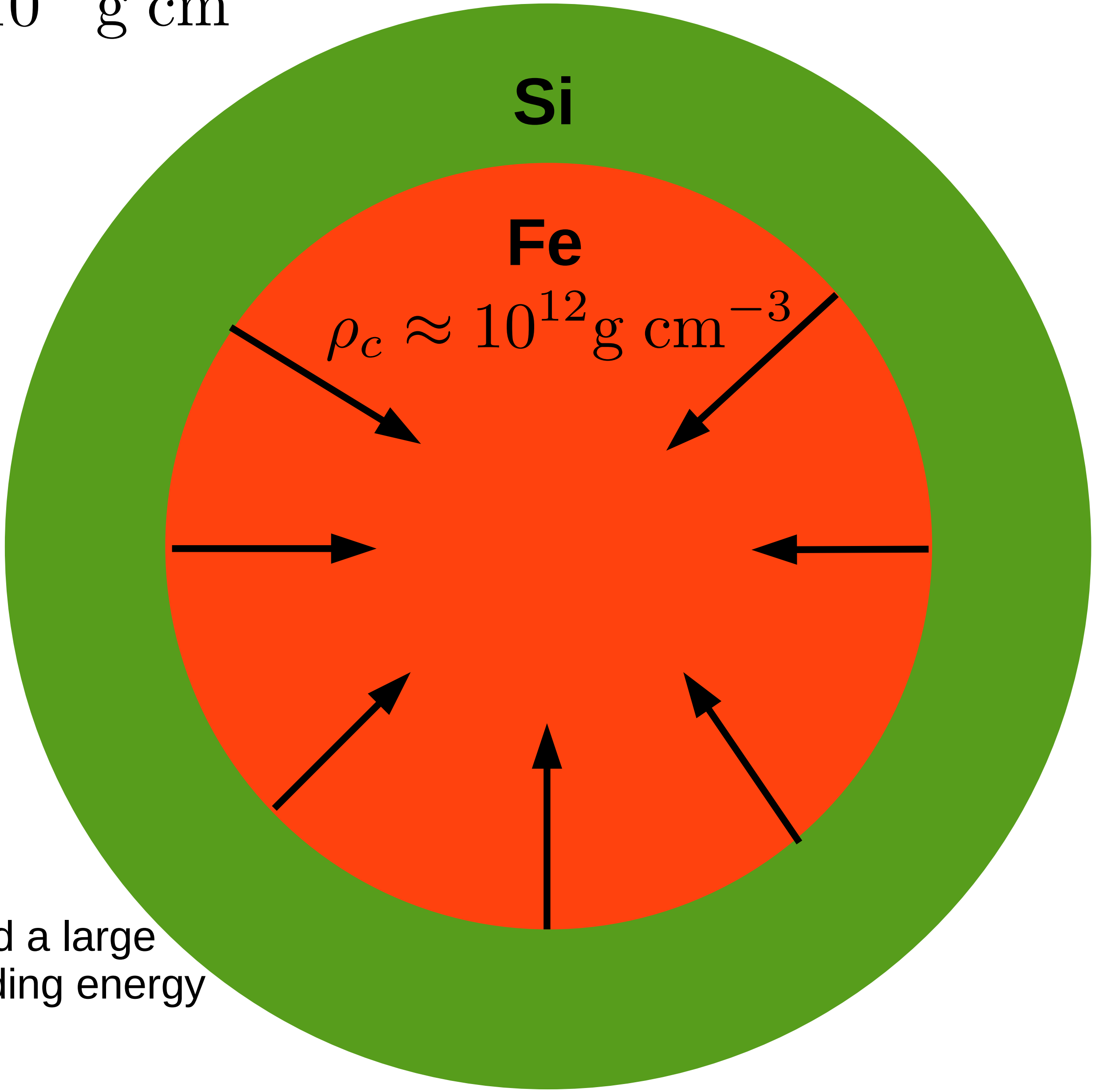
Collapse: trapping

Neutrino trapping: $\rho \gtrsim 10^{12}\mathrm{g\ cm}^{-3}$

$$\tau_{diff} > \tau_{dyn}$$

The outward "diffusion speed" equals or is less than the infall speed

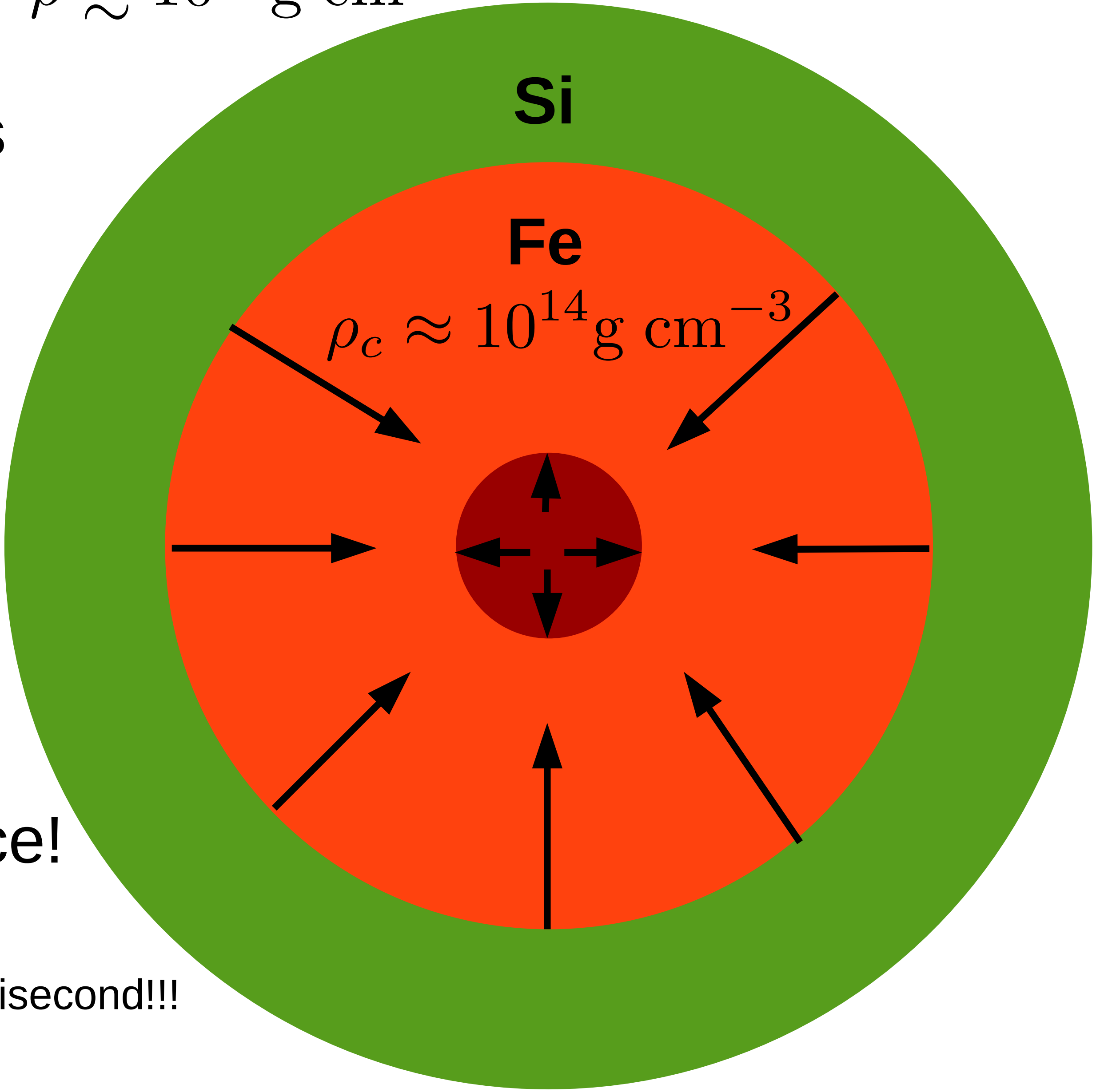
➡ The collapse proceeds adiabatically



Neutrinos as still copiously produced and a large fraction of the liberated gravitational binding energy is accumulated in the neutrinos

Bounce

At densities exceeding $\rho \gtrsim 10^{14}\mathrm{g\ cm}^{-3}$ the nuclei phase transition into nucleons that at such high densities experience strong nuclear repulsion



→ This halts and reverses the collapse: bounce!

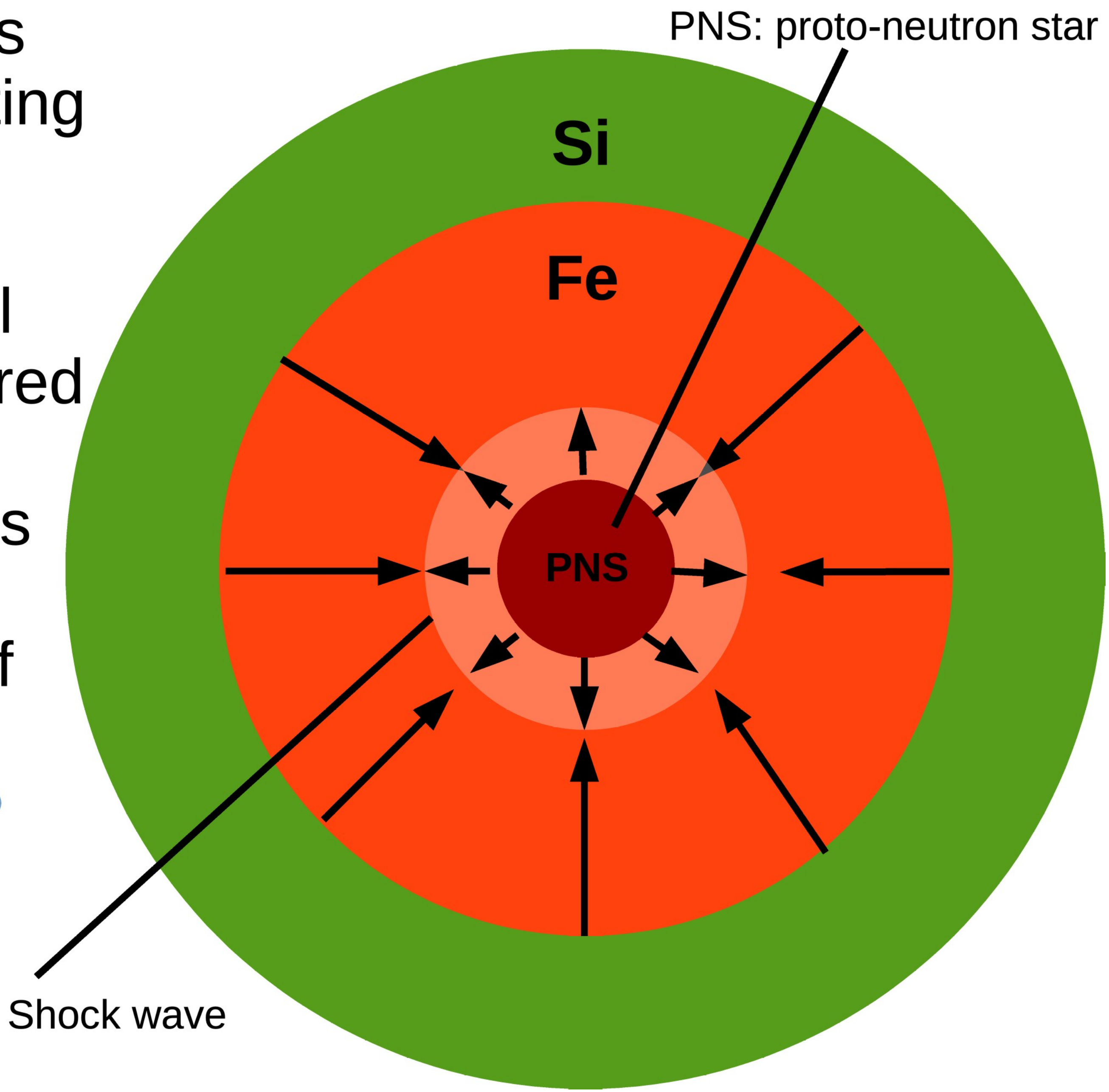
This happens within less than a millisecond!!!

Shock

The bounce launches an outward propagating strong shock wave

The mechanical energy transferred to the shock during bounce is enough to blow away the rest of the star!!!

SOLVED THE CCSN PROBLEM



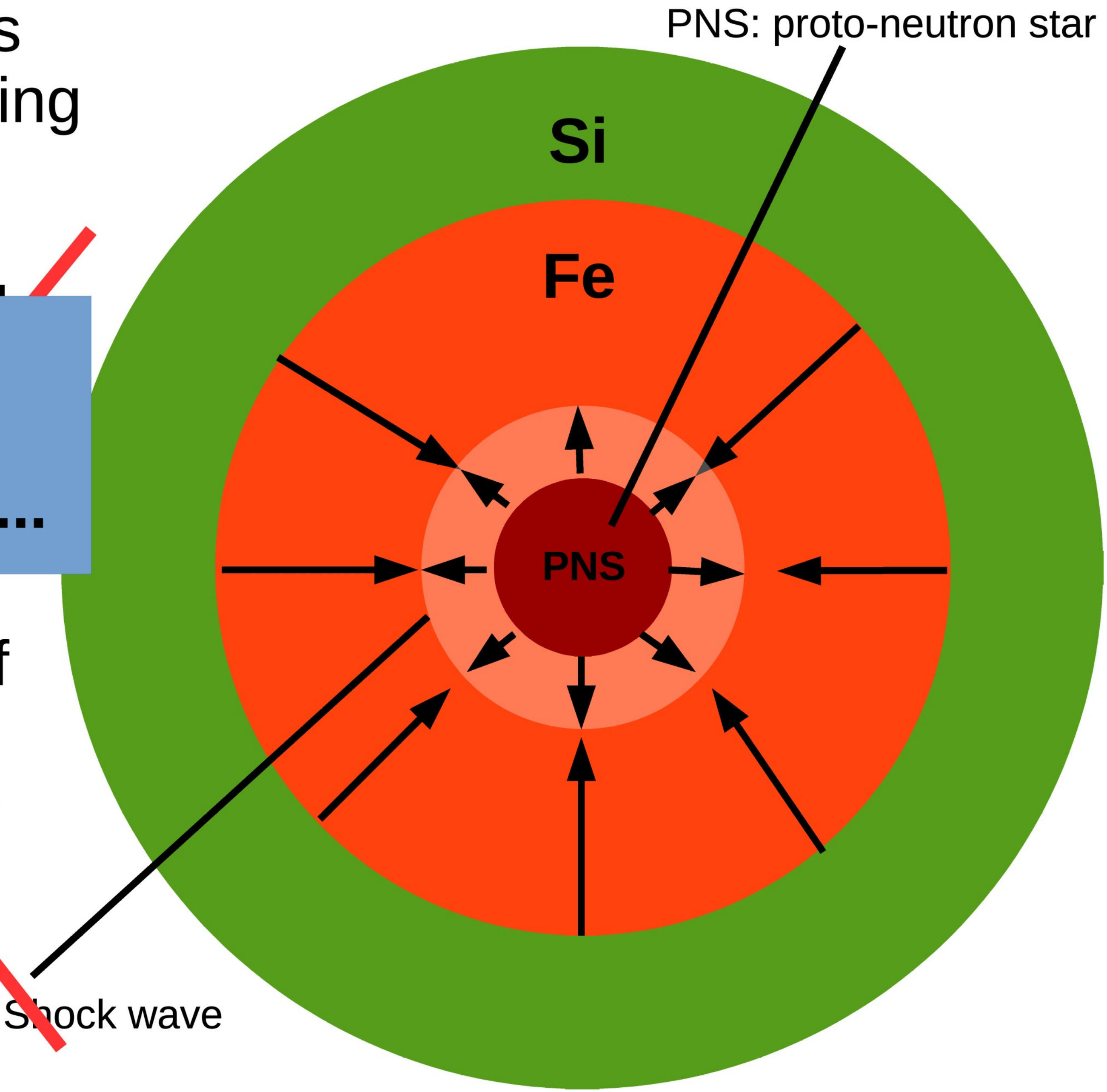
Shock

The bounce launches an outward propagating strong shock wave

The so-called prompt mechanism fails...

~~enough to blow away the rest of the star!!!~~

~~SOLVED THE CCSN PROBLEM~~



Shock stalling

The shock stalls due to:

1. Shock looses energy due to dissociation of iron nuclei

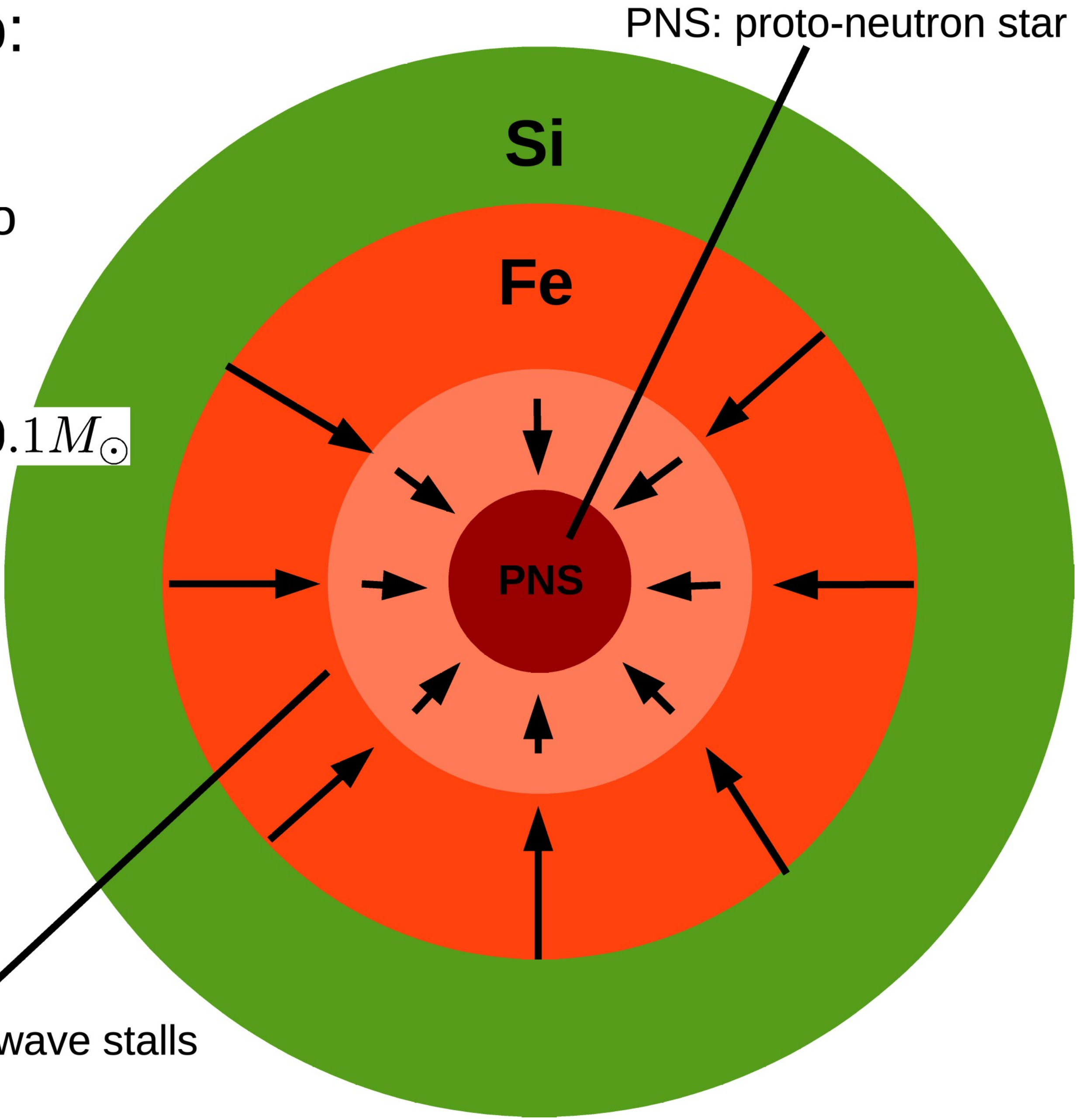
~8.8 MeV/nucleon

$$\sim (1.6 - 1.8) \times 10^{51} \mathrm{erg}/0.1 M_{\odot}$$

2. Neutrino losses due to electron captures on free protons

$$e^{-} + p \rightarrow n + \nu_e$$

Neutrino burst!!!



Shock “revival”

The stalled shock wave must regain energy to start the expansion against the ram pressure of the in-falling matter and finally blow away the outer mantle!

