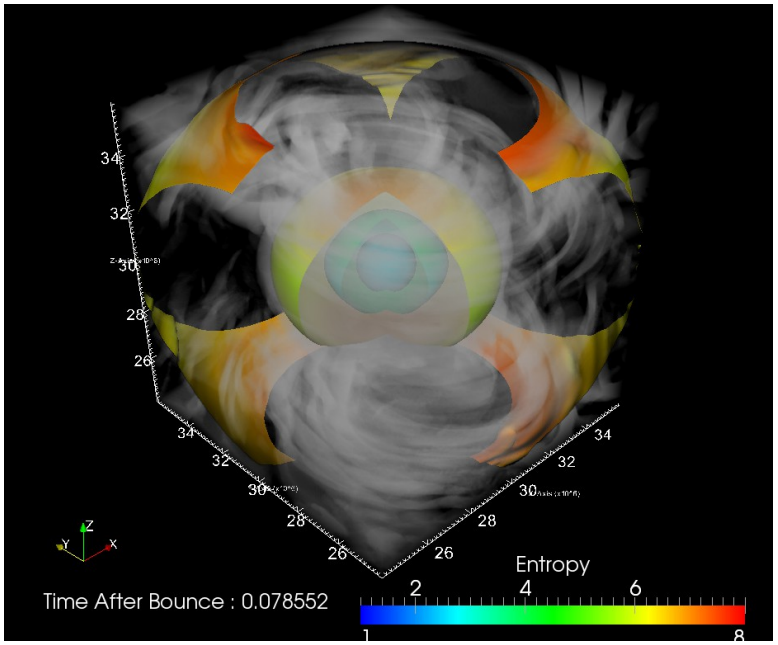
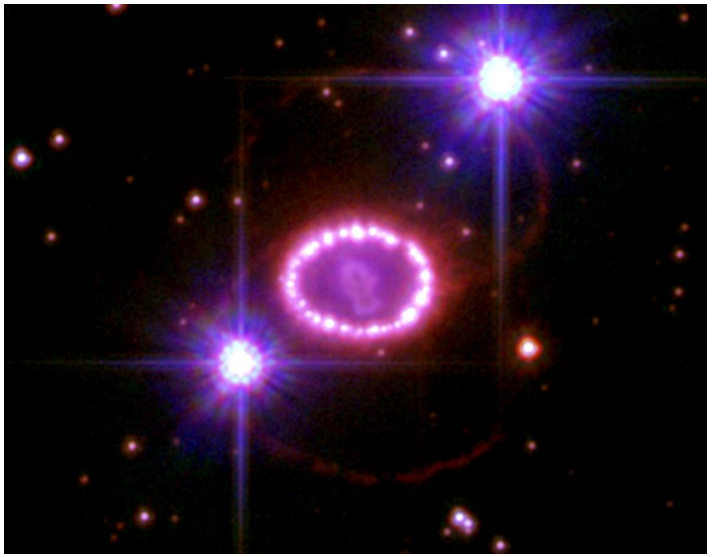
Computational astrophysics
The explosion mechanism of massive stars

Roger Käppeli

Seminar for Applied Mathematics SAM

ETH





Outline

- **Introduction**
  - A brief introduction of the problem
- **Physical model, numerical & computational aspects**
  - Physics ingredients & mathematical model
- **Simulation of magneto-rotational core-collapse**
  - MHD CCSN mechanism
  - Magnetic field amplification, formation and driving mechanism of bipolar outflow
  - Explosion energy, ejected mass and its composition
- **Conclusion**