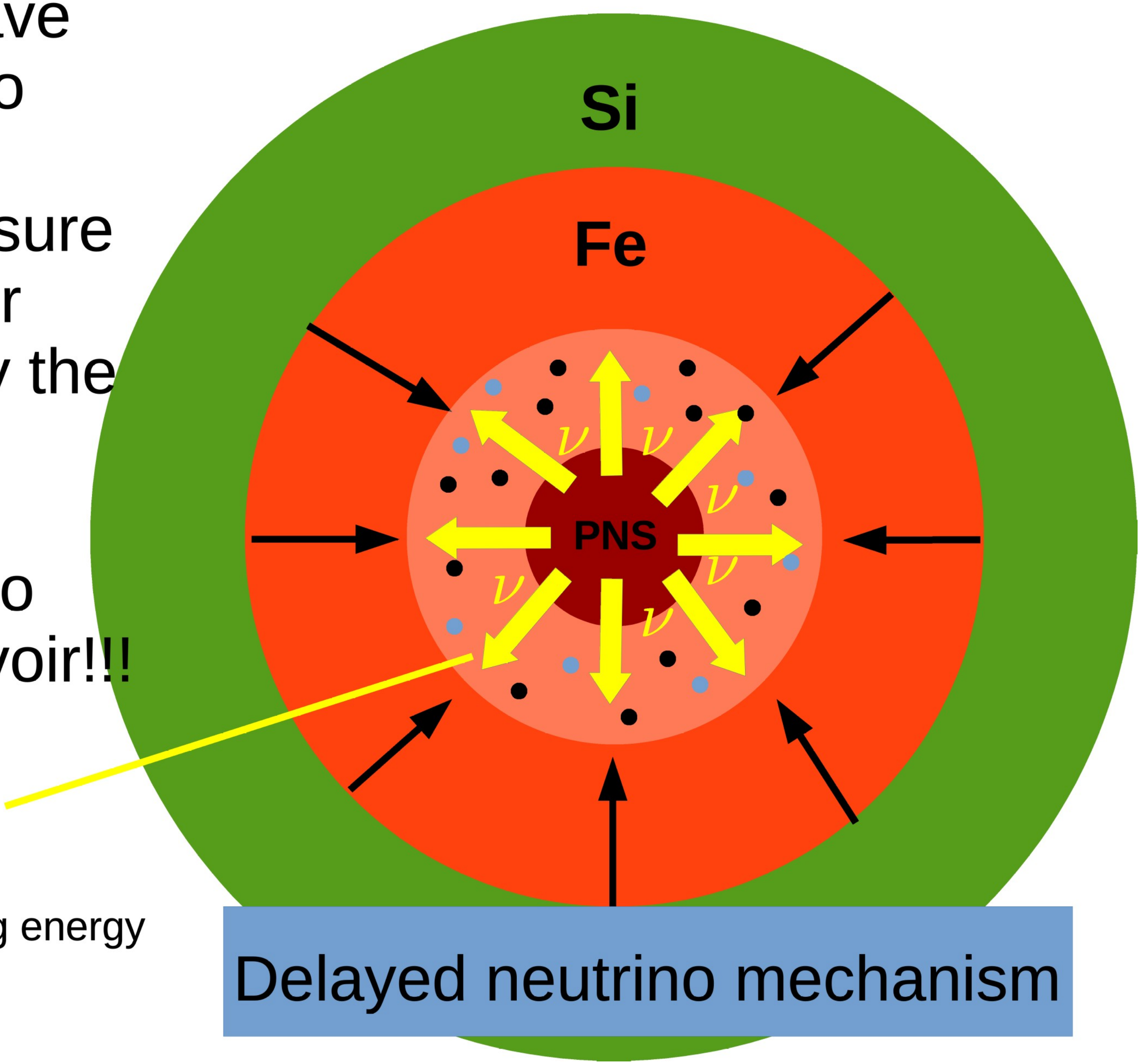
Tap the neutrino “energy” reservoir!!!

$$n + \nu_e \rightarrow e^- + p$$

$$p + \overline{\nu}_e \rightarrow e^+ + n$$

From the gravitational binding energy

● Neutrons ● Protons



Delayed neutrino mechanism

Shock “revival”

The stalled shock wave must regain energy to start the expansion against the ram pressure

The problem:

Find a way to explode robustly massive stars consistent with the available direct and indirect observables

$$n + \nu_e \to e^- + p$$

$$p + \overline{\nu}_e \to e^+ + n$$

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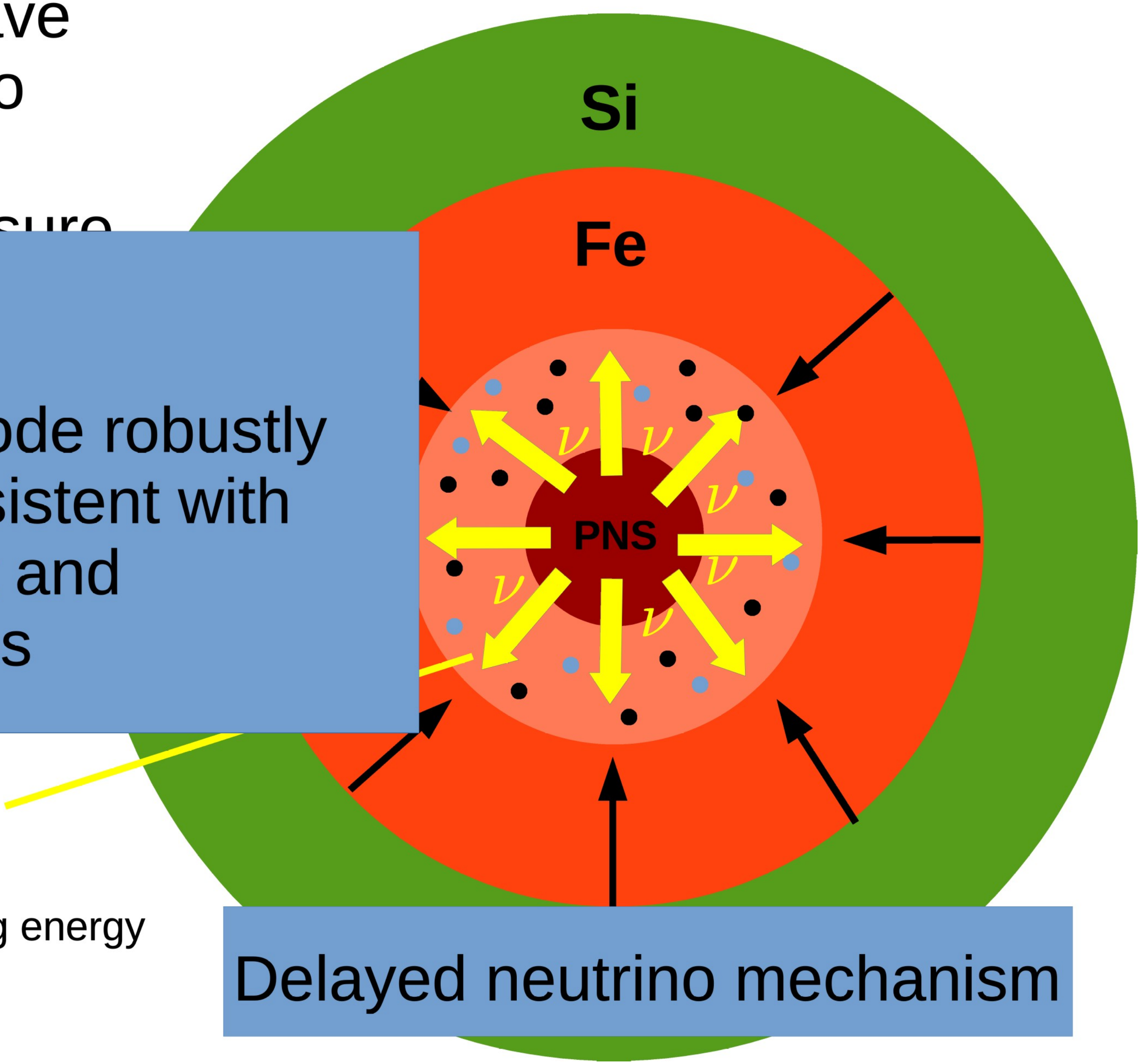

Delayed neutrino mechanism

CCSN Explosion Mechanism?

- Discussed explosion mechanisms:
  - “Enhanced” neutrino-driven explosion mechanism

    Hydro. instabilities: convection, Standing Accretion Shock Instabilities (SASI) e.g. Blondin et al. 2003, Blondin & Shaw 2007, Foglizzo et al. 2008, Iwakami et al. 2008, Marek & Janka 2009, Suwa et al. 2010, 2012, Takiwaki et al. 2013, Bruenn et al. 2013...
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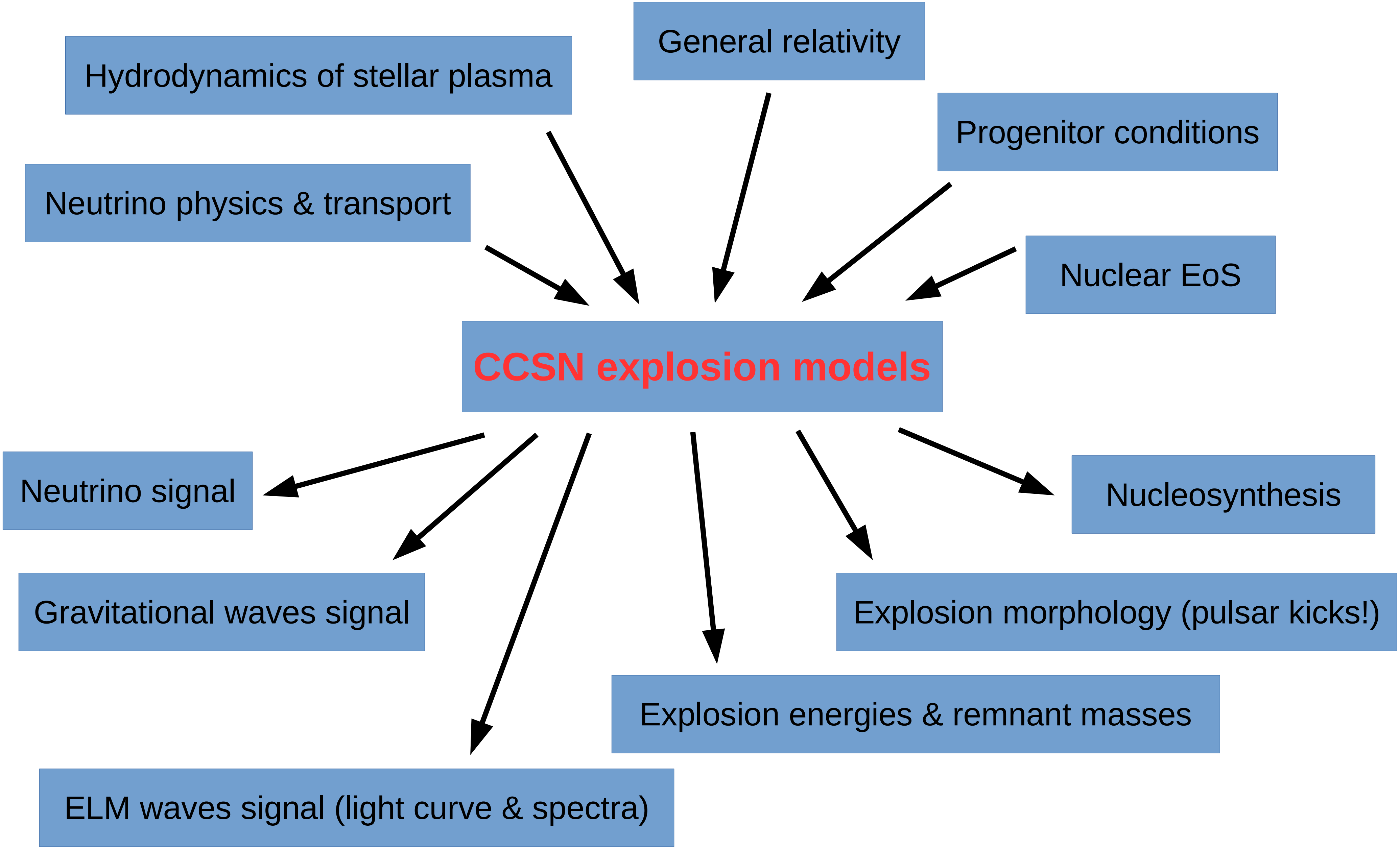
The overall challenge



Hydrodynamics of stellar plasma

General relativity

Progenitor conditions

Neutrino physics & transport

Nuclear EoS

CCSN explosion models

Neutrino signal

Nucleosynthesis

Gravitational waves signal

Explosion morphology (pulsar kicks!)

Explosion energies & remnant masses

ELM waves signal (light curve & spectra)

CCSN model

Model's ingredients wish list:

1)Multi-D hydro.	It's a 3D problem
2)Plasma physics	Stars have magnetic fields, e.g. Sun, ... pulsars & magnetars!
3)Weak interactions	Most of the released gravitational binding energy “available” in form of neutrinos!
4)Neutrino transport	
5)Nuclear physics	Equation of state describing matter at extreme conditions
6)General relativity	Very compact and very massive objects!
7)“Accurate” initial conditions	

🟩 **DOABLE**

🟧 **DIFFICULT**

🟥 **VERY DIFFICULT**

CCSN model: 1D (spherical symm.)

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NO explosions for $M > 11M_\odot$

Thompson et al. 2003, Rampp & Janka 2002, Liebendörfer et al. 2002/2005

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CCSN model: 2D (axisymmetry)

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CCSN model: 3D

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Enhanced-nu-CCSN model 3D

Actual model's ingredients list:

1) Multi-D hydro.
2) Plasma physics
3) Weak interactions
4) Neutrino transport
5) Nuclear physics
6) General relativity
7) “Accurate” initial conditions

Assume infinite conductivity

Parallel 3D ideal MHD code

IDSA Liebendörfer et al. 2009

EoS e.g. Lattimer & Swesty 1991, Shen et al. 1998, Hempel et al. 2011

Spherical effective GR potential Marek et al. 2006
+
2D axisymmetric Newton potential

The Radiation-MHD equations (2)

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_i}{\partial x_i} = 0$$

Mass

$$\frac{\partial (\rho v_i)}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho v_i v_j + P_* I_{ij} - b_i b_j\right) = -\rho \frac{\partial \phi}{\partial x_i} + \left(\rho \dot{v}_i\right)_\nu$$

Momentum

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_j}\left[\left(E + P_*\right) v_j - v_i b_i b_j\right] = -\rho v_i \frac{\partial \phi}{\partial x_i} + \left(\rho \dot{e}\right)_\nu$$

Energy

$$\frac{\partial \rho Y_e}{\partial t} + \frac{\partial Y_e \rho v_i}{\partial x_i} = \left(\rho \dot{Y}_e\right)_\nu$$

Electron #

$$\left(\boldsymbol{b} = \frac{\boldsymbol{B}}{\sqrt{4\pi}}\right)$$

Magnetic flux $$\frac{\partial b_i}{\partial t} = \frac{\partial}{\partial x_j}\left(v_i b_j - v_j b_i\right)$$

No monopoles $$\boldsymbol{\nabla} \cdot \boldsymbol{b} = 0$$

$$E = \frac{1}{2}\rho v^2 + \rho e + \frac{b^2}{2}$$

$$P_* = p + \frac{b^2}{2}$$

EoS: $p = p(\rho, e, \ldots)$

Solution Algorithm: MHD

- MHD (FISH) Käppeli et al. 2011
  - Split hydro. and magnetic variables update
  - Dimensional splitting: solves eqs in 1D
  - Uses dim.-split constrained transport for $\boldsymbol{\nabla} \cdot \boldsymbol{b} = 0$
  - Well-balanced scheme for hydrostatic equilibrium

    $\Rightarrow$ Preserves discrete HSE exactly by reconstruction in equilibrium variables

Käppeli & Mishra 2014, 2016, Käppeli 2017,
Grosheintz-Laval & Käppeli 2019, 2020

  - Hybrid MPI/OpenMP parallelisation for distributed/shared memory architectures

Solution Algorithm: Neutrinos

- In principle, should solve the relativistic Boltzmann eq

4-vector

4-momentum

Distribution function

$$\left(\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\right)\frac{\partial f}{\partial x^\alpha} + \left(\frac{\mathrm{d}p^\alpha}{\mathrm{d}\tau}\right)\frac{\partial f}{\partial p^\alpha} = \left(\frac{\delta f}{\delta \tau}\right)_{\mathrm{coll}}$$

e.g. Cercignani & Kremer 2002

Proper time

Absorption, emission, scattering in medium

Full transfer NOT feasible in 3D (3+3+1=7 dim. problem!)

- Approximations:
  - Parametrisation scheme: only collapse phase Liebendörfer 2005
  - Spectral Leakage scheme: only cooling Perego et al. in prep.

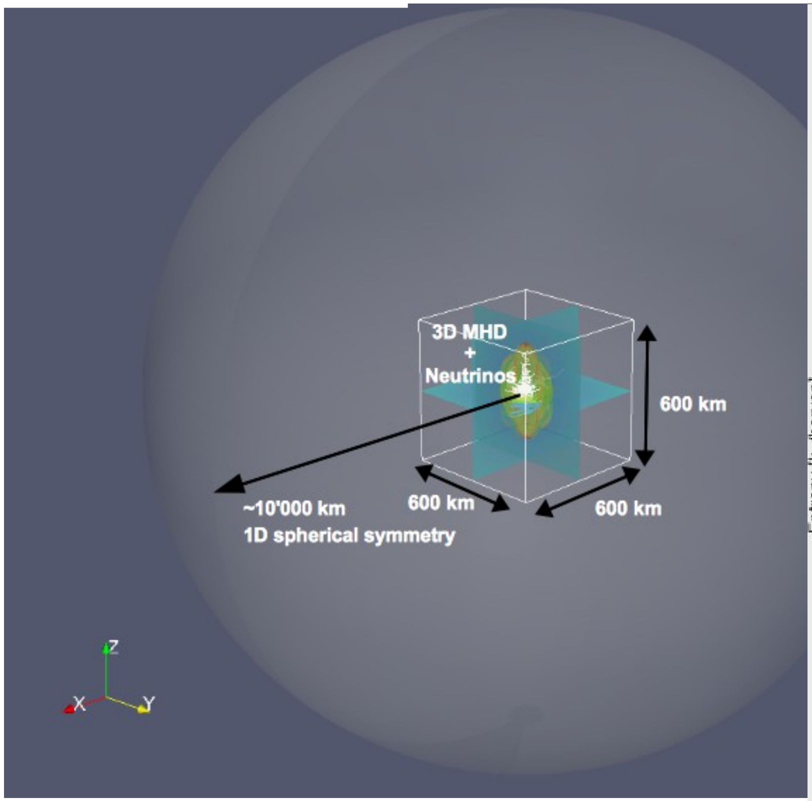
    Epstein & Pethick 1981, van Riper & Lattimer 1981, ..., Ruffert et al. 1996, Rosswog & Liebendörfer 2003
  - Isotropic Diffusion Source Approx. (IDSA) Liebendörfer et al. 2009

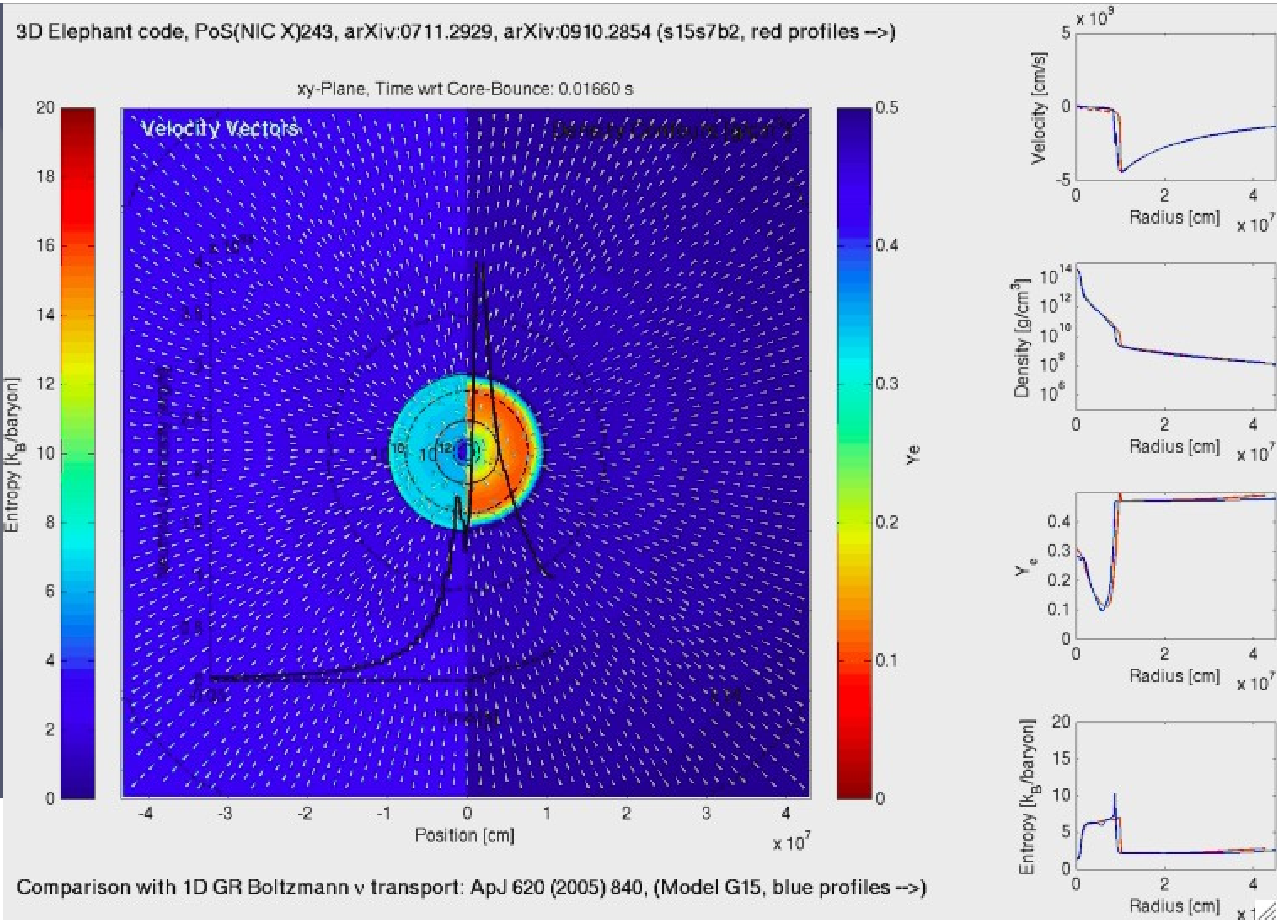
Computational cost

Enhanced-nu-CCSN model 3D

Computations @ CSCS Swiss National Supercomputing Centre

By M. Liebendoerfer





3D Elephant code, PoS(NIC X)243, arXiv:0711.2929, arXiv:0910.2854 (s15s7b2, red profiles -->)

Comparison with 1D GR Boltzmann ν transport: ApJ 620 (2005) 840, (Model G15, blue profiles -->)

MOVIE!

Outline

- **Introduction**
  - A brief introduction of the problem
- **Physical model, numerical & computational aspects**
  - Physics ingredients & mathematical model
- **Simulation of magneto-rotational core-collapse**
  - MHD CCSN mechanism
  - Magnetic field amplification, formation and driving mechanism of bipolar outflow
  - Explosion energy, ejected mass and its composition
- **More on numerical & computational aspects**
  - Well-balanced methods for hydrostatic equilibrium
- **Conclusion**

MHD-CCSN model

Actual model's ingredients list:

1) Multi-D hydro. →
2) Plasma physics →

Parallel 3D ideal MHD code

Assume infinite conductivity

3) Weak interactions →
4) Neutrino transport →

Spectral leakage scheme

developed by A. Perego

Rosswog & Liebendörfer 2003

“Not so bad”... 2D simulations shown that ν contribute only 10-25% to explosion energy

5) Nuclear physics →

EoS e.g. Lattimer & Swesty 1991, Shen et al. 1998, Hempel et al. 2011

6) General relativity →

Spherical effective GR potential Marek et al. 2006

+

2D axisymmetric Newton potential

7) “Accurate” initial conditions

The Radiation-MHD equations (2)

$$\frac{\partial\rho}{\partial t} + \frac{\partial\rho v_i}{\partial x_i} = 0$$

Mass

$$\frac{\partial(\rho v_i)}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho v_i v_j + P_* I_{ij} - b_i b_j\right) = -\rho\frac{\partial\phi}{\partial x_i} + \left(\dot{\rho v_i}\right)_\nu$$

Momentum

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_j}\left[\left(E + P_*\right)v_j - v_i b_i b_j\right] = -\rho v_i\frac{\partial\phi}{\partial x_i} + \left(\dot{\rho e}\right)_\nu$$

Energy

$$\frac{\partial\rho Y_e}{\partial t} + \frac{\partial Y_e\rho v_i}{\partial x_i} = \left(\rho\dot{Y}_e\right)_\nu$$

Electron #

$$\left(\boldsymbol{b} = \frac{\boldsymbol{B}}{\sqrt{4\pi}}\right)$$

Magnetic flux

$$\frac{\partial b_i}{\partial t} = \frac{\partial}{\partial x_j}\left(v_i b_j - v_j b_i\right)$$

No monopoles

$$\boldsymbol{\nabla}\cdot\boldsymbol{b} = 0$$

$$E = \frac{1}{2}\rho v^2 + \rho e + \frac{b^2}{2}$$

$$P_* = p + \frac{b^2}{2}$$

EoS: $p = p(\rho, e, \ldots)$

Solution Algorithm: MHD

- MHD (FISH) Käppeli et al. 2011
  - Split hydro. and magnetic variables update
  - Dimensional splitting: solves eqs in 1D
  - Uses dim.-split constrained transport for $\boldsymbol{\nabla} \cdot \boldsymbol{b} = 0$
  - Well-balanced scheme for hydrostatic equilibrium
    - $\Rightarrow$ Preserves discrete HSE exactly by reconstruction in equilibrium variables

Käppeli & Mishra 2014, 2016, Käppeli 2017,
Grosheintz-Laval & Käppeli 2019, 2020

  - Hybrid MPI/OpenMP parallelisation for distributed/shared memory architectures

Solution Algorithm: Neutrinos

- In principle, should solve the relativistic Boltzmann eq

4-vector

4-momentum

Distribution function

$$\left(\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\right)\frac{\partial f}{\partial x^\alpha} + \left(\frac{\mathrm{d}p^\alpha}{\mathrm{d}\tau}\right)\frac{\partial f}{\partial p^\alpha} = \left(\frac{\delta f}{\delta \tau}\right)_{\mathrm{coll}}$$

e.g. Cercignani & Kremer 2002

Absorption, emission, scattering in medium

Full transfer NOT feasible in 3D (3+3+1=7 dim. problem!)

- Approximations:
  - Parametrisation scheme: only collapse phase Liebendörfer 2005
  - Spectral Leakage scheme: only cooling Perego et al. in prep.

    Epstein & Pethick 1981, van Riper & Lattimer 1981, ..., Ruffert et al. 1996, Rosswog & Liebendörfer 2003
  - Isotropic Diffusion Source Approx. (IDSA) Liebendörfer et al. 2009

Computational cost

Role of Rotation & Magnetic Field

Pre-collapse

- Rotation (???)
- B (???)

Distribution
in Fe core ???

Observations: e.g.
Thompson et al. 2003
Donati & Landstreet 2009

Stellar evolution models:
Heger et al. 2005
Hirschi et al. 2004 & 2005

Successful
Explosion...

Post-collapse

- Pulsar
Magnetar

Rotation (???)

B (???)

Taylor et al. 1993
Kouveliotou et al. 1998
Mereghetti 2008

- Observable
Asymmetries

Wang & Wheeler 2008
Kjaer et al. 2010

Role of Rotatio & M netic Field

Pre-colla

-
- B (

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Thomps

Donat

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Heger et al. 200

Hirschi et al. 200

etar

n (???)

(???)

al. 1993

u et al. 1998

Meregnetti 08

- Observable Asymmetries

Wang & Wheeler 2008

Kjaer et al. 2010

Rotation & Magnetic fields present before and after explosion!

Influence of Rotation & B on explosion???

If strong effects, is it common or only (very) rare?

MHD CCSN Mechanism

- Rotational energy of Proto-Neutron Star (PNS)

$$T_{\rm rot} = \frac{1}{2} I_{\rm PNS} \Omega_{\rm PNS}^2$$
$$\approx 1 \times 10^{51}\ {\rm ergs} \times \left(\frac{M}{1.5 M_\odot}\right) \left(\frac{P}{2\ {\rm ms}}\right)^{-2} \left(\frac{R}{10\ {\rm km}}\right)^2$$

$\Longrightarrow$ Requires fast rotation!

- Idea: Extract “free” energy stored in differential rotation with
  - Viscosity Thompson et al. 2005
  - Magnetic Field

Typical CCSN explosion energy $E_{\rm expl} \sim 10^{51}$ erg

MHD CCSN Mechanism

- Rotational energy of Proto-Neutron Star (PNS)

“Free” energy in differential rotation?

$$E_{\mathrm{rot,free}} = T_{\mathrm{rot}}\left(L\right) - T_{\mathrm{rot,solid}}\left(L\right)$$

Angular momentum

Appreciable fraction of energy can be extracted by magnetic field and maybe trigger an explosion

Note: Structure assumed constant for the two realisations $\Longrightarrow$ Only approx.!

- I

al

Typical CCSN explosion energy $E_{\mathrm{expl}} \sim 10^{51}\ \mathrm{erg}$

Simulation of MHD CCSN

- Simulation parameters
  - L&S EoS K=180 MeV
  - Rotation laws:

1) Solid body $\Omega = \Omega_{0,c}$

2) Shellular $\Omega = \Omega_{0,c}\frac{R_0^2}{r^2+R_0^2}$

Popular in axisym. → 3) Cylindrical I $\Omega = \Omega_{0,c}\frac{X_0^2}{r^2\sin^2(\theta)+X_0^2}$

Popular in Japan → 4) Cylindrical II $\Omega = \Omega_{0,c}\frac{X_0^2}{r^2\sin^2(\theta)+X_0^2}\frac{Z_0^4}{r^4\cos^4(\theta)+Z_0^4}$

$r = \sqrt{x^2+y^2+z^2}$

$\cos(\theta) = z/r$

R_0, X_0, Z_0 Degree of diff. rotation

  - Magnetic field:

1) Uniform poloidal

2) Dipole-like poloidal

Simulation of MHD CCSN

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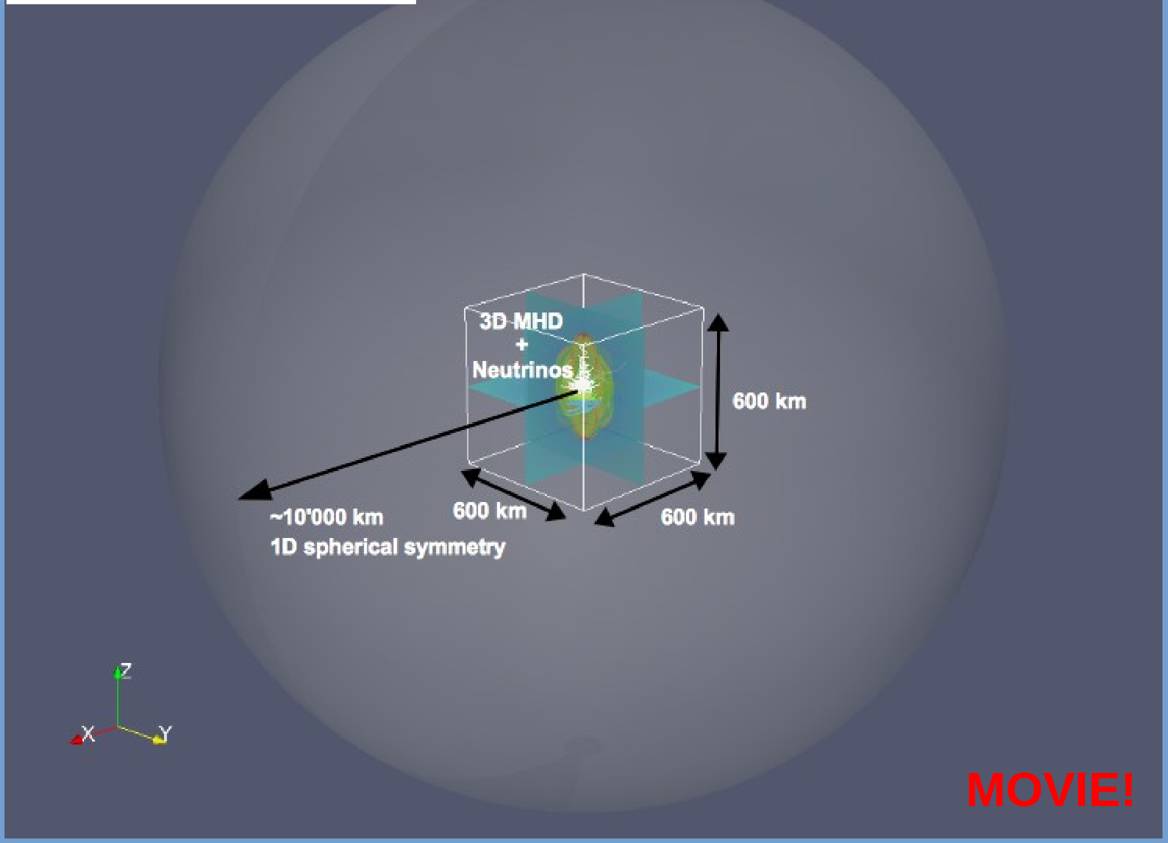
Simulation of MHD CCSN

Computations @ CSCS
Swiss National Supercomputing Centre



MOVIE!

Initial conditions:

$$\Omega_{0,c} = \pi \text{ rad/s}$$

$$B_{\text{pol}} = 5 \times 10^{12}$$

Constant 1 km resolution!

$+ z^2$

$+ Z_0^4$

ion

Magnetic field amplification

- Flux compression $B \propto \rho^{2/3}$ → Factor ≈ 1000

  Works well during collapse... Actually the main amplification

- Winding $\dot{B}_{\rm tor} \sim B_{\rm pol} \dfrac{\partial \Omega}{\partial \ln r}$

  With differential rotation, linear growth with time

- Magneto-Rotational Instability (MRI)

  With differential rotation, exponential growth with time, VERY small wavelengths...

- Dynamo?

Formation and driving mechanism of bipolar outflow

- Differential rotation winds poloidal magnetic field into toroidal field increasing the magnetic energy and pressure

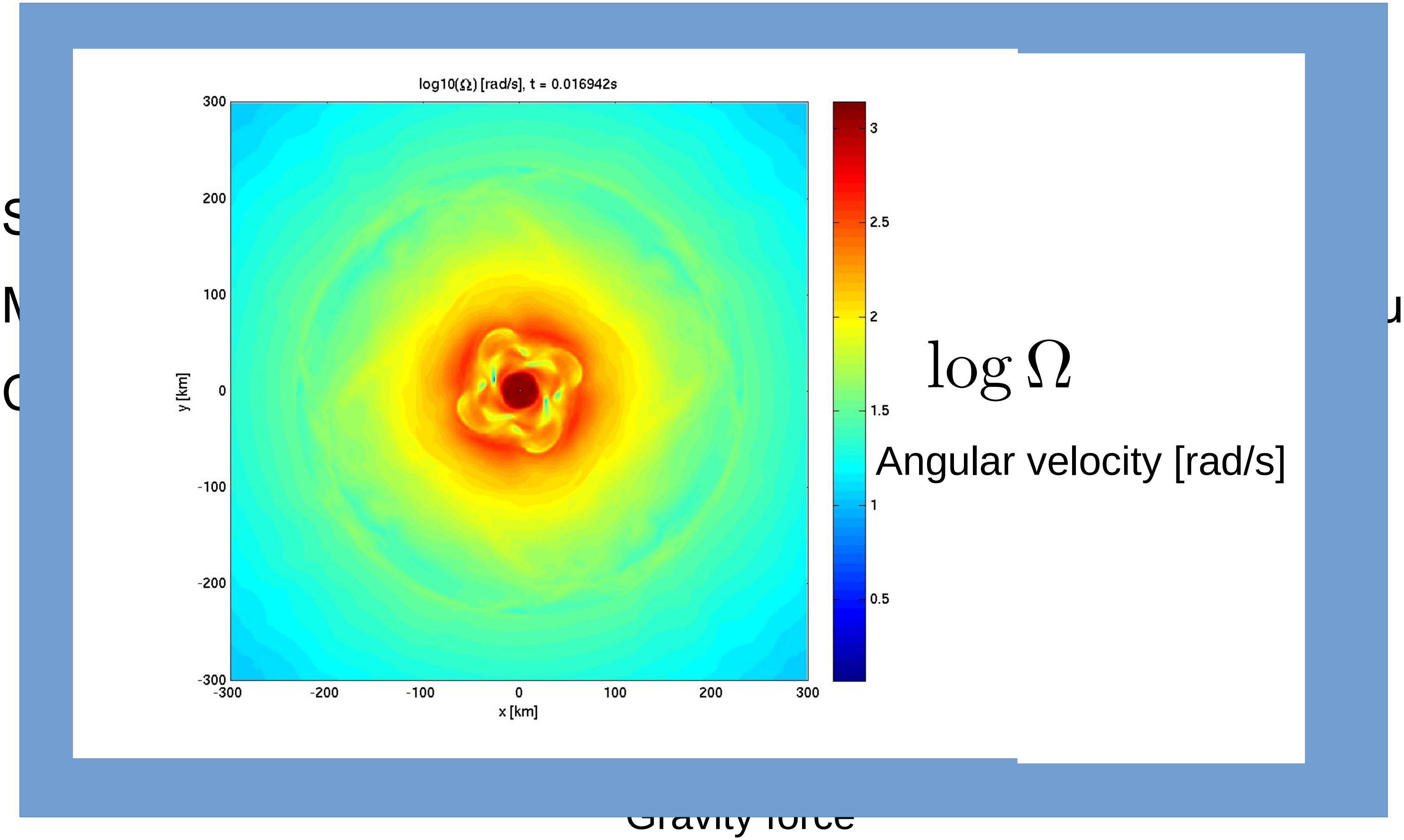
$$P_{\mathrm{mag}} = \frac{B^2}{8\pi}$$

- Strongly magnetised regions appear along the rotational axis
- Magnetic pressure quickly reaches and exceeds the gas pressure
- Consider the equation of motion

$$\rho\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \underset{\text{Pressure force}}{-\nabla p} - \underset{\text{Gravity force}}{\rho\nabla\phi} - \underbrace{\boldsymbol{b}\times(\nabla\times\boldsymbol{b})}_{\text{Lorentz force}}$$

Formation and driving mechanism of bipolar outflow

- Differential rotation winds poloidal magnetic field into toroidal field increasing the magnetic energy and pressure
- S
- M ure
- C



$\log \Omega$

Angular velocity [rad/s]

Gravity force

Formation and driving mechanism of bipolar outflow

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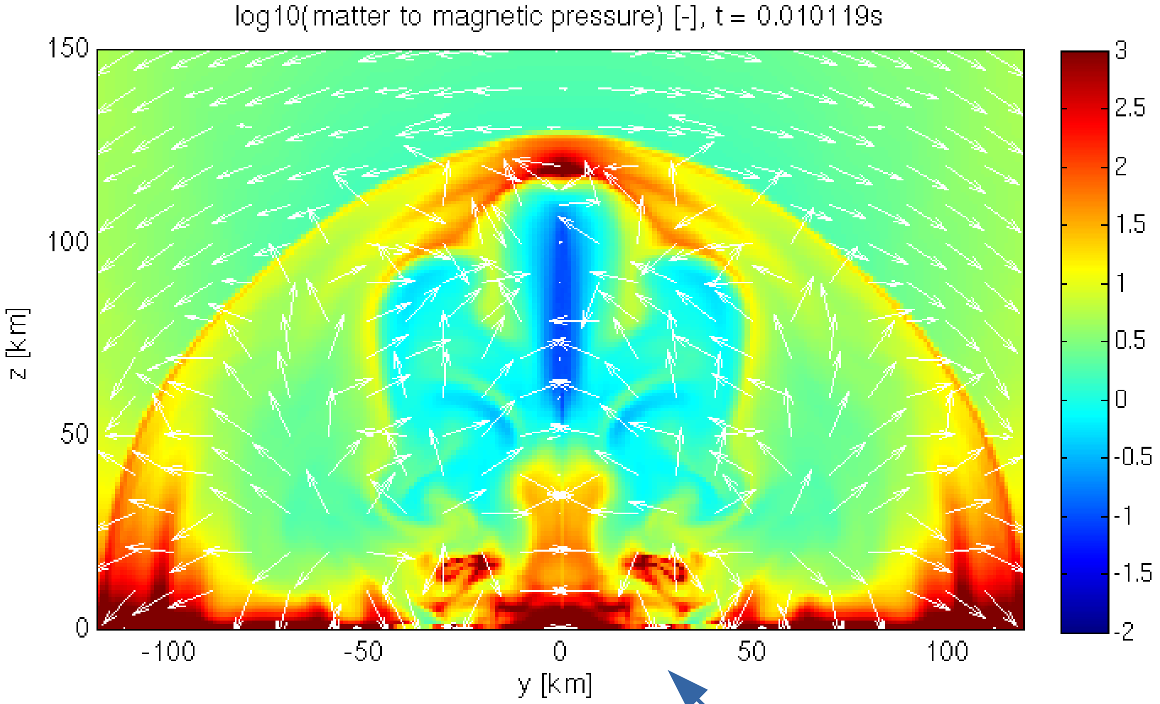
Forma utflow

- Differ eld
  increa



$\log\left(P_{\text{mat}}/P_{\text{mag}}\right)$

- Stron
- Magn ure
- Cons

$$\rho\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\nabla p - \rho\nabla\phi - \underbrace{\boldsymbol{b}\times(\nabla\times\boldsymbol{b})}_{\text{Lorentz force}}$$

Pressure force

Gravity force

Explosion energy, ejected mass and its composition

- Bipolar jets quickly expand & transport energy and stellar material outward against the gravitational attraction of the PNS
- Very neutron rich matter is lifted... r-process?
- Approximately determine explosion energy and ejected mass