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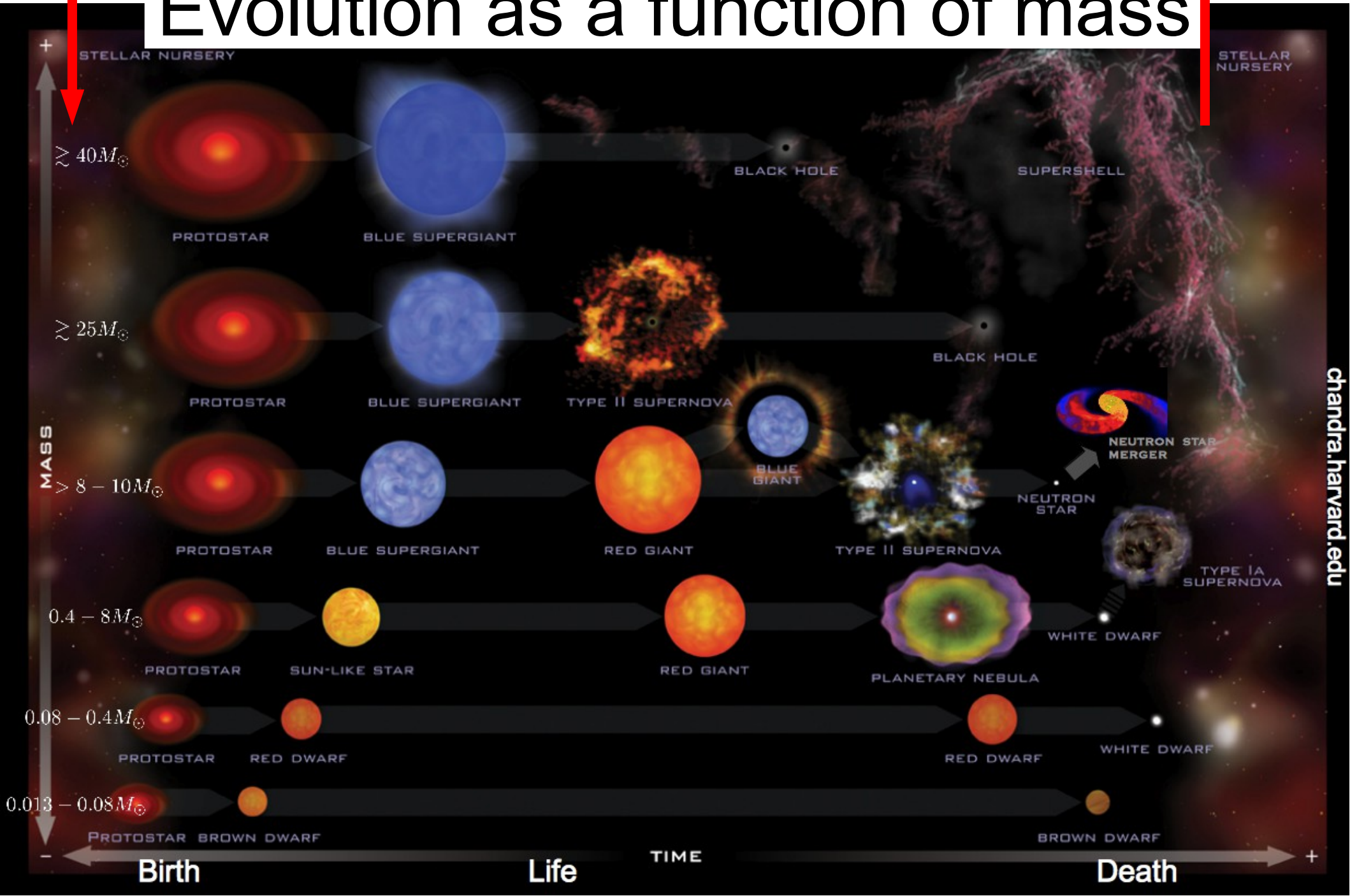
Stellar life cycle

Evolution as a function of mass



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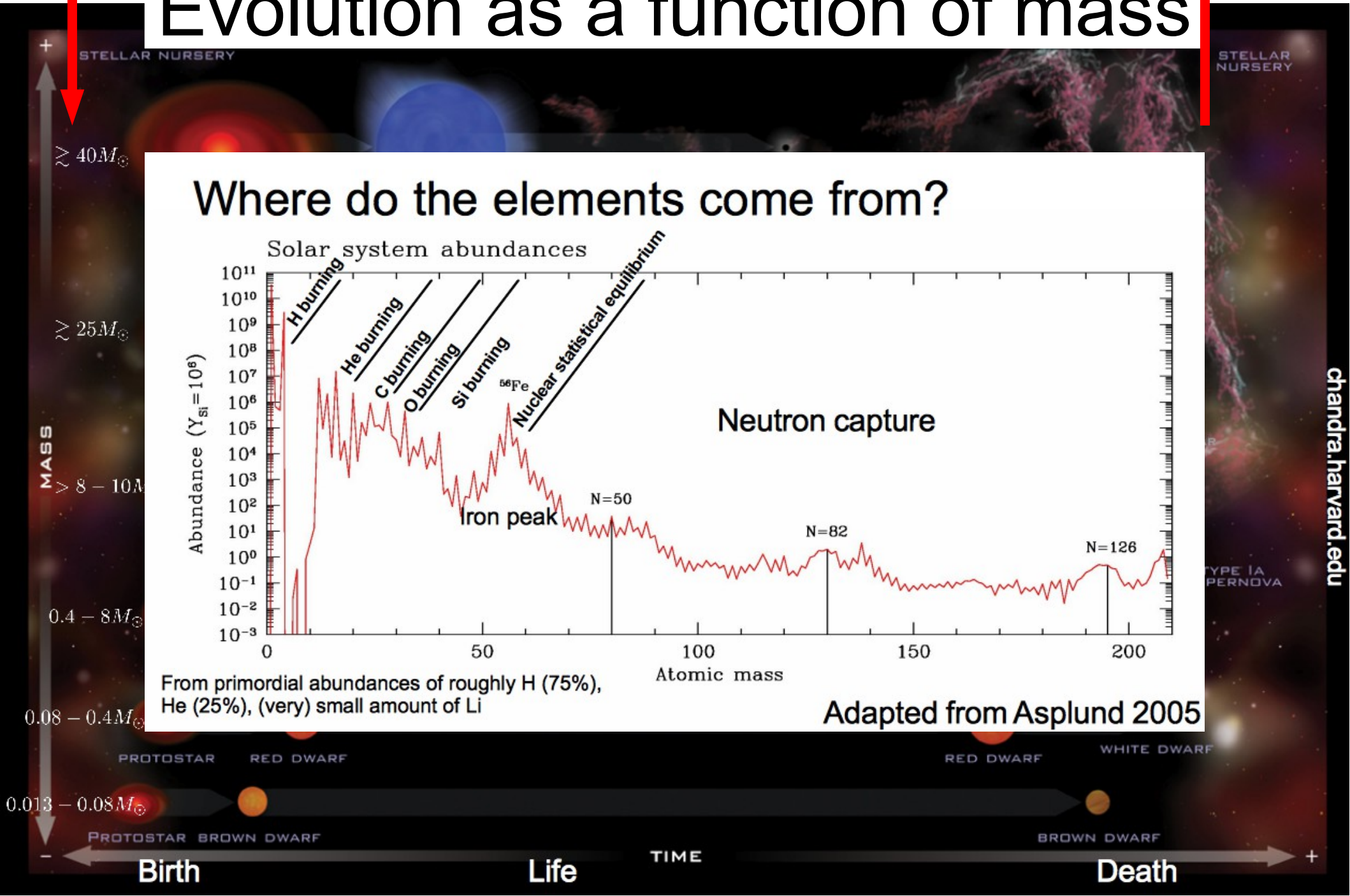