Specific total energy $$\epsilon = e_{\text{int}} + \frac{v^2}{2} + \frac{b^2}{2\rho} + \phi > 0$$

when shock reaches upper boundary of 3D domain 700 x 700 x 1400 km

$$t_f = 33\text{ms}$$

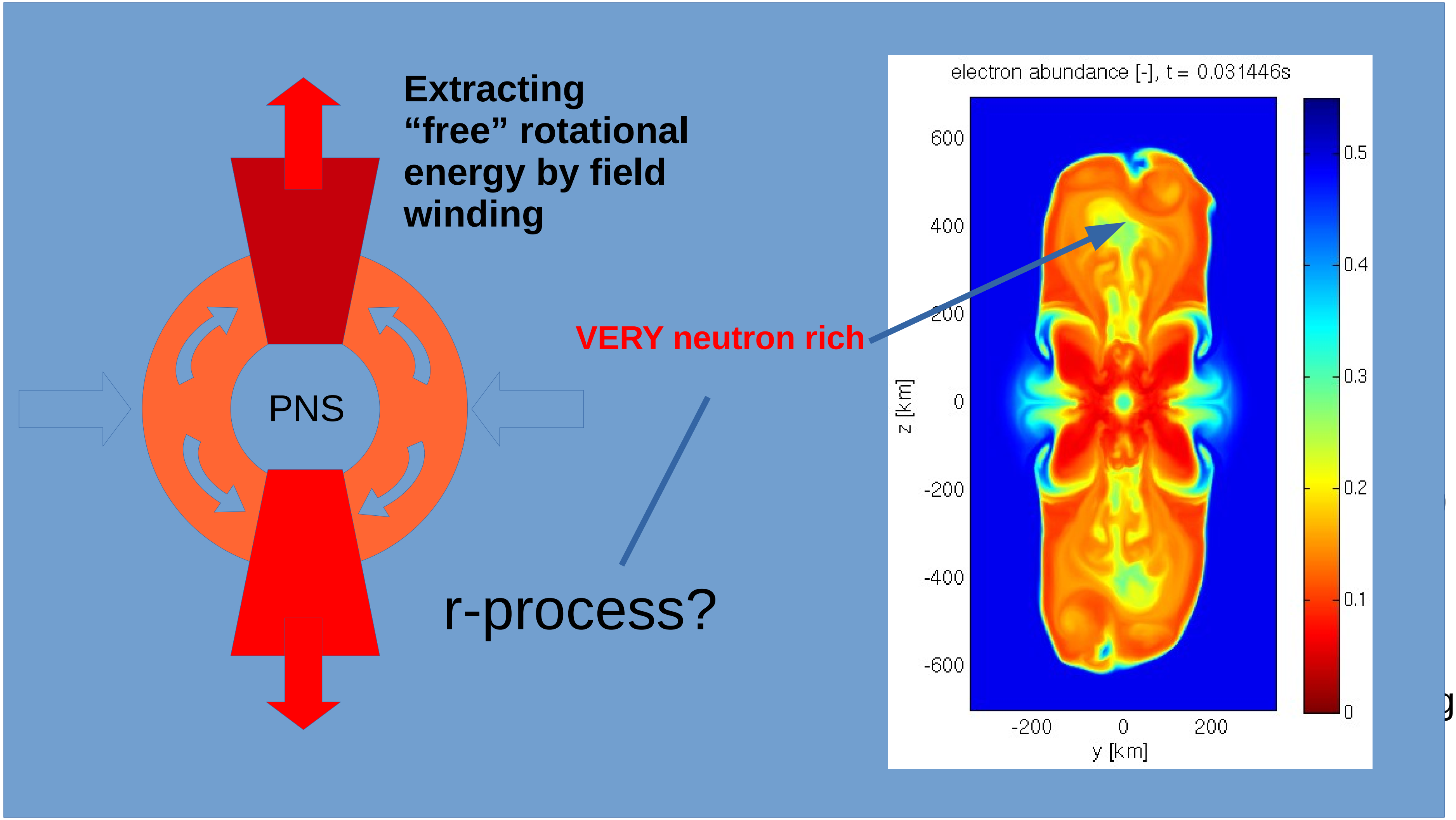
Prompt time scale...

$$M_{\text{ej}} = 6.72 \times 10^{-3} M_{\odot}$$

$$E_{\text{exp}} = 8.45 \times 10^{+49}\text{erg} \lesssim 10^{51}\text{erg}$$

Still growing!

Explosion energy, ejected mass and its composition



Extracting “free” rotational energy by field winding

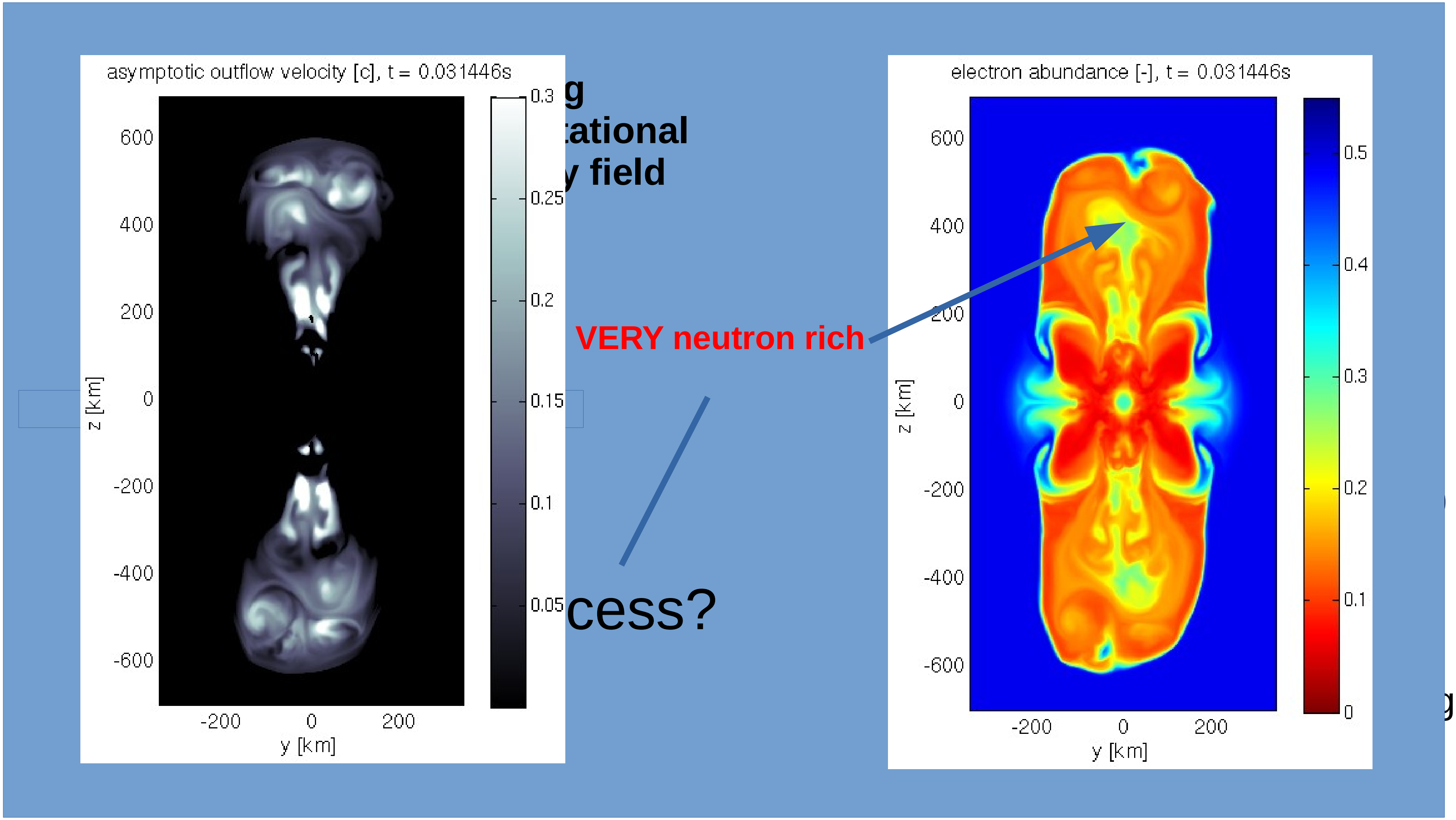
VERY neutron rich

r-process?

km

g!

Explosion energy, ejected mass and its composition



VERY neutron rich

g
tational
y field

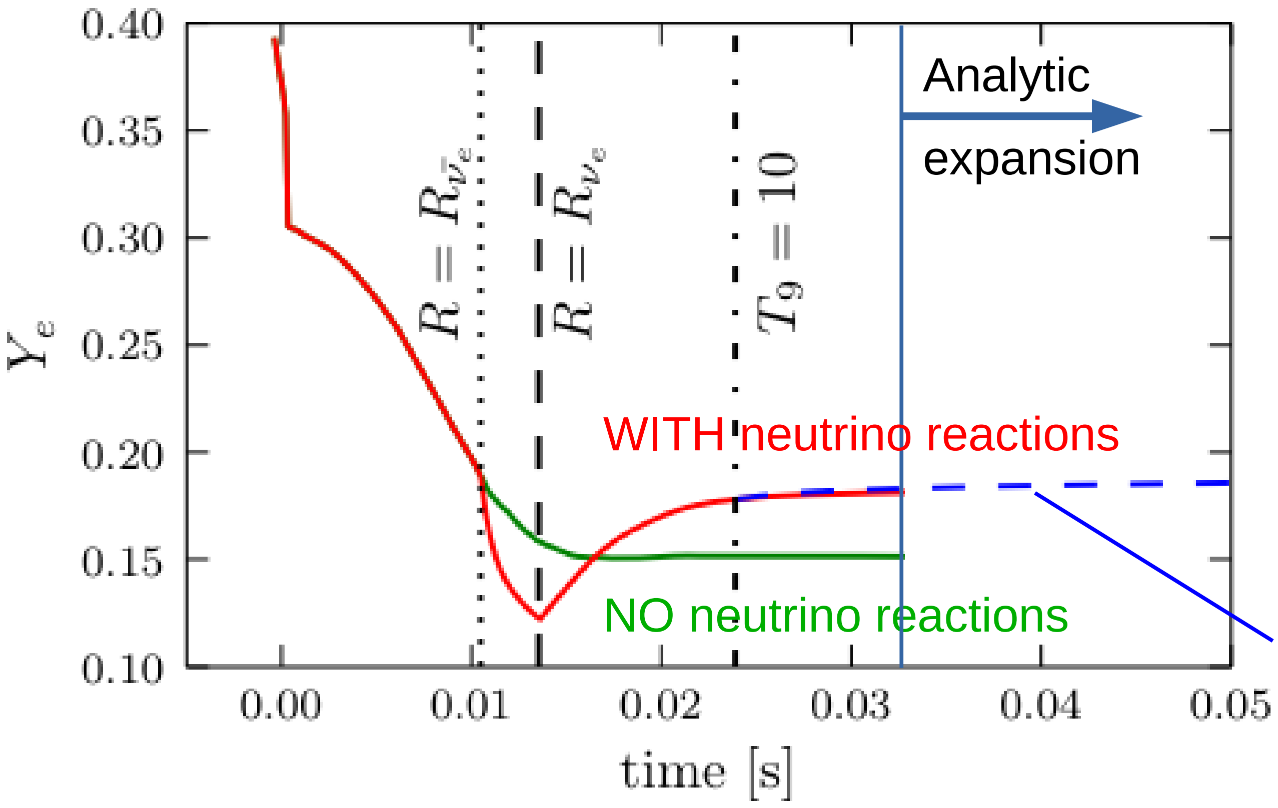
km

cess?

g!

Composition of the ejecta

- Included tracer particles to track the evolution of thermodynamic conditions in a Lagrangian manner
- Electron fraction is a key input for the nucleosynthesis and strongly depends on the challenging ν transport



Evolve the electron fraction with integrated neutrino luminosities outside of the neutrino spheres using approx. emission/absorption on nucleons Janka 2001

Inlcuding neutrino reactions in network (Fröhlich et al. 2006)

Winteler et al. 2012

!!! POST-PROCESSING !!!

Composition of the ejecta

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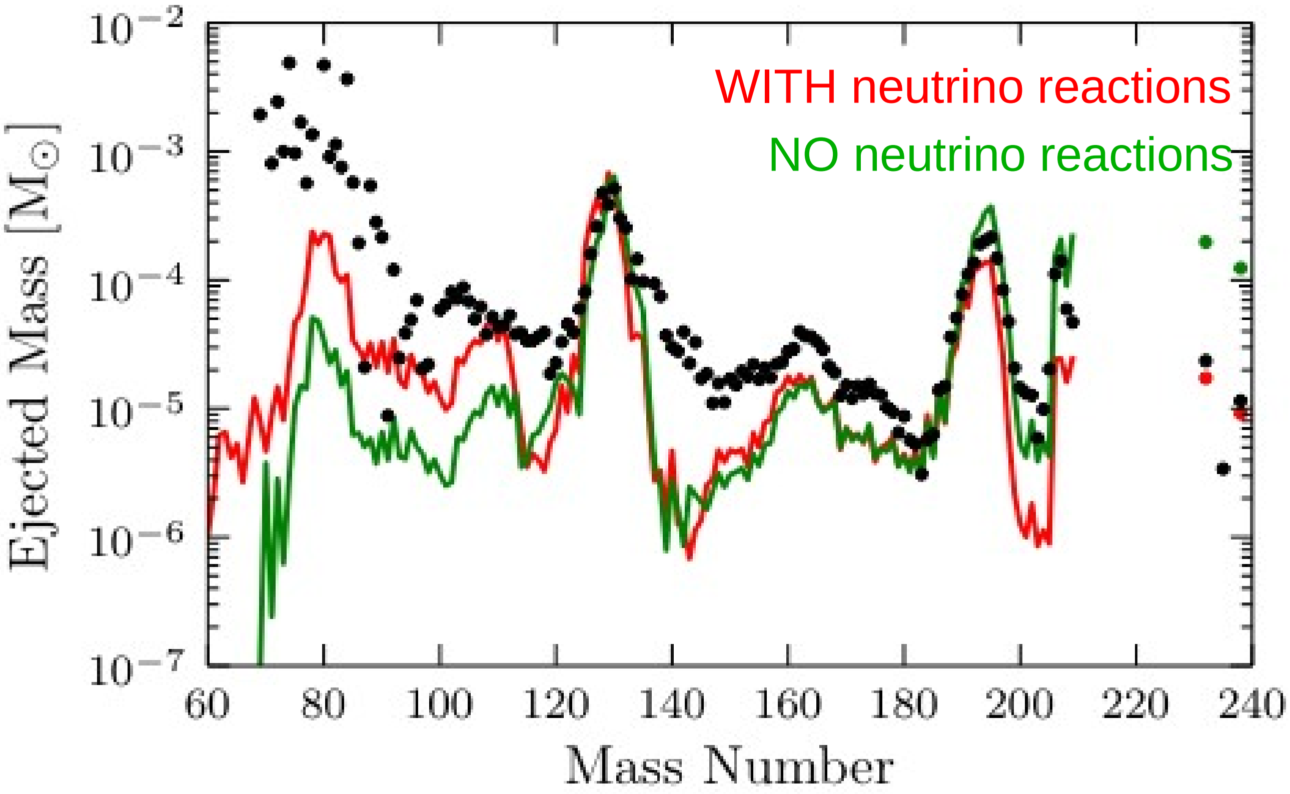
- Electron fraction is a key input for the nucleosynthesis and strongly depends on the challenging ν transport



Mass integrated abundances

Winteler et al. 2012

!!! POST-PROCESSING !!!

Discussion of MHD-CCSN

- (Too) Fast initial rotation rate?

Stellar evolution with magnetic fields $\Omega \sim 0.3 \ \mathrm{rad/s}$

Heger et al. 2005

Stellar evolution without magnetic fields $\Omega \sim \pi \ \mathrm{rad/s}$

e.g. Heger et al. 2000, Hirschi et al. 2004

- (Too) much r-process matter ejected?

Eject $M_{\mathrm{r,ej}} = 5.64 \times 10^{-3} M_\odot$ of r-process material...

But: if all CCSN exploded with the MHD mechanism,
then r-process overproduced by factor $100-1000$

- (Too) strong initial magnetic field ~ 1000

Some stars may have large field strengths... e.g. magnetars
strong magnetic white dwarfs

e.g. Wickramasinghe & Ferrario 2000

- (Very) Short simulation time?

Only 33 ms !!!

MHD CCSN only rare event with special conditions

Woosley & Heger 2006

Consistent with large star-to-star scatter of r-process abundances at low metallicity

e.g. Cowan & Sneden 2006

Numerical issues... Angular momentum conservation

Currently working on angular momentum preserving schemes

Resolution of MRI...

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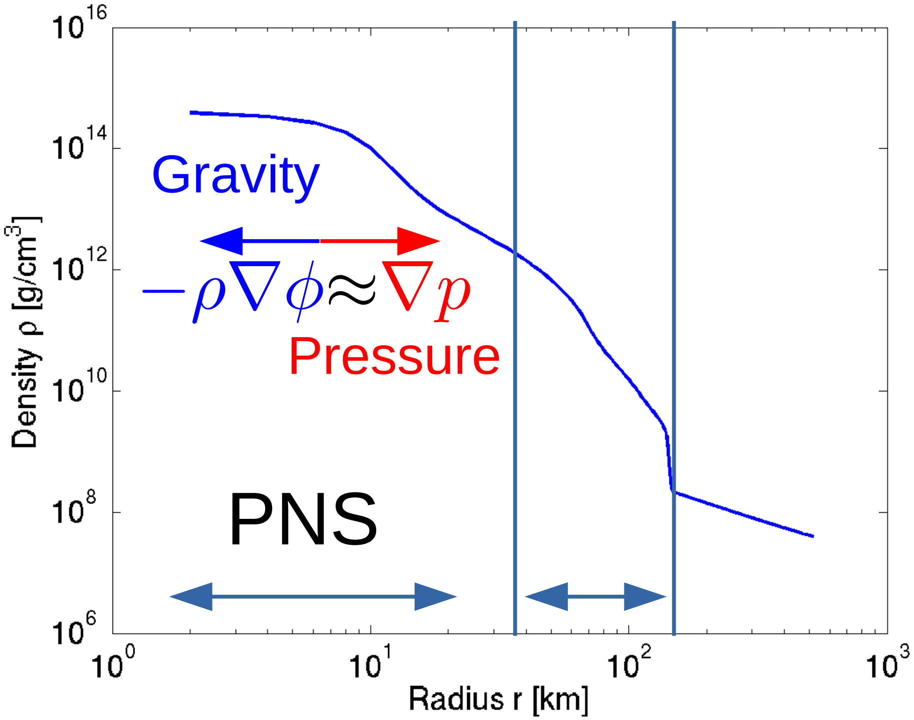
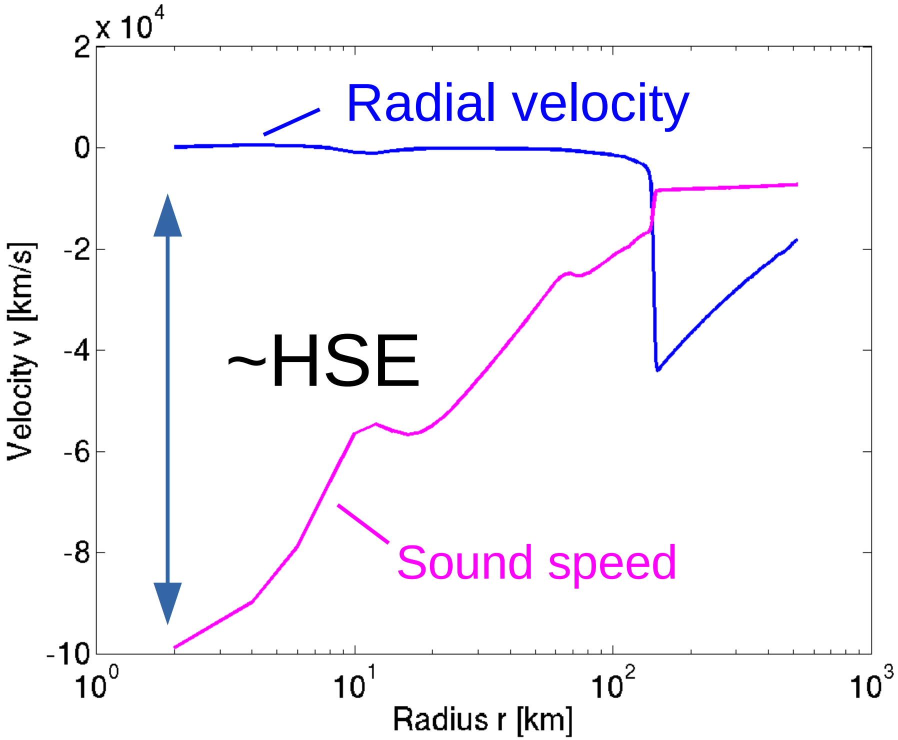
Core-collapse Supernova

- The problem:





Ability to maintain near hydrostatic equilibrium for a long time!

$$\tau_{\rm dyn} = (G\bar{\rho})^{-1/2} \approx 1{\rm ms} \quad \Longleftrightarrow \quad \tau_{\rm expl} \gtrsim 100{\rm ms}$$

Other applications

- Waves in stellar atmospheres

  The waves amplitude may be much smaller when compared to the stratification stemming from gravity...