Where do the elements come from?

From primordial abundances of roughly H (75%), He (25%), (very) small amount of Li

Adapted from Asplund 2005

Stellar life cycle

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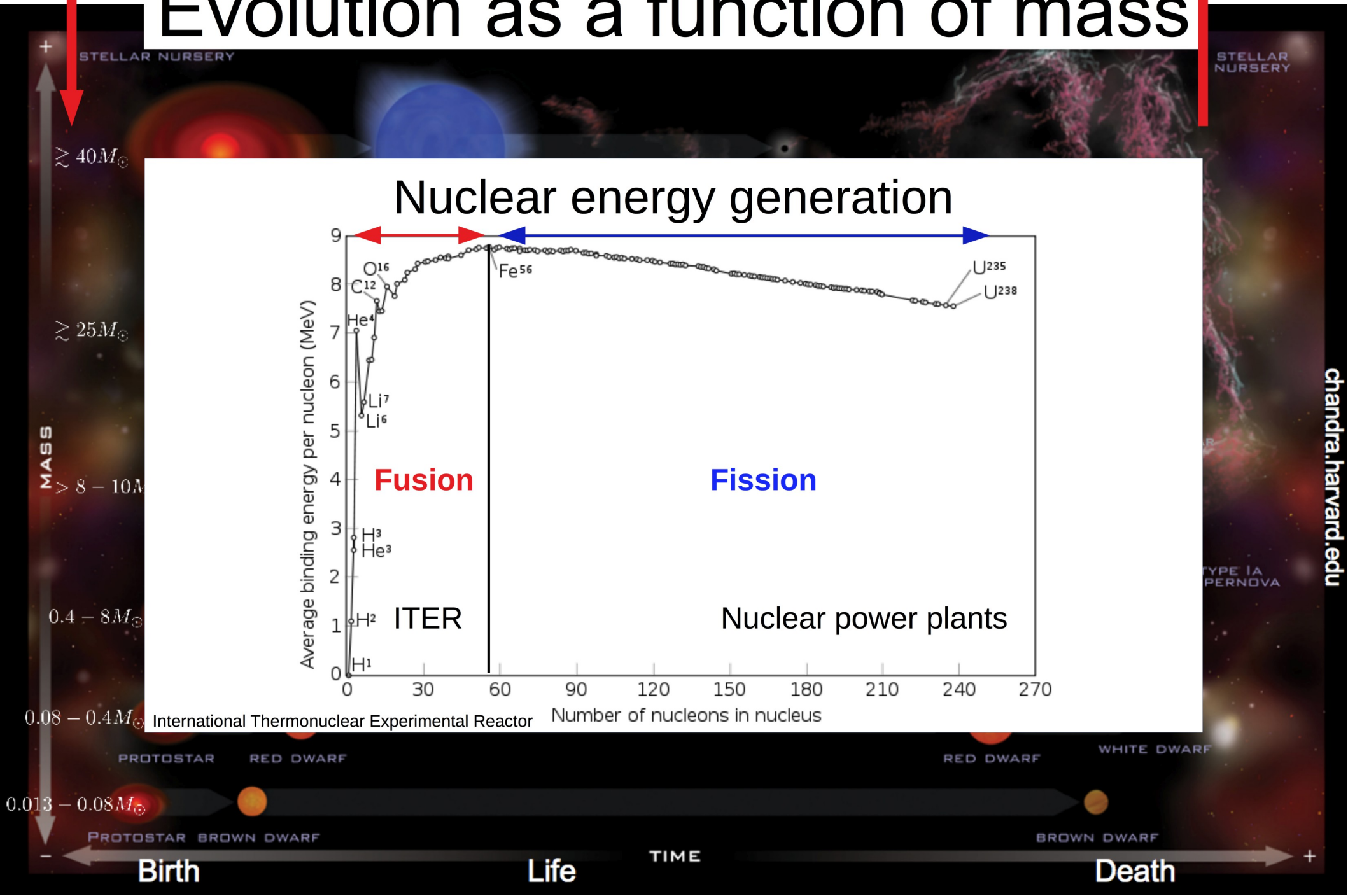
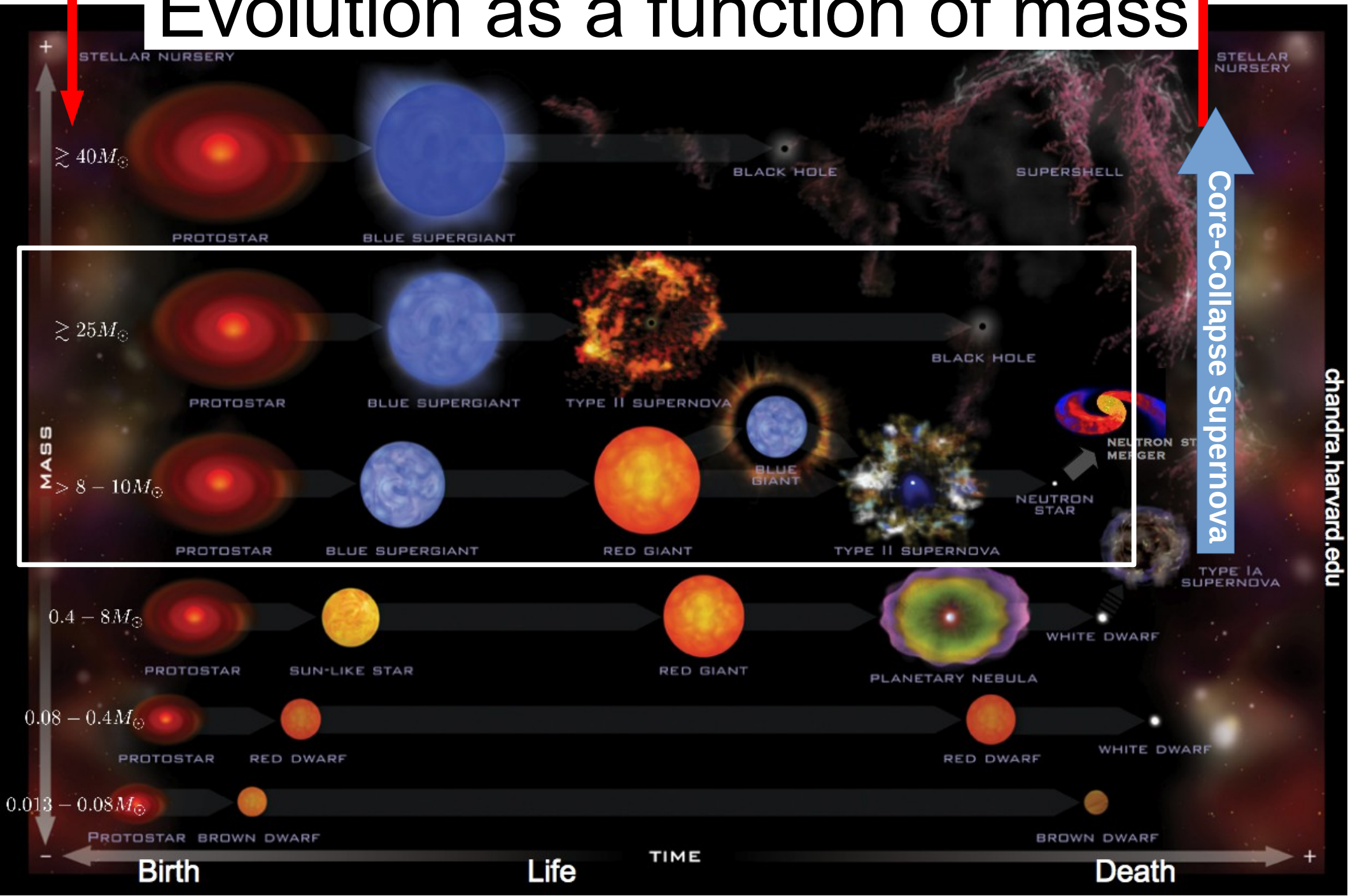