- Stellar “evolution”
  Stars evolve mostly quietly very close to a hydrostatic state... E.g. convection is only a small perturbation of the stationary state

- Climate modelling on exoplanets
  Atmospheric motions happen on a hydrostatic background

Hydrostatic equilibrium

- Consider 1D hydrodynamics eqs with gravity

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial x} = \boldsymbol{S}$$

$$\boldsymbol{u} = \left[\begin{array}{c} \rho \\ \rho v \\ E \end{array} \right] \quad \boldsymbol{F} = \left[\begin{array}{c} \rho v \\ \rho v^2 + p \\ (E+p)v \end{array} \right] \quad \boldsymbol{S} = -\left[\begin{array}{c} 0 \\ \rho \\ \rho v \end{array} \right] \frac{\partial \phi}{\partial x}$$

- Classical solution algorithm:
  - Solve homogeneous Eqs. with Godunov type method (i.e. solve Riemann problem)
  - Account for source term in second step (split/unsplit)

Hydrostatic equilibrium (2)

- Classical solution algorithm:

$$\boldsymbol{u}_i^{n+1} = \boldsymbol{u}_i^n - \frac{\Delta t}{\Delta x}\left(\boldsymbol{F}_{i+1/2}^n - \boldsymbol{F}_{i-1/2}^n\right) + \Delta t \boldsymbol{S}_i^n$$

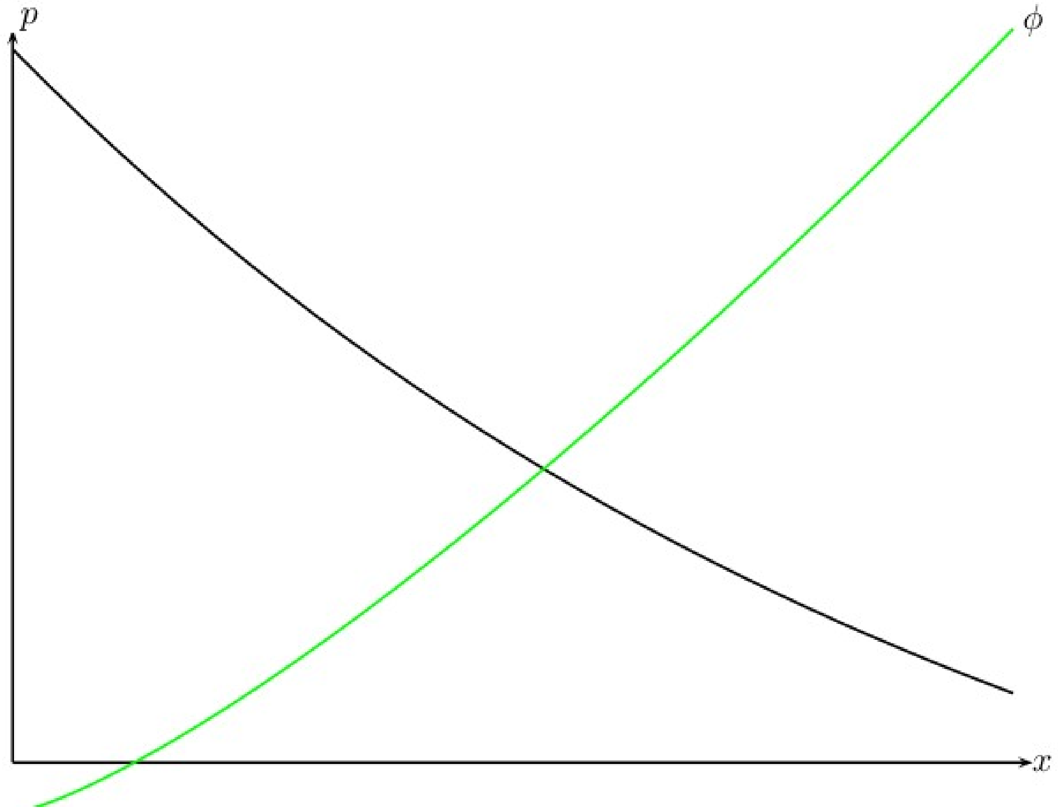
- Numerical flux $\boldsymbol{F}_{i\pm1/2}^n = \mathcal{F}(\boldsymbol{u}_{i\pm1/2}^{n,L}, \boldsymbol{u}_{i\pm1/2}^{n,R})$ from (approximate) Riemann solver, e.g.
  - (Local) Lax-Friedrichs Lax (1954), Rusanov (1961)
  - HLL (C) Harten, Lax and van Leer (1983), Toro et al. (1994)
  - Roe Roe (1981)
  - ...

Hydrostatic equilibrium (3)

Interested in hydrostatic equilibrium:

$$\frac{\partial \boldsymbol{F}}{\partial x} = \boldsymbol{S} \Rightarrow \quad \frac{\partial p}{\partial x} = -\rho \frac{\partial \phi}{\partial x}$$

EoS: $p = p(\rho, e)$



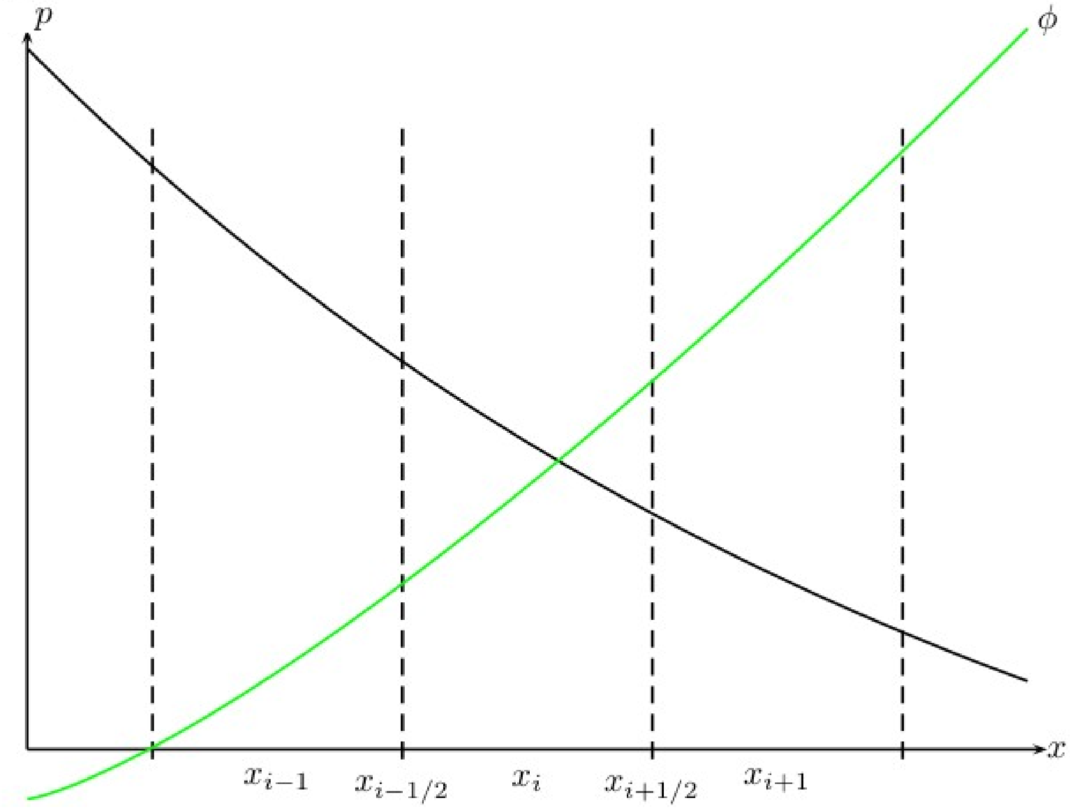
Hydrostatic equilibrium (4)

Interested in hydrostatic equilibrium:

$$\frac{\partial \boldsymbol{F}}{\partial x} = \boldsymbol{S} \Rightarrow \quad \frac{\partial p}{\partial x} = -\rho \frac{\partial \phi}{\partial x}$$

EoS: $p = p(\rho, e)$

Discretise in cells $\left[x_{i-1/2}, x_{i+1/2}\right]$

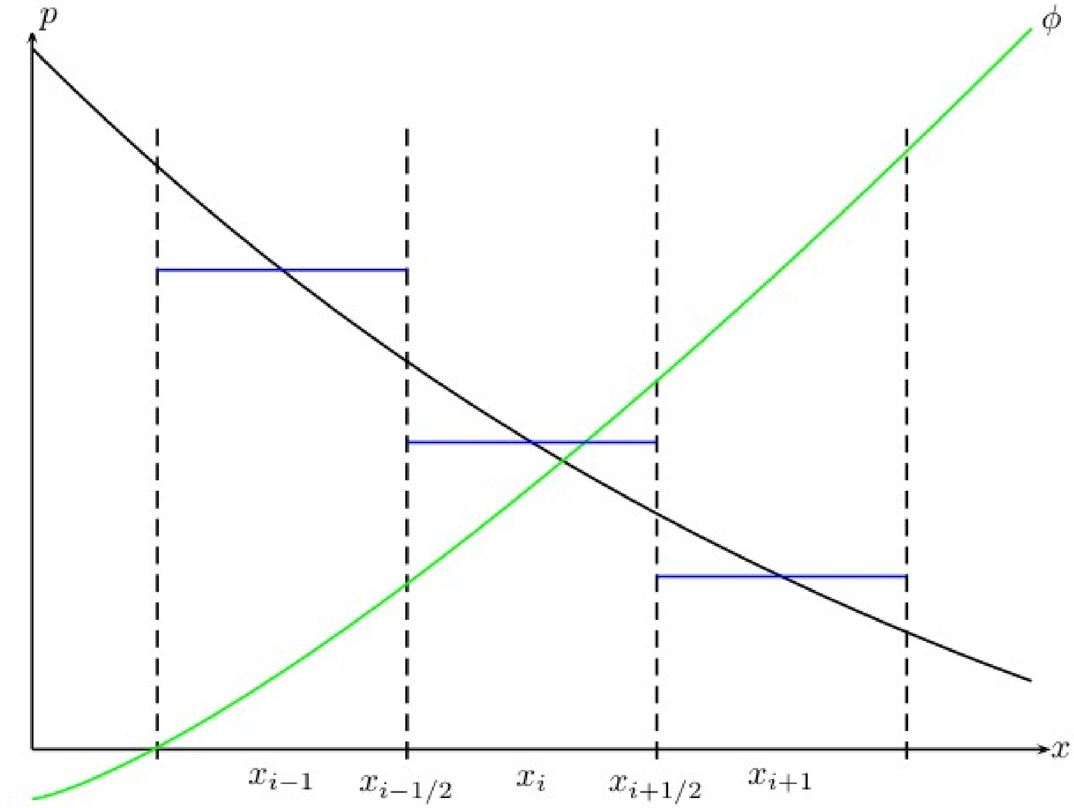


Hydrostatic equilibrium (5)

Interested in hydrostatic equilibrium:

$$\frac{\partial \boldsymbol{F}}{\partial x} = \boldsymbol{S} \Rightarrow \quad \frac{\partial p}{\partial x} = -\rho \frac{\partial \phi}{\partial x}$$

EoS: $p = p(\rho, e)$



Discretise in cells $\left[x_{i-1/2}, x_{i+1/2}\right]$

Define cell averages

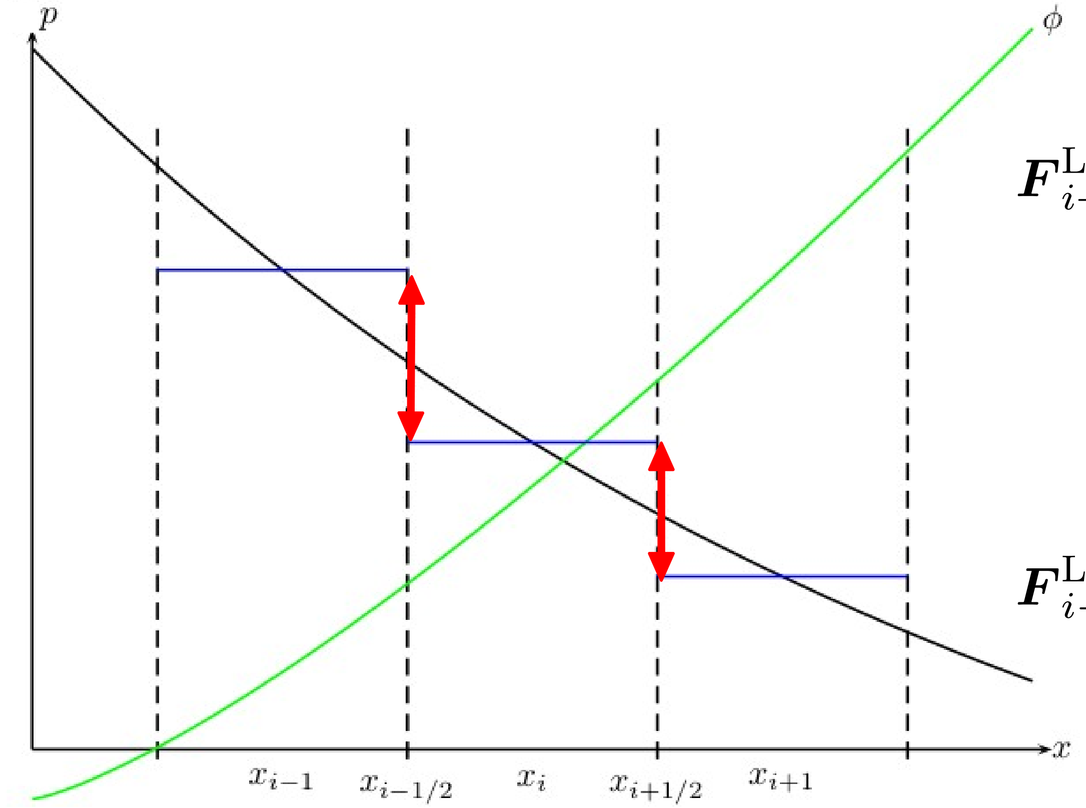
$$\boldsymbol{u}_i = \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \boldsymbol{u}(x, t^n) x$$

$$\boldsymbol{S}_i = \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \boldsymbol{S}\left(\boldsymbol{u}(x, t)\right) x$$

Hydrostatic equilibrium (6)

Interested in hydrostatic equilibrium:

$$\frac{1}{\Delta x}\left(\boldsymbol{F}^n_{i+1/2} - \boldsymbol{F}^n_{i-1/2}\right) \overset{?}{=} \boldsymbol{S}^n_i$$



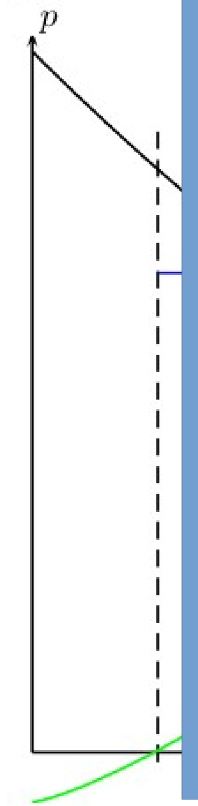
$$\boldsymbol{F}^{\mathrm{LxF}}_{i+1/2} = \frac{1}{2}\left(\boldsymbol{F}_i + \boldsymbol{F}_{i+1}\right) - \frac{S_{\max}}{2}\underbrace{\left(\boldsymbol{u}_{i+1} - \boldsymbol{u}_i\right)}$$

Contains also gravity induced gradient!

$$\boldsymbol{F}^{\mathrm{LxF}}_{i-1/2} = \frac{1}{2}\left(\boldsymbol{F}_{i-1} + \boldsymbol{F}_i\right) - \frac{S_{\max}}{2}\overbrace{\left(\boldsymbol{u}_i - \boldsymbol{u}_{i-1}\right)}$$

Hydrostatic equilibrium (6)

Intere
equil



Hydrostatic atmosphere in a constant gravitational field

$$\phi(x) = gx \qquad \rho(x) = \left[\rho_0^{\gamma-1} - \frac{g}{K}\frac{\gamma-1}{\gamma}x\right]^{\frac{1}{\gamma-1}} \qquad p = \frac{p_0}{\rho_0^{\gamma}}\rho^{\gamma} = K\rho^{\gamma}$$

$$x \in [0,2]$$

Error in pressure:

(after 2 sound crossing times!!!)

N	1st	2ndTVD
128	2.1E-02	6.5E-05
256	1.1E-02	1.6E-05
512	5.3E-03	4.1E-06
1024	2.6E-03	1.0E-06
2048	1.3E-03	2.6E-07

$$Err = \frac{1}{N}\sum_i |p_i - p_i^0|$$

HLLC numerical flux

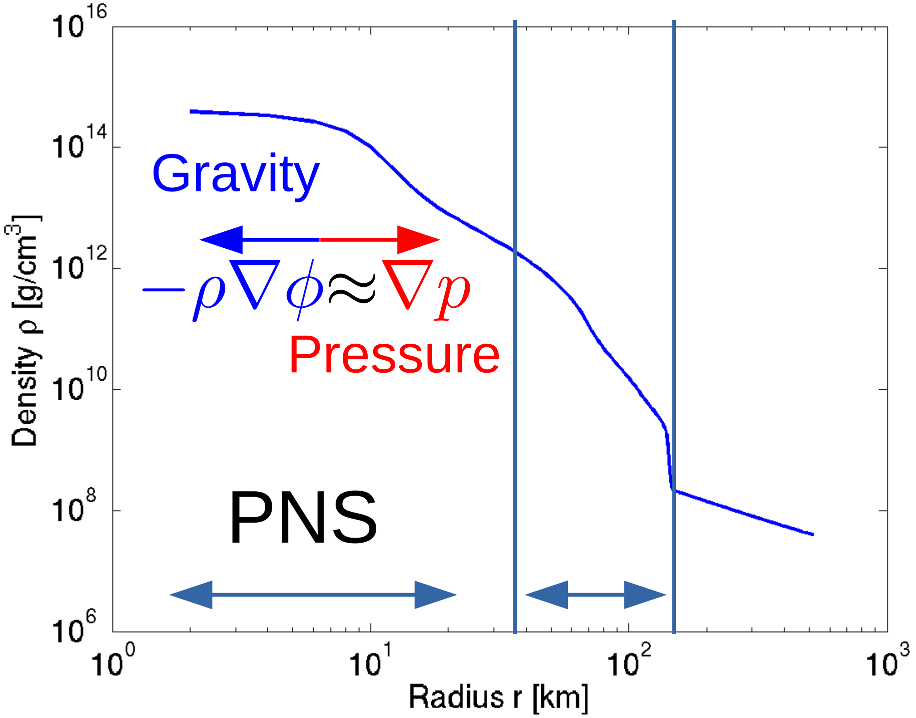
$_1 - \boldsymbol{u}_i)$

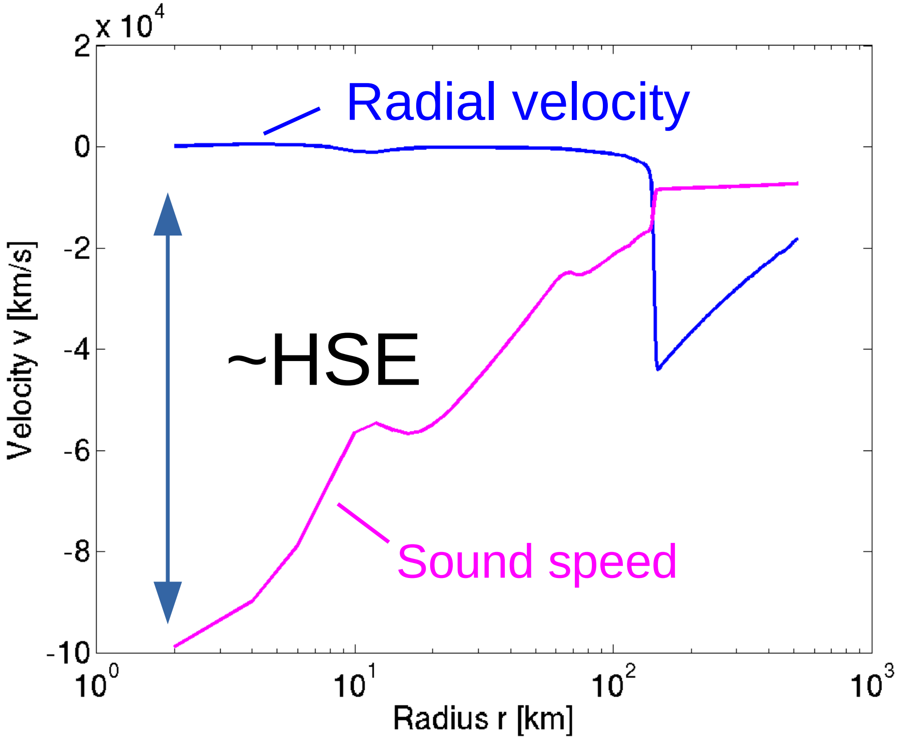
avity
!

$- \boldsymbol{u}_{i-1})$

Core-collapse Supernova

- The problem:





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Well-balanced schemes

- Solutions:
  - Define a **global** stationary state $u_0(\boldsymbol{x})$ **at each time step** and evolve $u(\boldsymbol{x}) - u_0(\boldsymbol{x})$
  - Steady state preserving reconstructions, well-balanced schemes

e.g. LeVeque (1998), LeVeque & Bale (1998), Botta et al. (2004), Fuchs et al. (2010), Xing & Shu (2013), Vides et al. (2013), Desveaux et al. (2014), Chandrashekar & Klingenberg (2015), Desveaux et al. (2015), Li & Xing (2015,2016,2018) Chandrashekar & Zenk (2017), ...

See also Mellema et al. (1991), Zingale et al. (2002), Kastaun (2006), Freytag et al. (2012), Gosse (2015)

Note: there are many, many more... especially for shallow-water eqs!!!

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Fuchs et al. (2010), Xing & Shu (2013), Vides et al. (2013),
Desveaux et al. (2014), Chandrashekar & Klingenberg (2015),

Requirements

- Equilibrium (usually) not known in advance (self-gravity)
- Extensible for general EoS
- (At least) second order accuracy
- Preserve robustness of base shock capturing scheme

Well-balanced scheme (2)

Interested in **numerical** hydrostatic equilibrium:

$$\frac{1}{\Delta x}\left(\boldsymbol{F}^n_{i+1/2} - \boldsymbol{F}^n_{i-1/2}\right) = \boldsymbol{S}^n_i$$

Standard centered differences

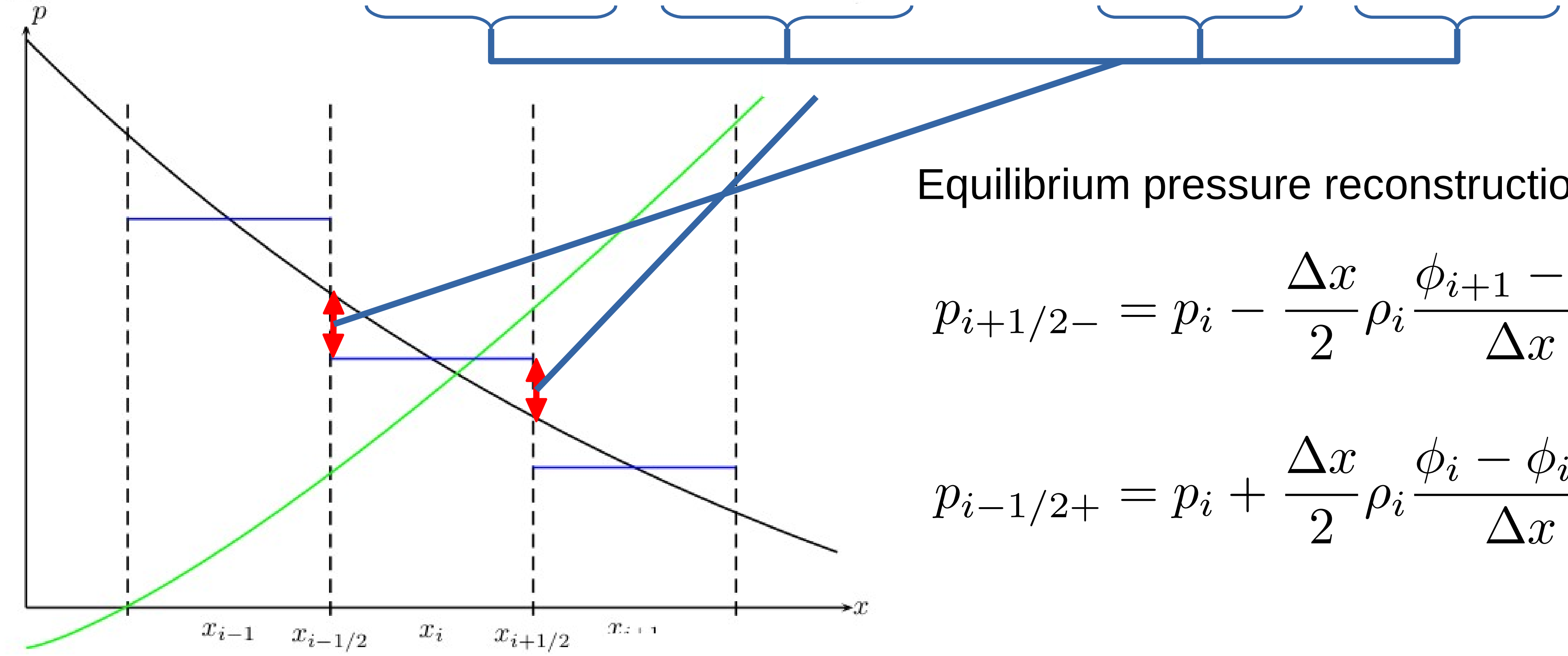
$$\frac{\partial p}{\partial x} + O(\Delta x^2) = \frac{p_{i+1/2} - p_{i-1/2}}{\Delta x} = -\rho_i \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x} = -\rho \frac{\partial \phi}{\partial x} + O(\Delta x^2)$$

$$\frac{(p_{i+1/2} - p_i) - (p_{i-1/2} - p_i)}{\Delta x} = -\frac{\rho_i}{2} \frac{(\phi_{i+1} - \phi_i) - (\phi_{i-1} - \phi_i)}{\Delta x}$$

Well-balanced scheme (3)

Interested in **numerical** hydrostatic equilibrium:

$$\frac{p_{i+1/2}-p_i}{\Delta x}-\frac{p_{i-1/2}-p_i}{\Delta x}=-\frac{\rho_i}{2}\left(\frac{\phi_{i+1}-\phi_i}{\Delta x}-\frac{\phi_{i-1}-\phi_i}{\Delta x}\right)$$



Equilibrium pressure reconstruction:

$$p_{i+1/2-}=p_i-\frac{\Delta x}{2}\rho_i\frac{\phi_{i+1}-\phi_i}{\Delta x}$$

$$p_{i-1/2+}=p_i+\frac{\Delta x}{2}\rho_i\frac{\phi_i-\phi_{i-1}}{\Delta x}$$

Well-balanced scheme (3)

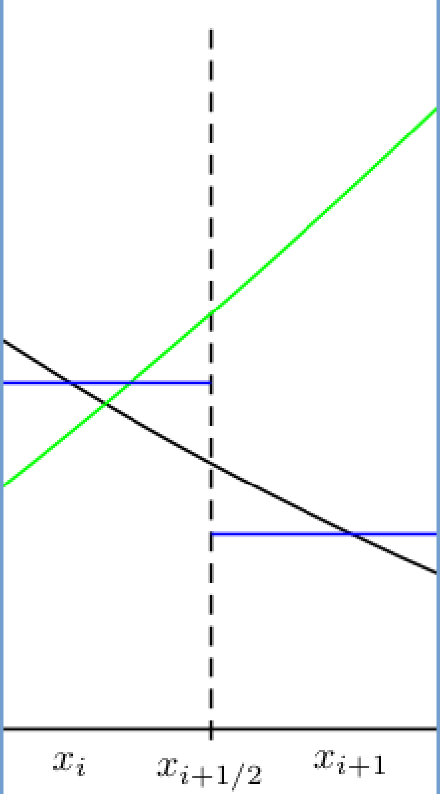
Interested in **numerical** hydrostatic equilibrium:

n:

ϕ_i

-1

Equilibrium?



$$p_{i+1/2-} \overset{!}{=} p_{i+1/2+}$$

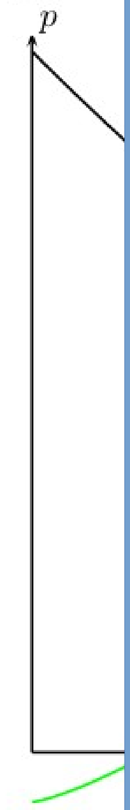
$$\Downarrow$$

$$\frac{p_{i+1}-p_i}{\Delta x} = -\frac{\rho_i+\rho_{i+1}}{2}\,\frac{\phi_{i+1}-\phi_i}{\Delta x}$$

Discrete HydroStatic Equilibrium

Well-balanced scheme (3)

Interested in **numerical** hydrostatic equilibrium:

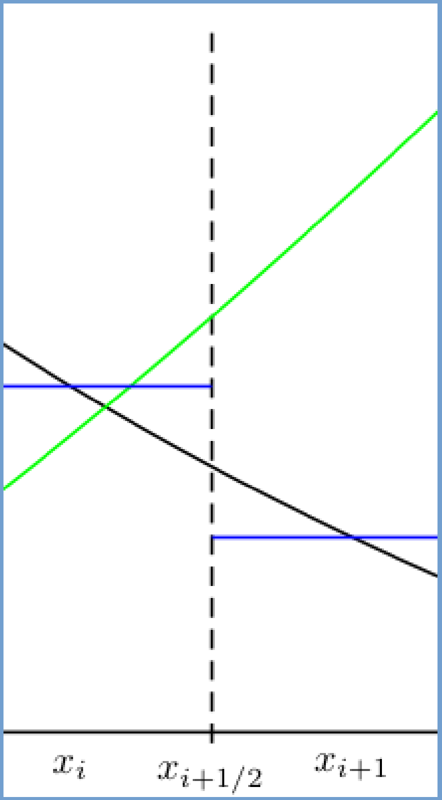


n:

ϕ_i

-1

Equilibrium?



$$p_{i+1/2-} \overset{!}{=} p_{i+1/2+}$$

Requirement on Riemann solver:

$$\boldsymbol{F}^n_{i\pm1/2} = \boldsymbol{\mathcal{F}}\left(\begin{bmatrix} \rho_{i+1/2-} \\ 0 \\ p_{i+1/2} \end{bmatrix}, \begin{bmatrix} \rho_{i+1/2+} \\ 0 \\ p_{i+1/2} \end{bmatrix} \right) = \begin{bmatrix} 0 \\ p_{i+1/2} \\ 0 \end{bmatrix}$$

e.g. HLLC, Roe

Discrete HydroStatic Equilibrium

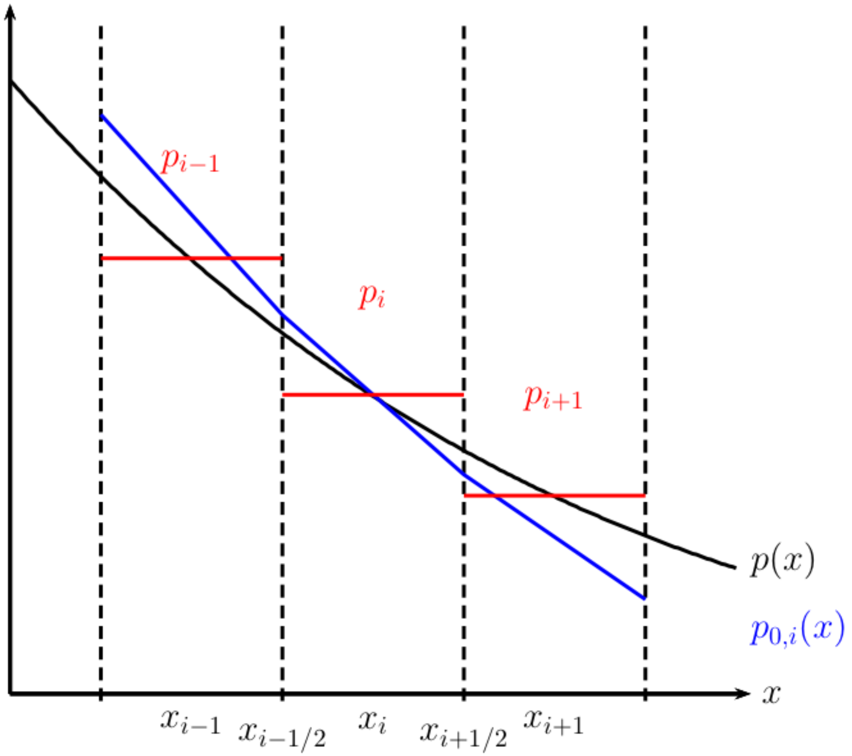
Higher-order extension

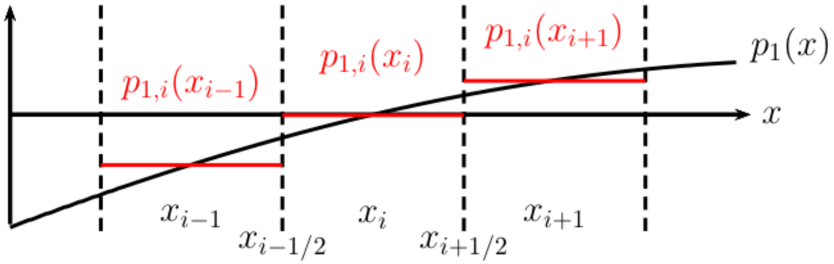
- Second order extension: $r_{1,i}(x_j) = r_j - r_{0,i}(x_j)$

$r =$ pressure, density

Eq. perturbation Data Equilibrium

Stencil: $j = ..., i-1, i, i+1, ...$





Apply a high-order reconstruction on perturbation!
E.g. piecewise-linear, ...

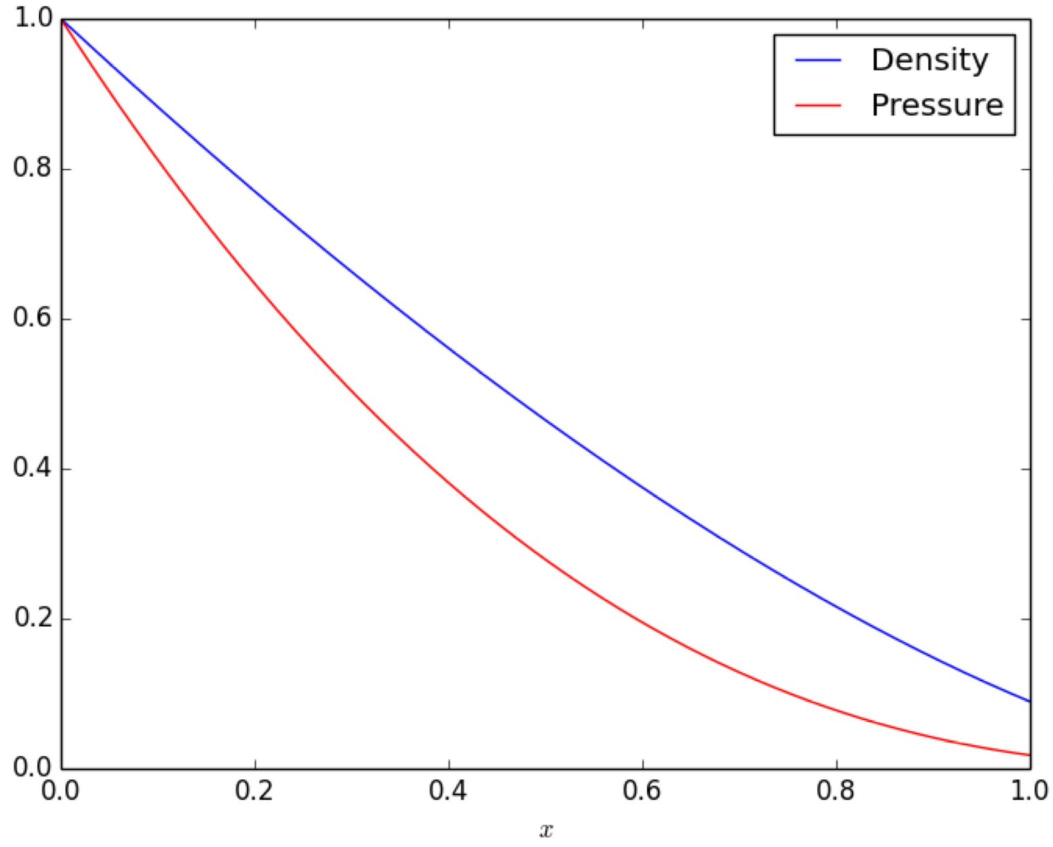
Example 1

Hydrostatic atmosphere in a constant gravitational field

$$\phi(x) = gx \qquad \frac{p_{i+1} - p_i}{\Delta x} = -\frac{\rho_i + \rho_{i+1}}{2}\frac{\phi_{i+1} - \phi_i}{\Delta x} \qquad p = \frac{p_0}{\rho_0^\gamma}\rho^\gamma = K\rho^\gamma$$

$$x \in [0,1] \qquad g = 2 \qquad \gamma = 5/3$$

$K = const$ ~ entropy



Error in pressure:

MC limiter

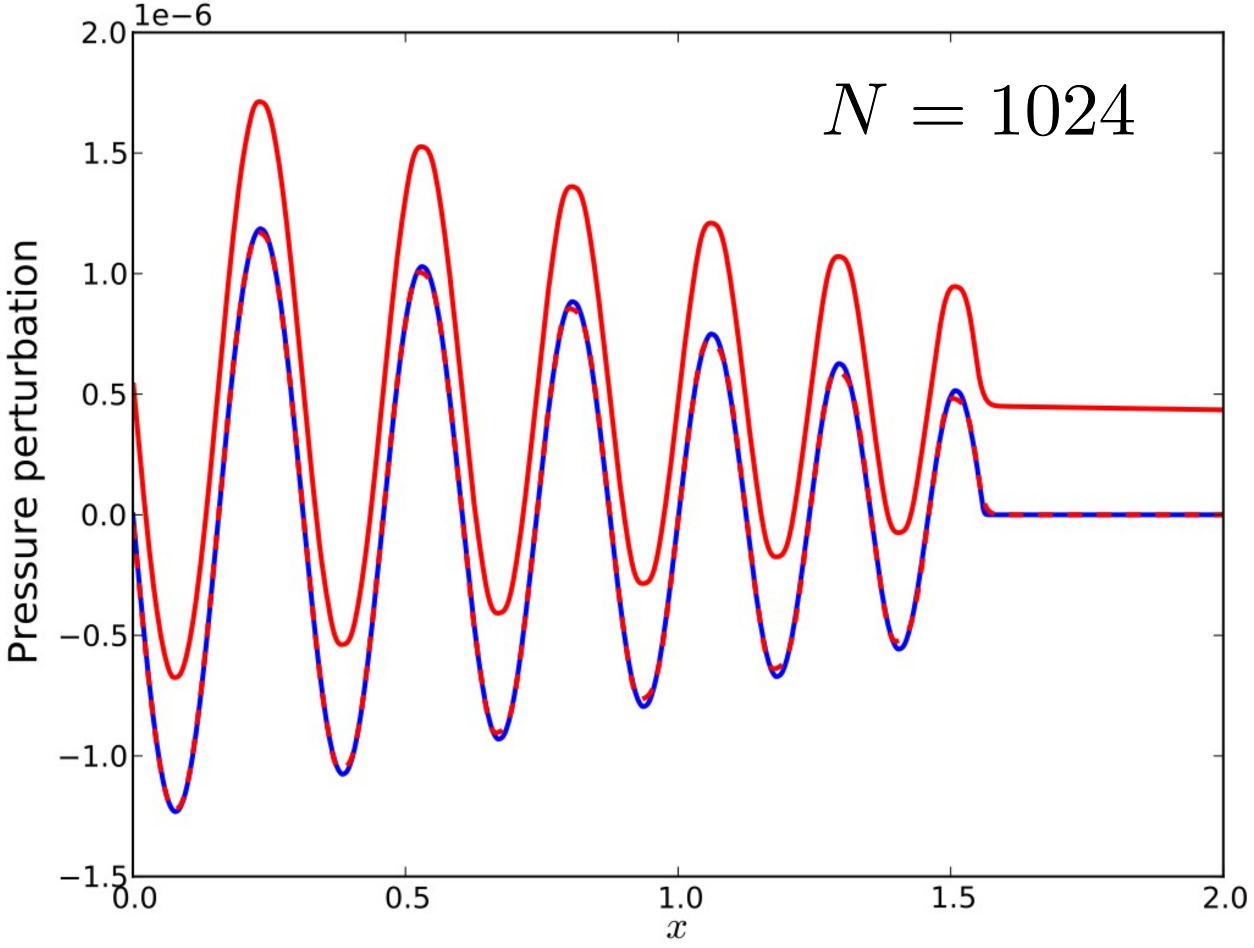
N	1st	2ndTVD
128	2.4E-02 / **2.7E-16**	2.6E-07 / **5.9E-16**
256	1.2E-02 / **6.2E-15**	3.3E-08 / **3.1E-16**
512	5.9E-03 / **2.7E-14**	4.2E-09 / **3.4E-15**
1024	3.0E-03 / **2.0E-14**	5.2E-10 / **1.5E-15**
2048	1.5E-03 / **5.5E-14**	6.5E-11 / **1.3E-14**
rate	1.00 / -	3.00 / -

$$Err = \frac{1}{N}\sum_i |p_i - p_i^0|$$

Example 2

Hydrostatic atmosphere in a constant gravitational field

+ small perturbation $v(t, x=0) = 10^{-6} \sin(8\pi t)$



Error in pressure:

N	2ndTVD
128	3.1E-05 / **1.9E-07**
256	7.8E-06 / **6.8E-08**
512	2.0E-06 / **2.5E-08**
1024	4.8E-07 / **8.5E-09**
2048	1.2E-07 / **4.1E-09**
rate	2.01 / 1.40

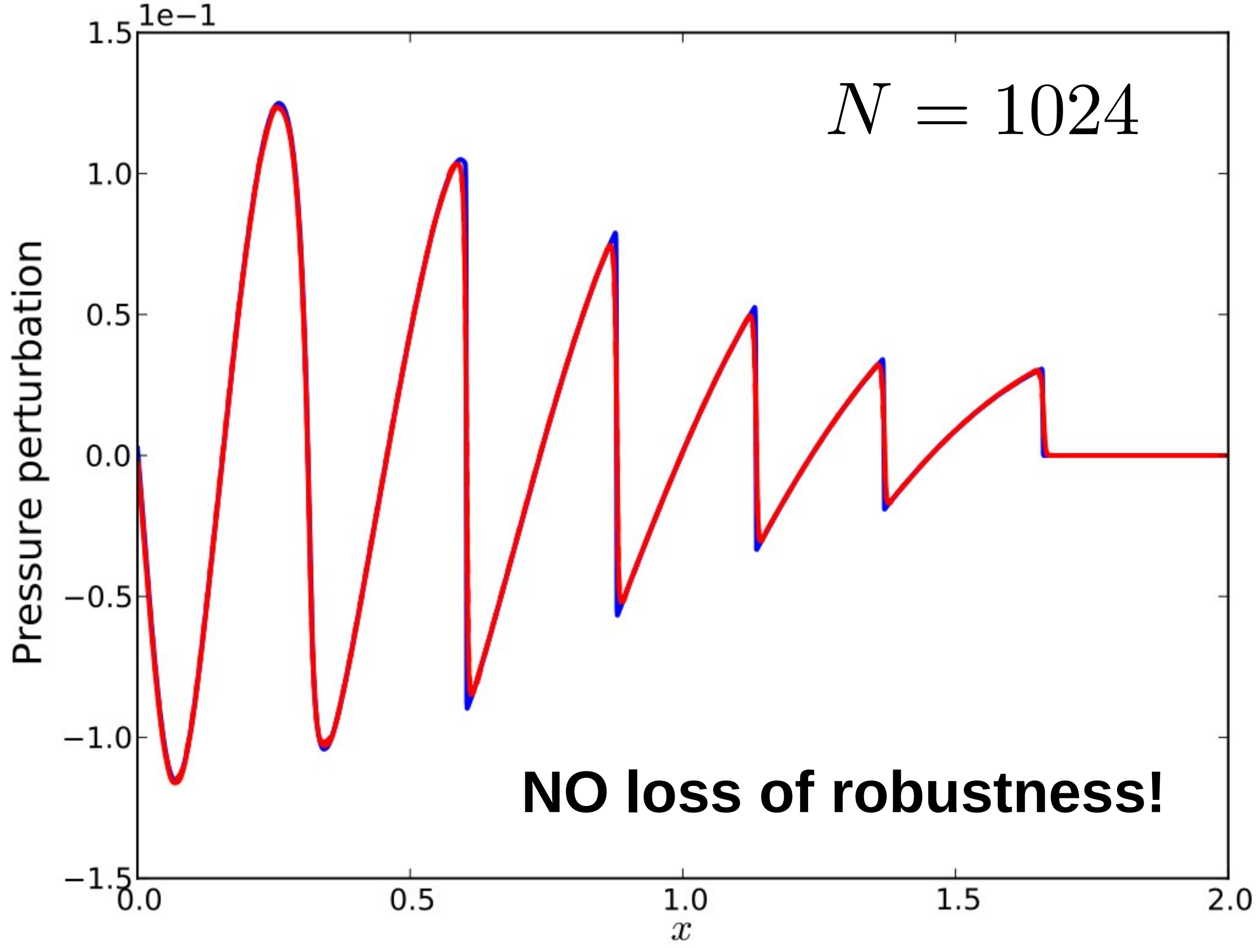
!

$$Err = \frac{1}{N}\sum_i |p_i - p_i^0|$$

Example 2 (2)

Hydrostatic atmosphere in a constant gravitational field

+ large perturbation $v(t, x=0) = 0.1 \sin(8\pi t)$



NO loss of robustness!

Error in pressure:

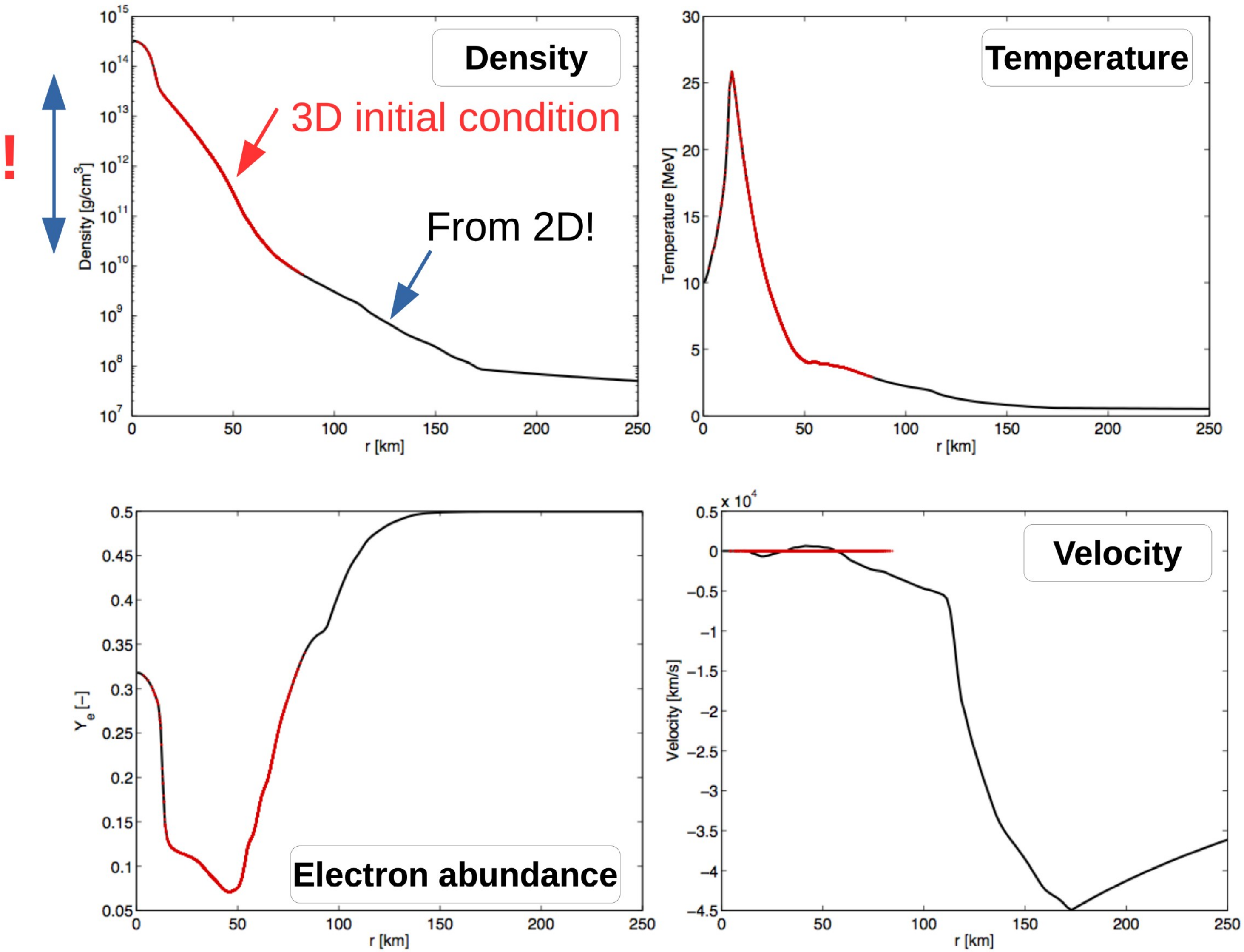
N	2ndTVD
128	9.8E-03 / **1.1E-02**
256	4.1E-03 / **4.9E-03**
512	1.9E-03 / **2.0E-03**
1024	8.7E-04 / **8.0E-04**
2048	5.5E-04 / **3.3E-04**
rate	1.05 / 1.28

$$Err = \frac{1}{N} \sum_i |p_i - p_i^0|$$

“CCSN” simulation

Actually, just the simulation of a stationary PNS!

3D (Cartesian)



3D initial condition

From 2D!

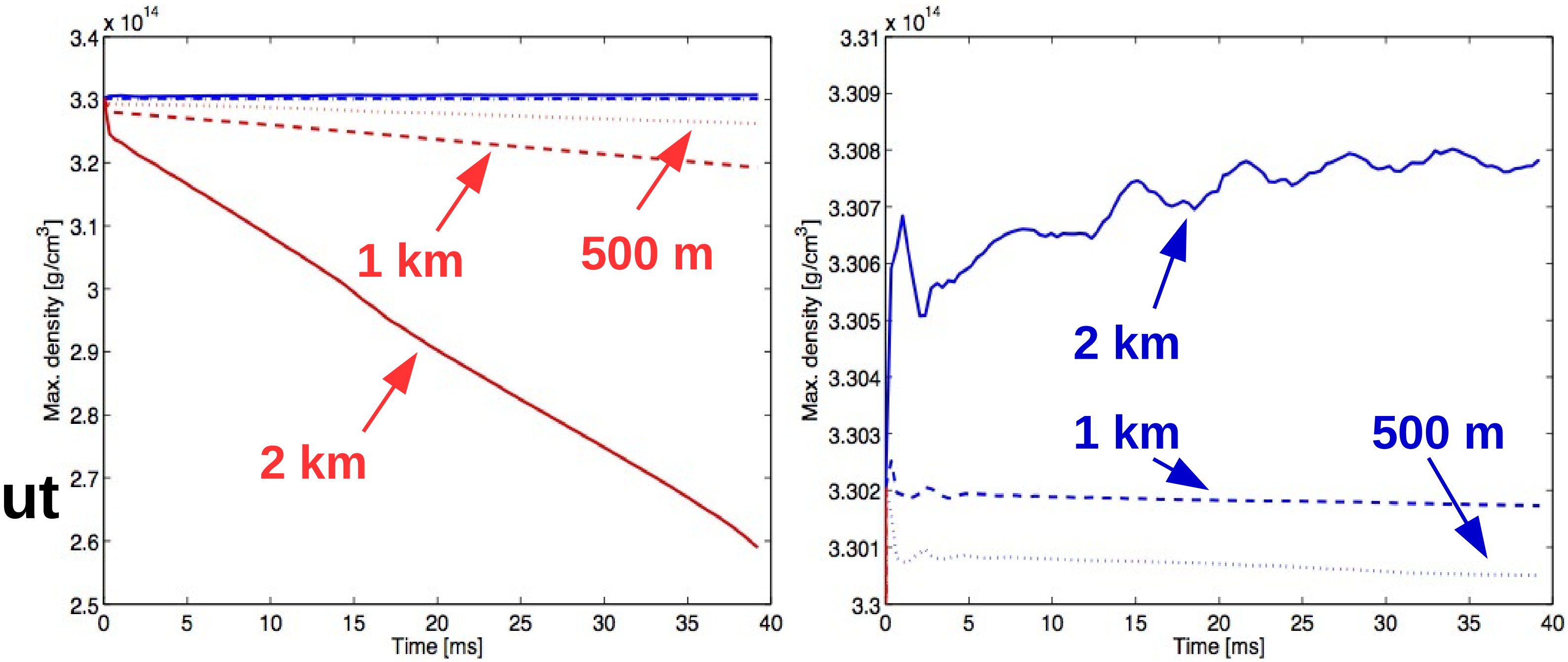
“No” physics, but “realistic” EoS

“CCSN” simulation

Actually, just the simulation of a stationary PNS!

3D (Cartesian)

Maximal density evolution



“No” physics, but “realistic” EoS

“Real” CCSN simulation
evolve for 10-20 times more!

— **NO HSE**

— **WITH HSE**

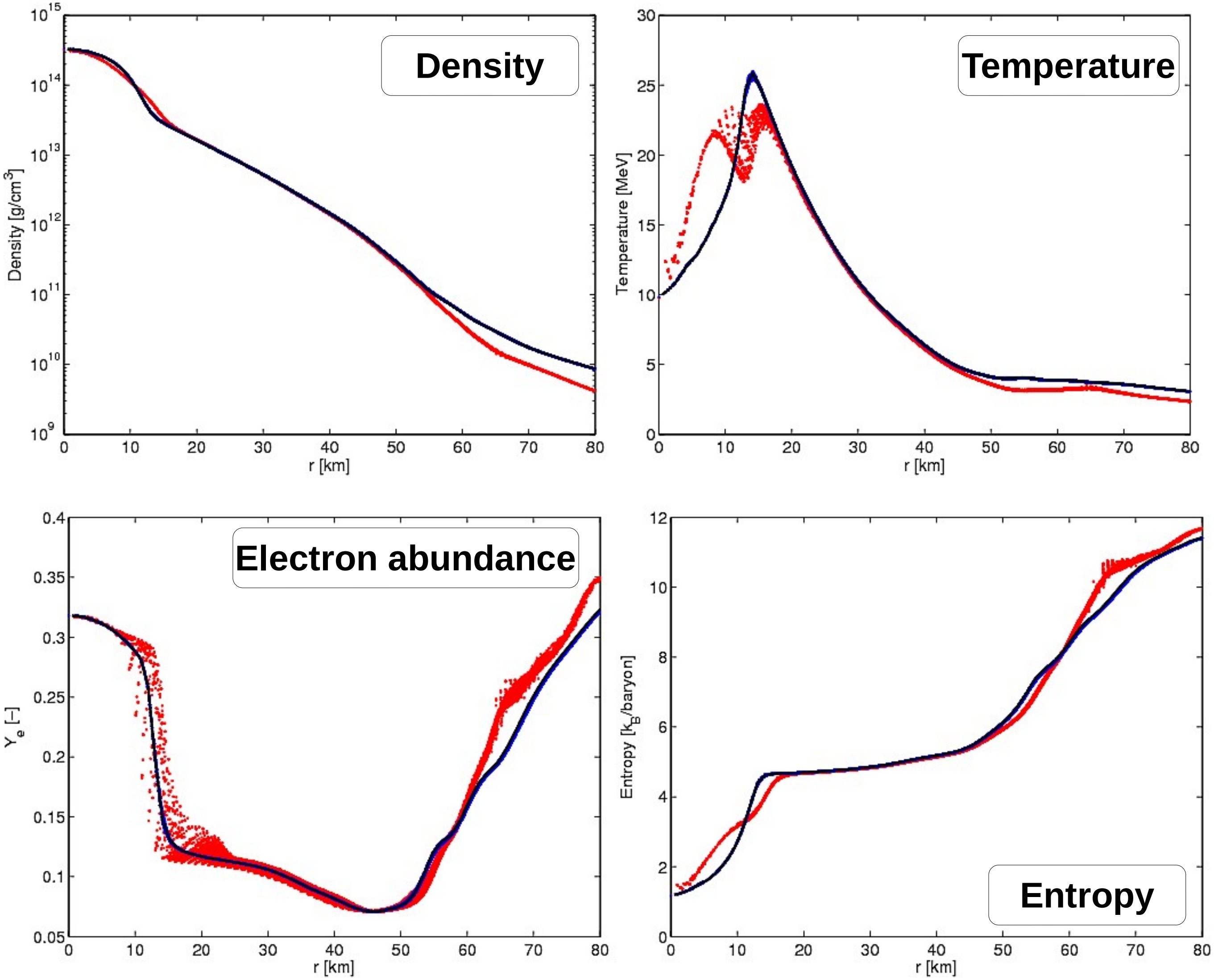
For explicit schemes: keep CFL condition mind!

“CCSN” simulation

Actually, just the simulation of a stationary PNS!

3D (Cartesian)

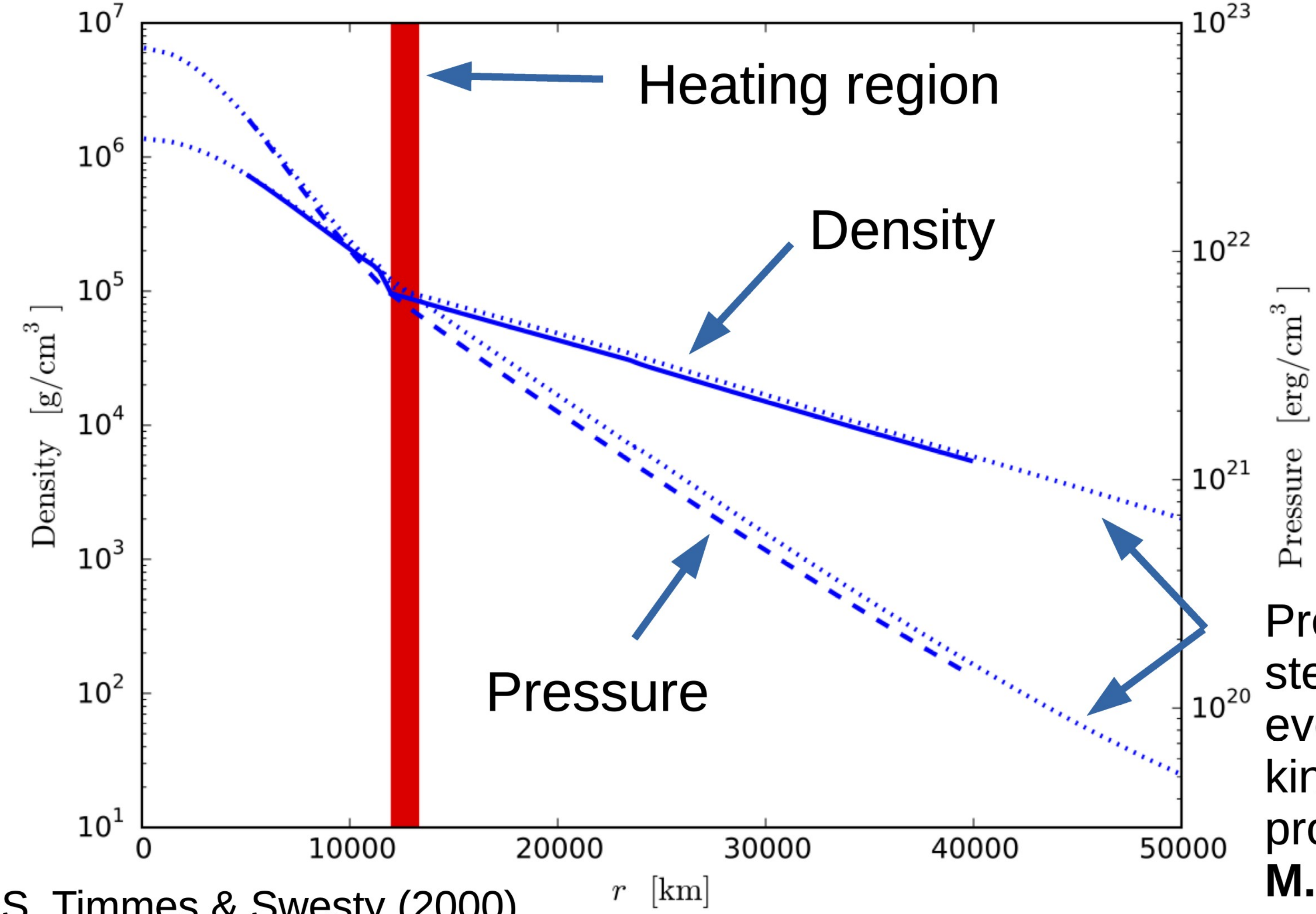
“No” physics, but “realistic” EoS



REFERENCE **NO HSE** **WITH HSE**

Stellar convection

- Convective shell “carbon burning” **HIGHLY SIMPLIFIED!!!**

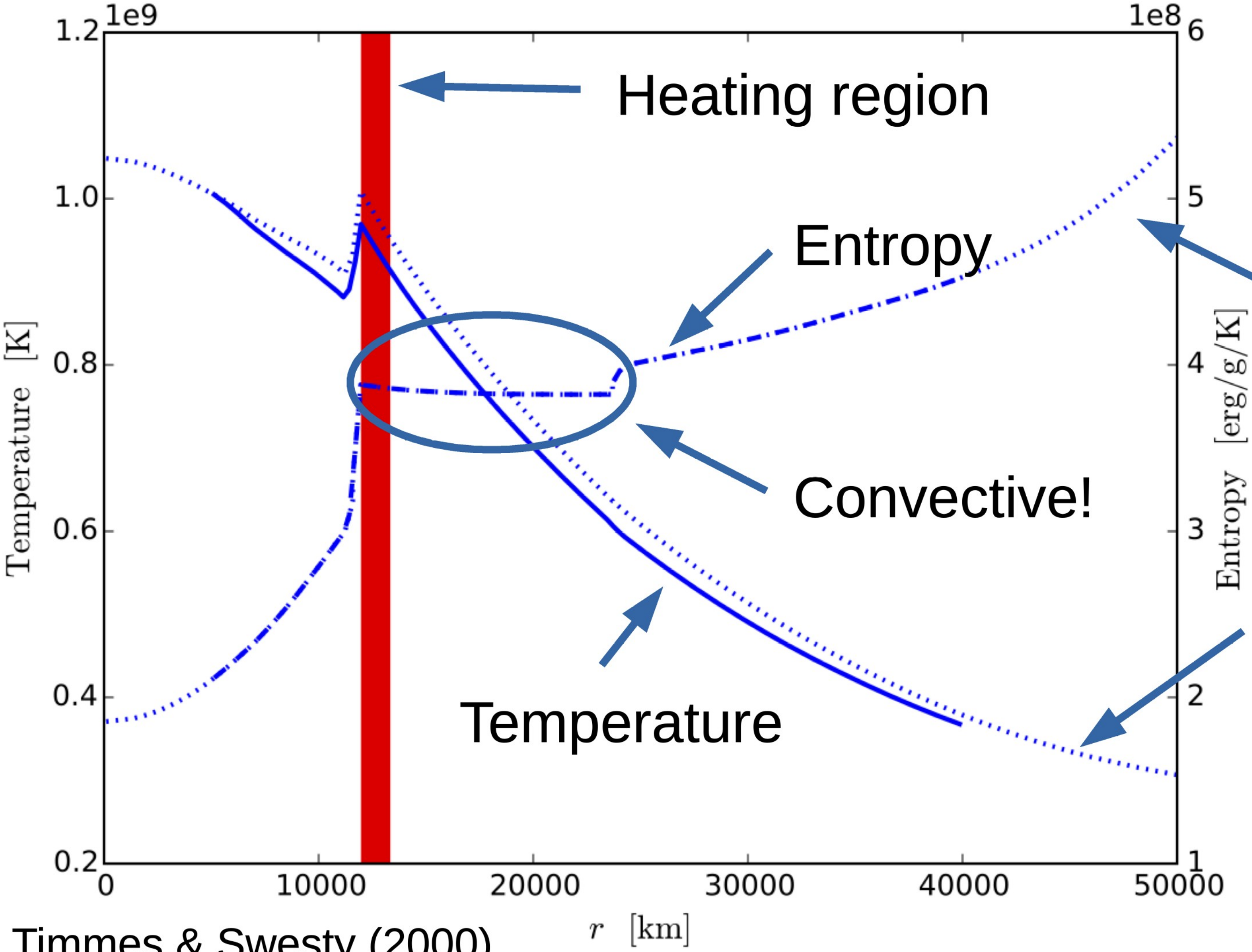


Profiles from stellar evolution kindly provided by **M. Pignatari**

Helmholtz EoS, Timmes & Swesty (2000)