Nuclear energy generation

International Thermonuclear Experimental Reactor

Stellar life cycle

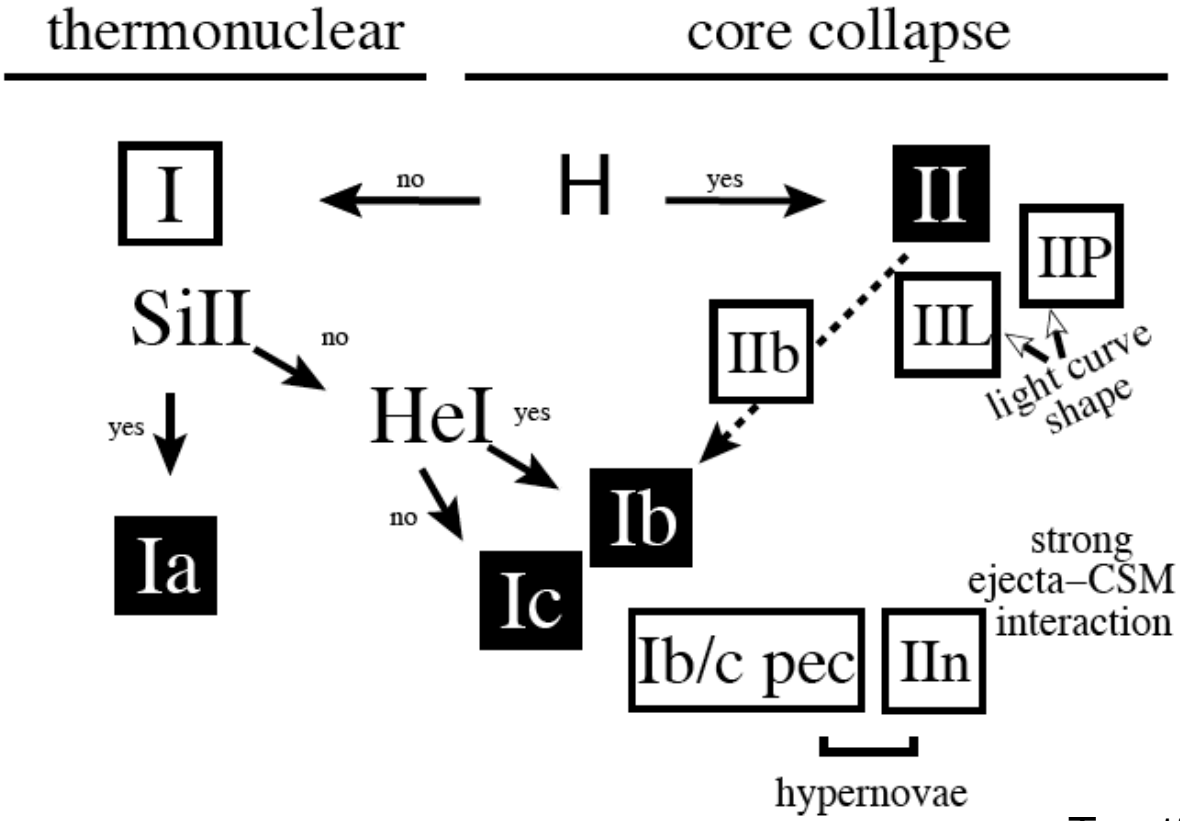
Evolution as a function of mass



Supernovae classification

- Taxonomical/Morphological approach

Like botanists and zoologists, find observable characteristics that eventually provide a deeper physical understanding. However, not all necessarily meaning full...



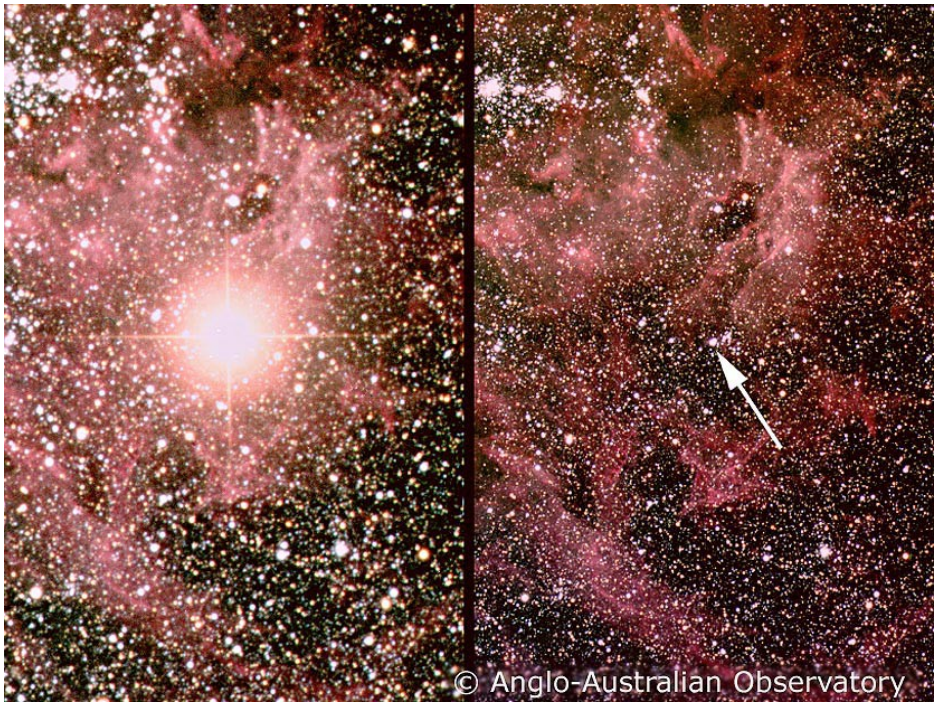
Tarutto 2003

Core-collapse supernova

$$1\ \mathrm{erg} = 10^{-7}\mathrm{J}$$

- Huge energy scales
  - ~1e+53 erg neutrinos
  - ~1e+51 erg mechanical
  - ~1e+48 erg elm
  - ~1e+41 erg visible elm
- Observables
  - Elm
  - Neutrinos
  - Gravitational waves

SN1987A



After Before

Neutrino & GW astronomy!!!

SN1987A in the Large Magellanic Cloud (a nearby galaxy) ~ 163'000 light-years away!

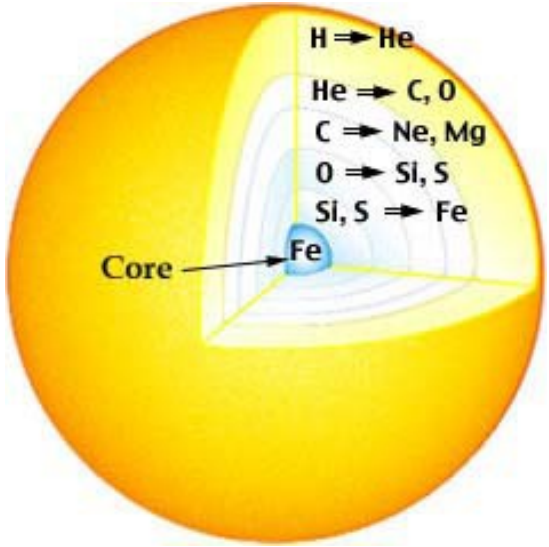
Core-collapse supernova

- General idea:
  - Implosion of iron core of massive star $M \gtrsim 8M_\odot$ at the end of thermonuclear evolution
  - Explosion powered by gravitational binding energy of forming compact remnant:

$$E_b \approx 3 \times 10^{53} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{R}{10\mathrm{km}}\right)^{-1} \mathrm{erg}$$

M Mass of remnant

R Radius of remnan



GRAVITY BOMB!