Stellar convection

- Convective shell “carbon burning” **HIGHLY SIMPLIFIED!!!**



Heating region

Entropy

Convective!

Temperature

Profiles from stellar evolution kindly provided by **M. Pignatari**

Helmholtz EoS, Timmes & Swesty (2000)

Stellar convection

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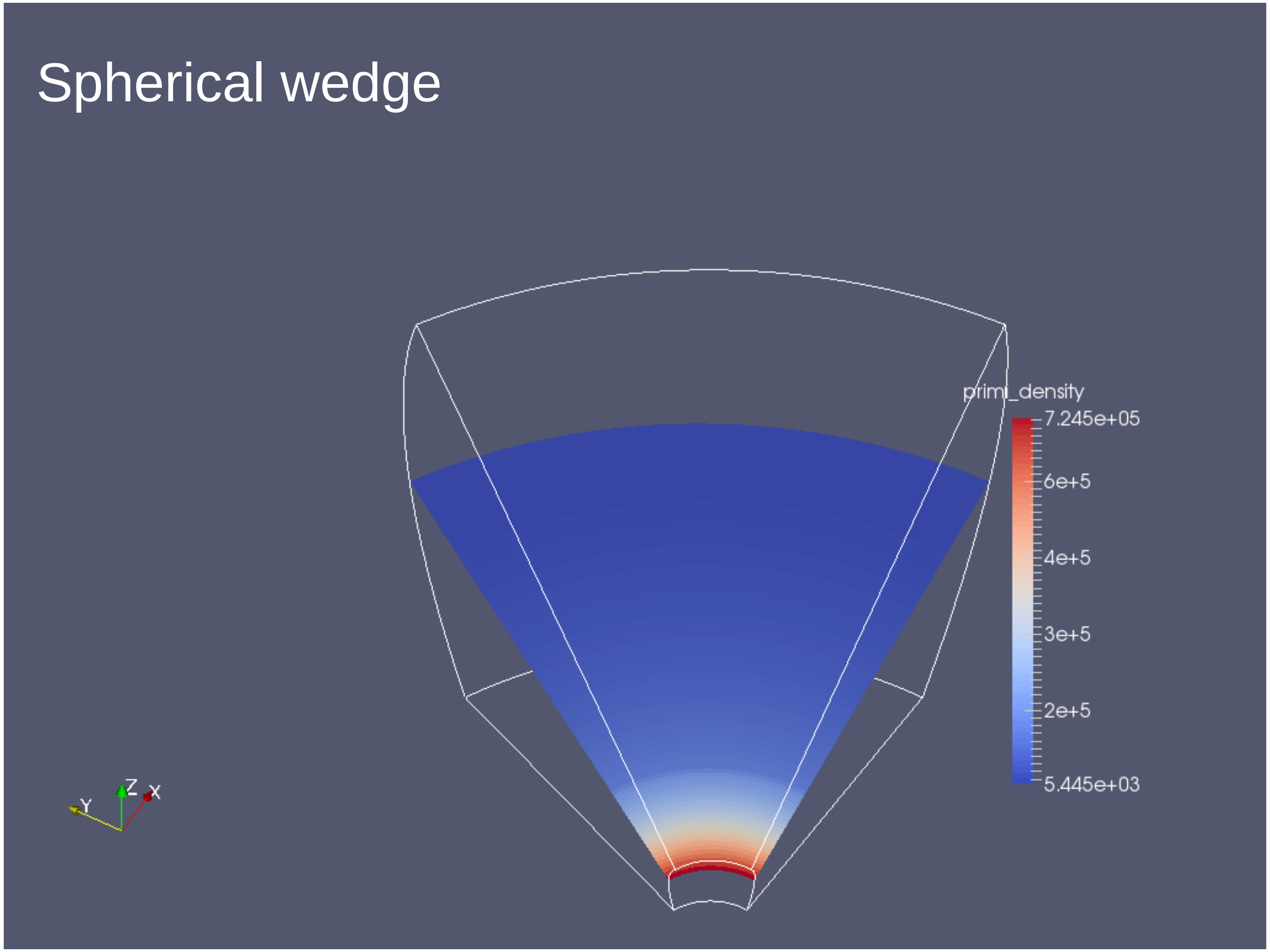
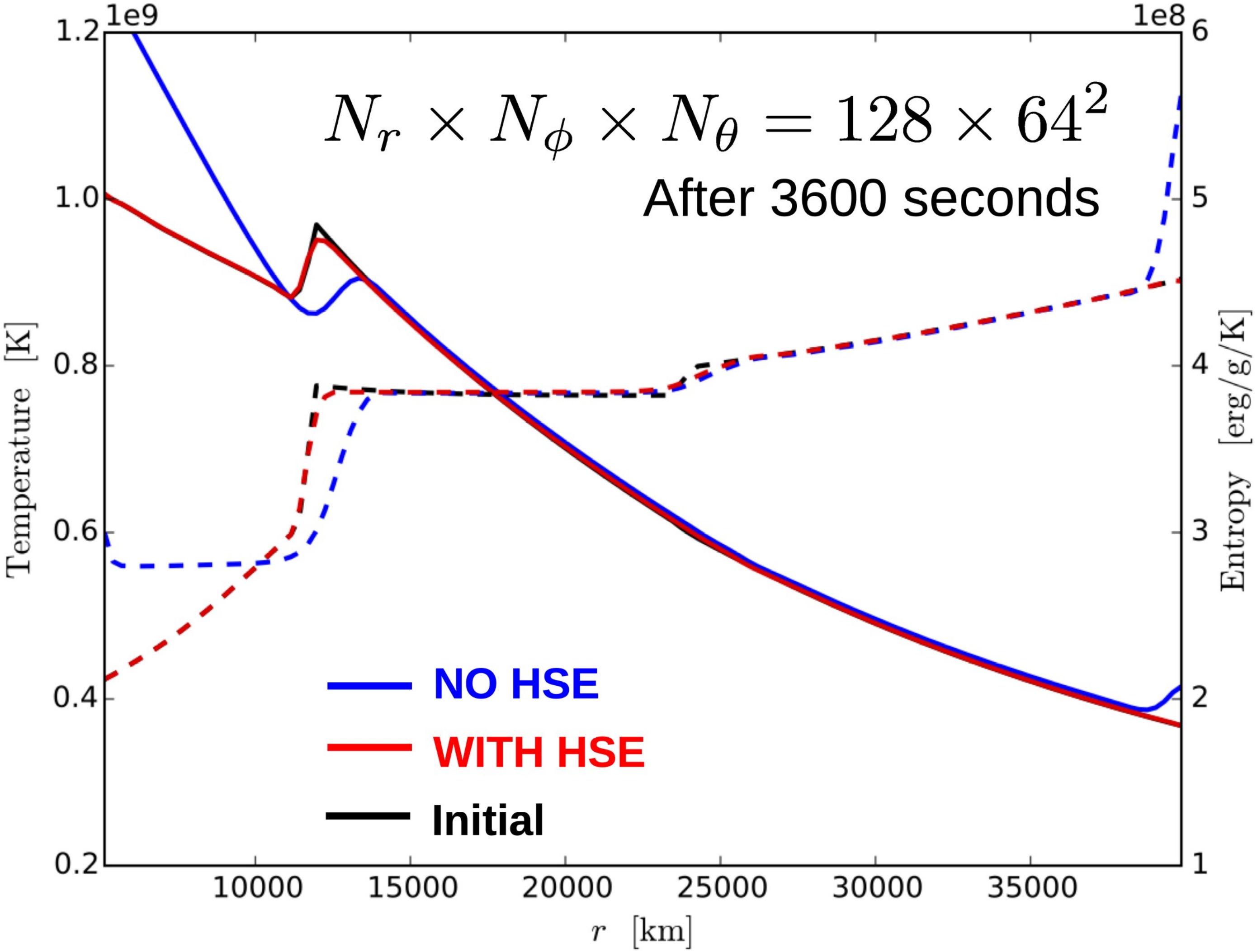


Stellar convection

- Convective shell “carbon burning” **HIGHLY SIMPLIFIED!!!**

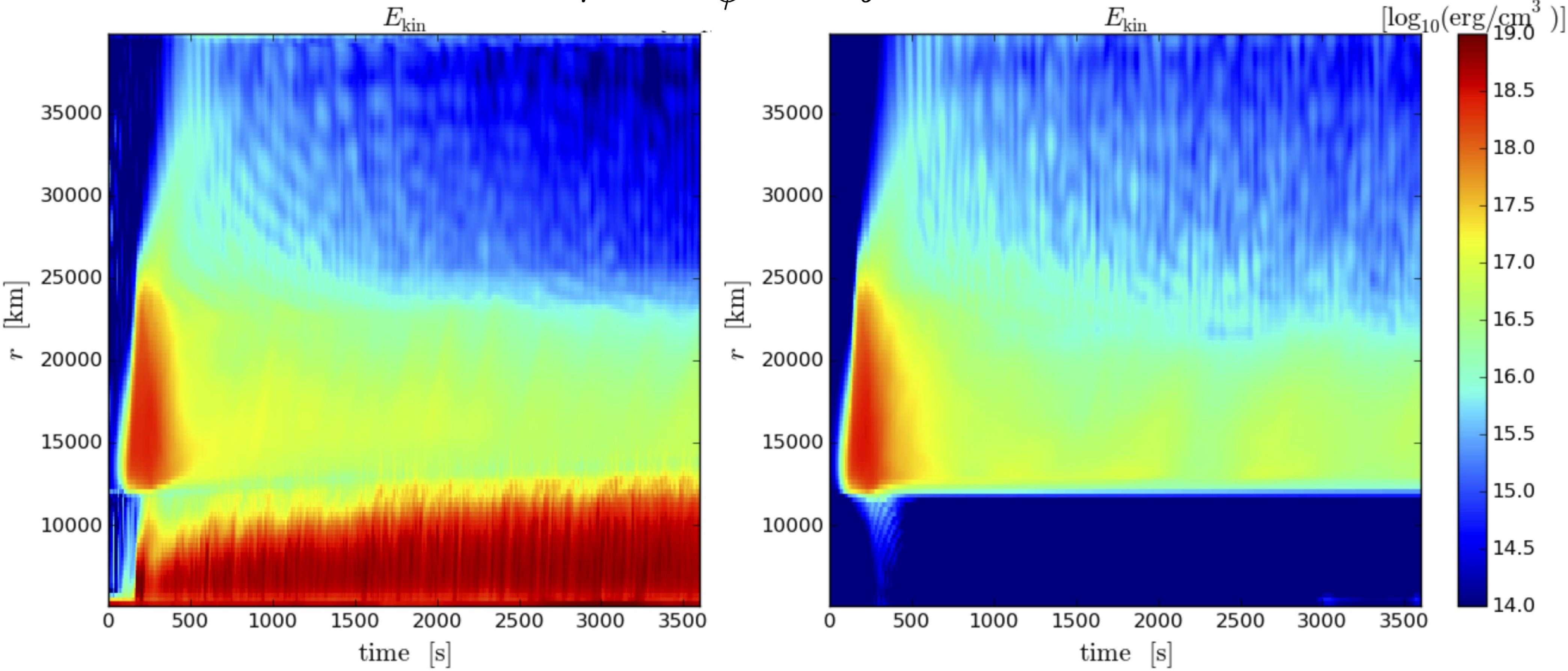


Stellar convection

- Convective shell “carbon burning” **HIGHLY SIMPLIFIED!!!**

Kinetic energy

$$N_r \times N_\phi \times N_\theta = 128 \times 64^2$$



Outline

- **Introduction**
  - A brief introduction of the problem
- **Physical model, numerical & computational aspects**
  - Physics ingredients & mathematical model
- **Simulation of magneto-rotational core-collapse**
  - MHD CCSN mechanism
  - Magnetic field amplification, formation and driving mechanism of bipolar outflow
  - Explosion energy, ejected mass and its composition
- **More on numerical & computational aspects**
  - Well-balanced methods for hydrostatic equilibrium
- **Conclusion**

Conclusion

- A brief introduction to the problem of CCSN
- Simulation of CCSN a challenging field for physics/computational science/numerics
- Highly sophisticated 1D models do not explode
  2D & ultimately 3D models with comparable sophistication are needed (what is important?)
- New physics?
- MHD mechanism works and performs a strong r-process nucleosynthesis
  BUT: it s probably not the standard explosion mechanism!

The End, Thanks!

Solution Algorithm: Neutrinos

- In principle, should solve the relativistic Boltzmann eq

4-vector

4-momentum

Distribution function

Spectral Leakage scheme: only cooling

Electron or positron capture on nucleons and nuclei

$$e^- + p \leftrightarrow n + \nu_e$$
$$e^+ + n \leftrightarrow p + \overline{\nu}_e$$
$$e^- + (A,Z) \leftrightarrow (A,Z-1) + \nu_e$$

Elastic coherent scattering of neutrinos on nucleons and nuclei

$$\nu + N \leftrightarrow \nu + N$$
$$\nu + (A,Z) \leftrightarrow \nu + (A,Z)$$

Cercignani

emer 2002

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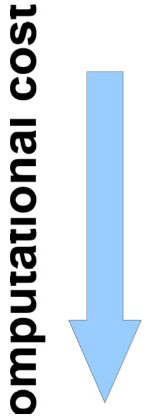
- Appr



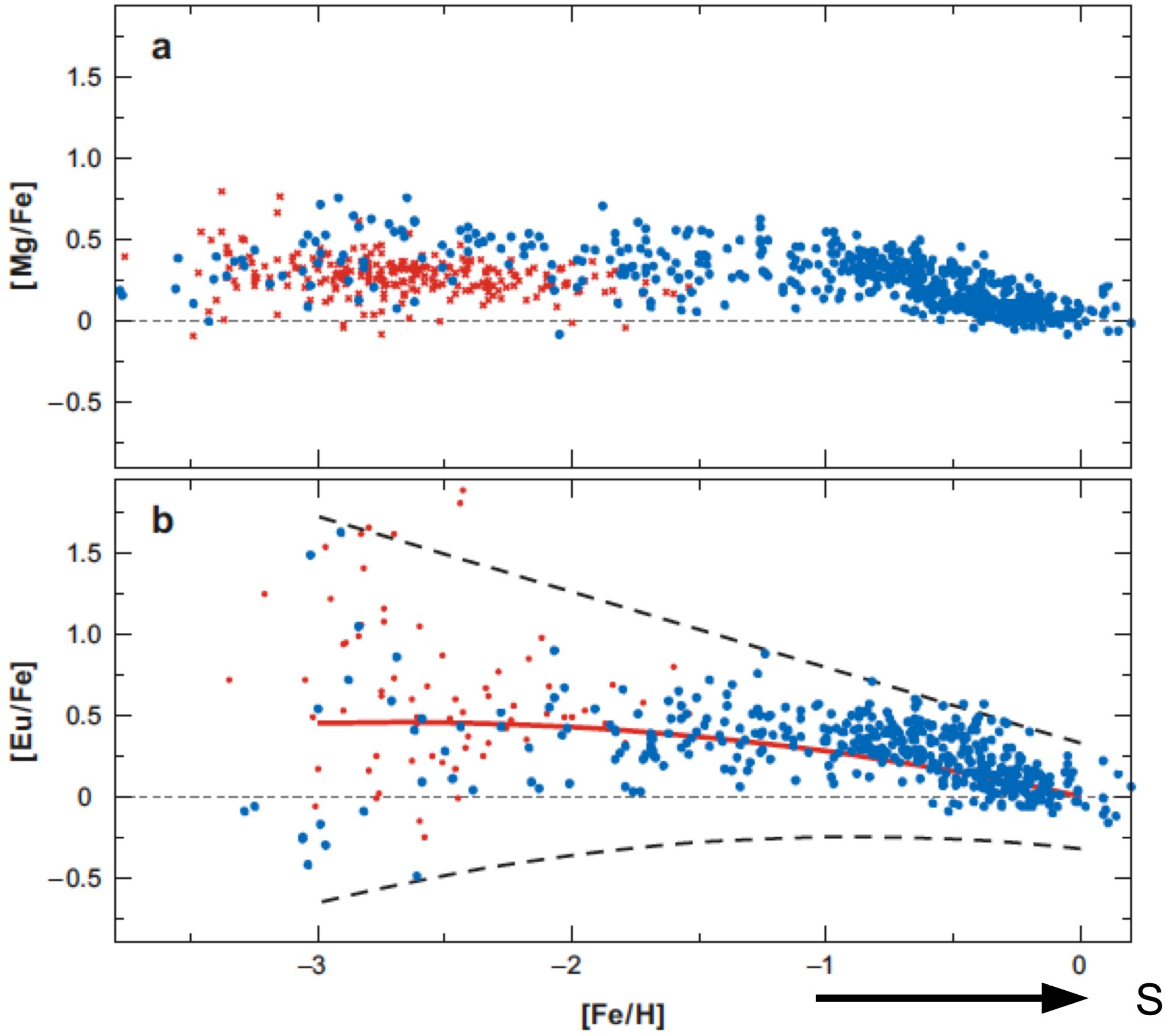
  - Parametrisation scheme: only collapse phase Liebendörfer 2005
  - Spectral Leakage scheme: only cooling A. Perego et al. in prep.

    Epstein & Pethick 1981, van Riper & Lattimer 1981, ..., Ruffert et al. 1996, Rosswog & Liebendörfer 2003
  - Isotropic Diffusion Source Approx. (IDSA) Liebendörfer et al. 2009

Early galactic r-process nucleosythesis

α -element commonly produced in massive star

Typical r-process element



SNIa

From Sneden, Cowan & Gallino 2008

Neutrino Reactions Leakage

Electron or positron capture on nucleons and nuclei

$$e^- + p \leftrightarrow n + \nu_e$$

$$e^+ + n \leftrightarrow p + \overline{\nu}_e$$

$$e^- + (A,Z) \leftrightarrow (A,Z-1) + \nu_e$$

Elastic coherent scattering of neutrinos on nucleons and nuclei

$$\nu + N \leftrightarrow \nu + N$$

$$\nu + (A,Z) \leftrightarrow \nu + (A,Z)$$