Conditions at onset of collapse

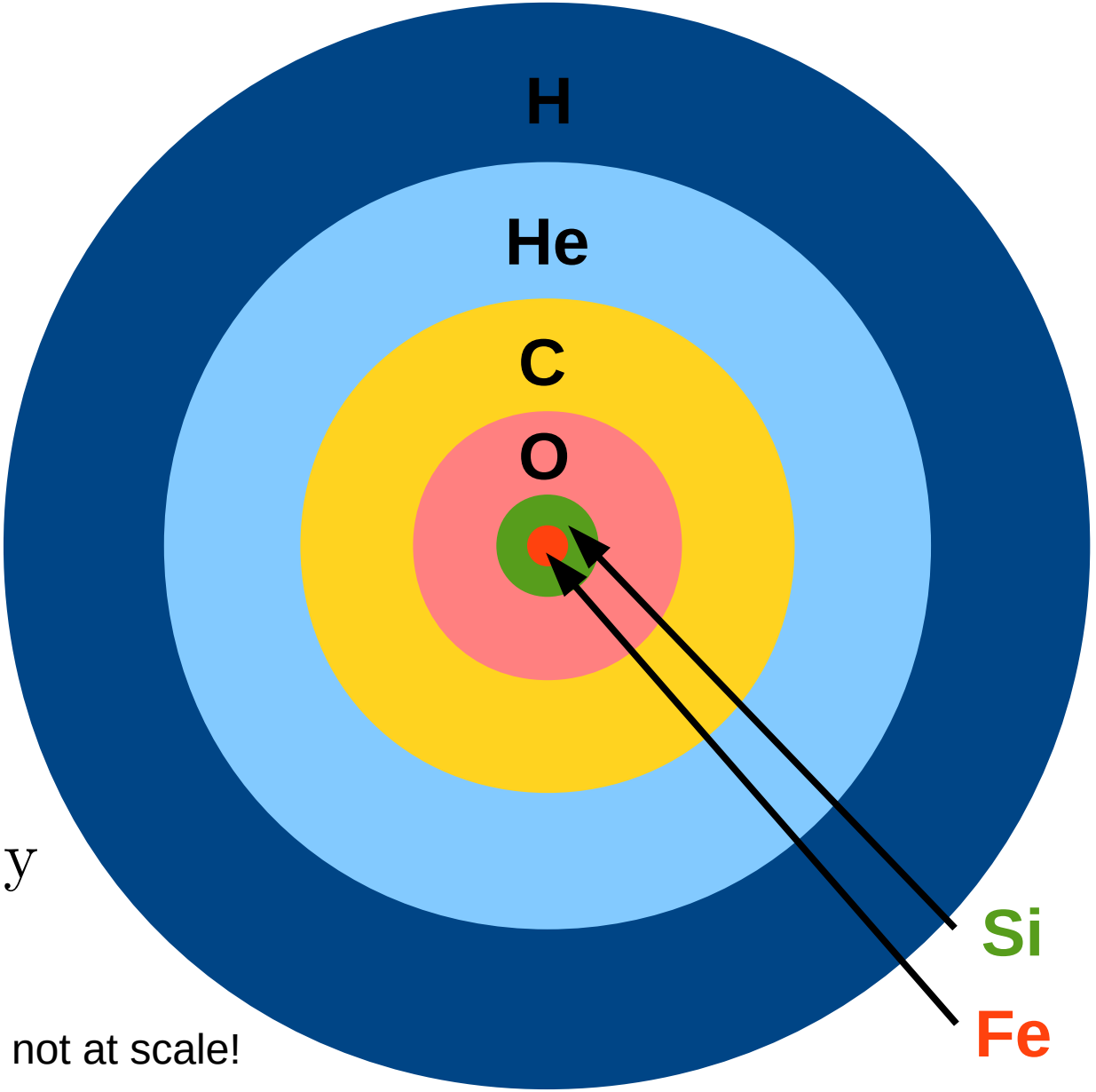
Evolved massive star prior to collapse

Onion-like structure due to nuclear burning stages

E.g.: Mass ~ $15 M_\odot$

Age ~ 2×10^6 y

Size ~ 10^8 km



Shells not at scale!

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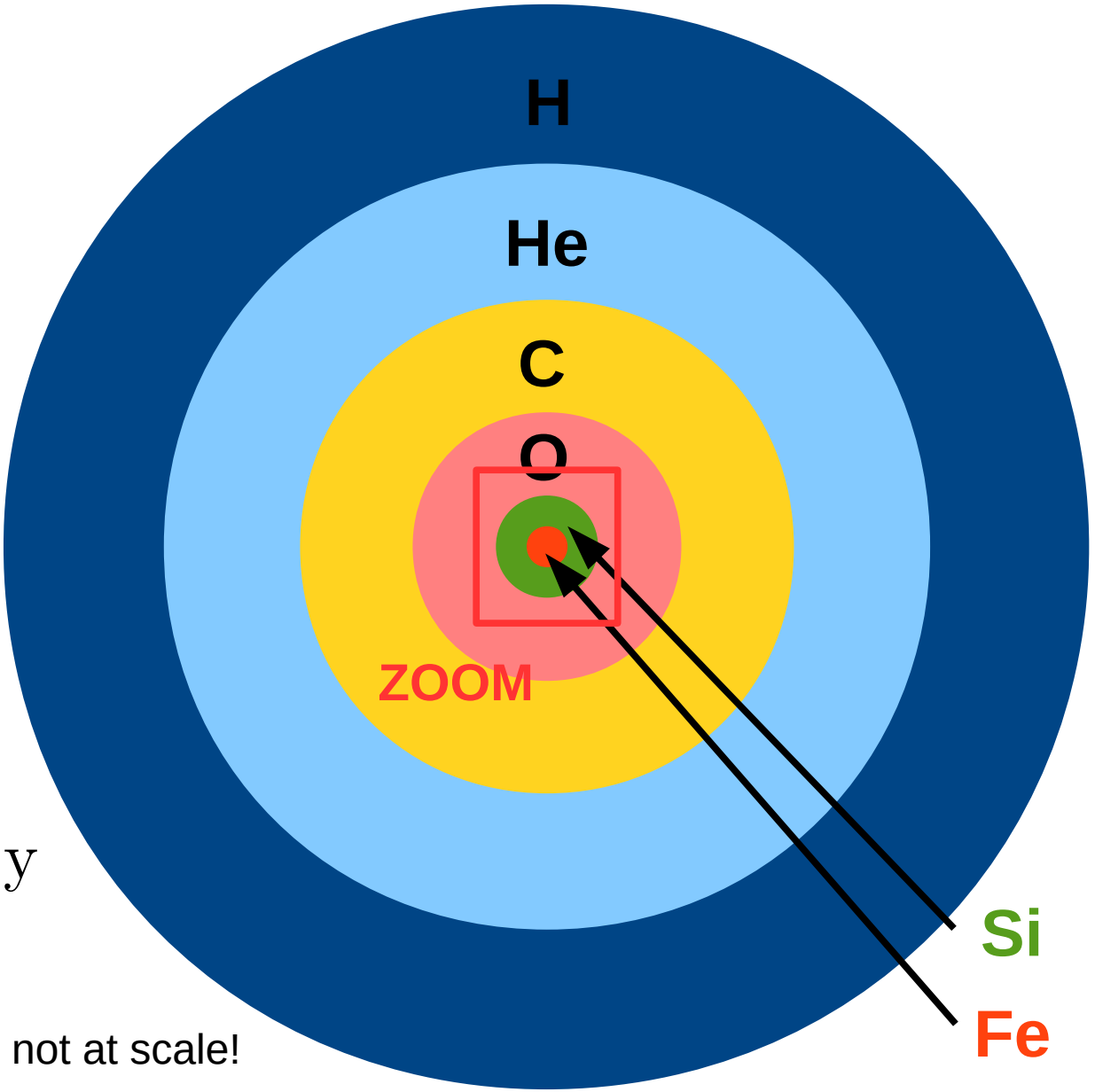


Shells not at scale!

Conditions at onset of collapse

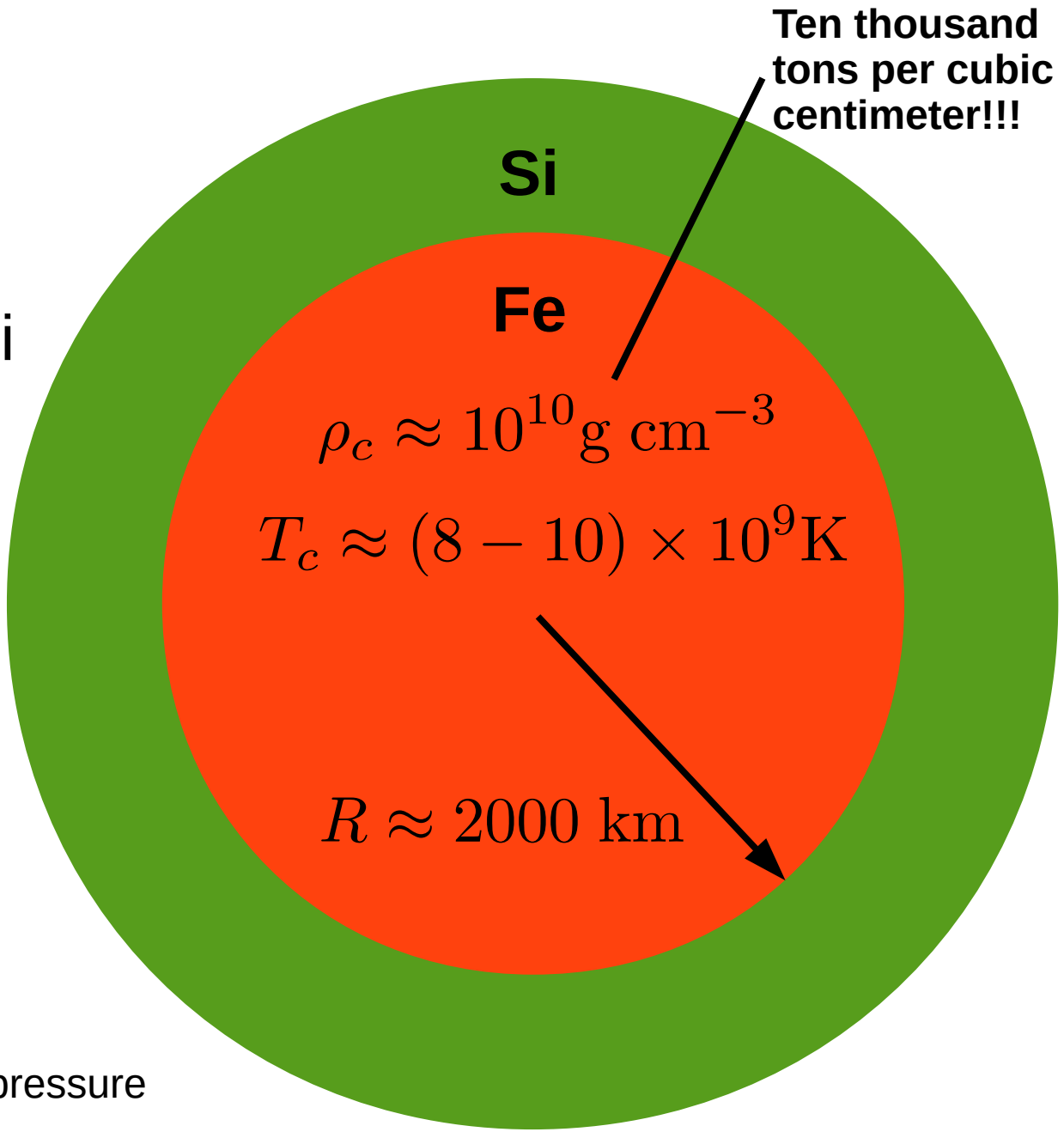
Core made of ashes from Silicone burning... Mainly iron group nuclei

IRON CORE

Iron core stabilized against gravity by relativistic and degenerate electrons

$$P_e \gg P_{ion} \gg P_{rad}$$

Electron Ion Photon pressure



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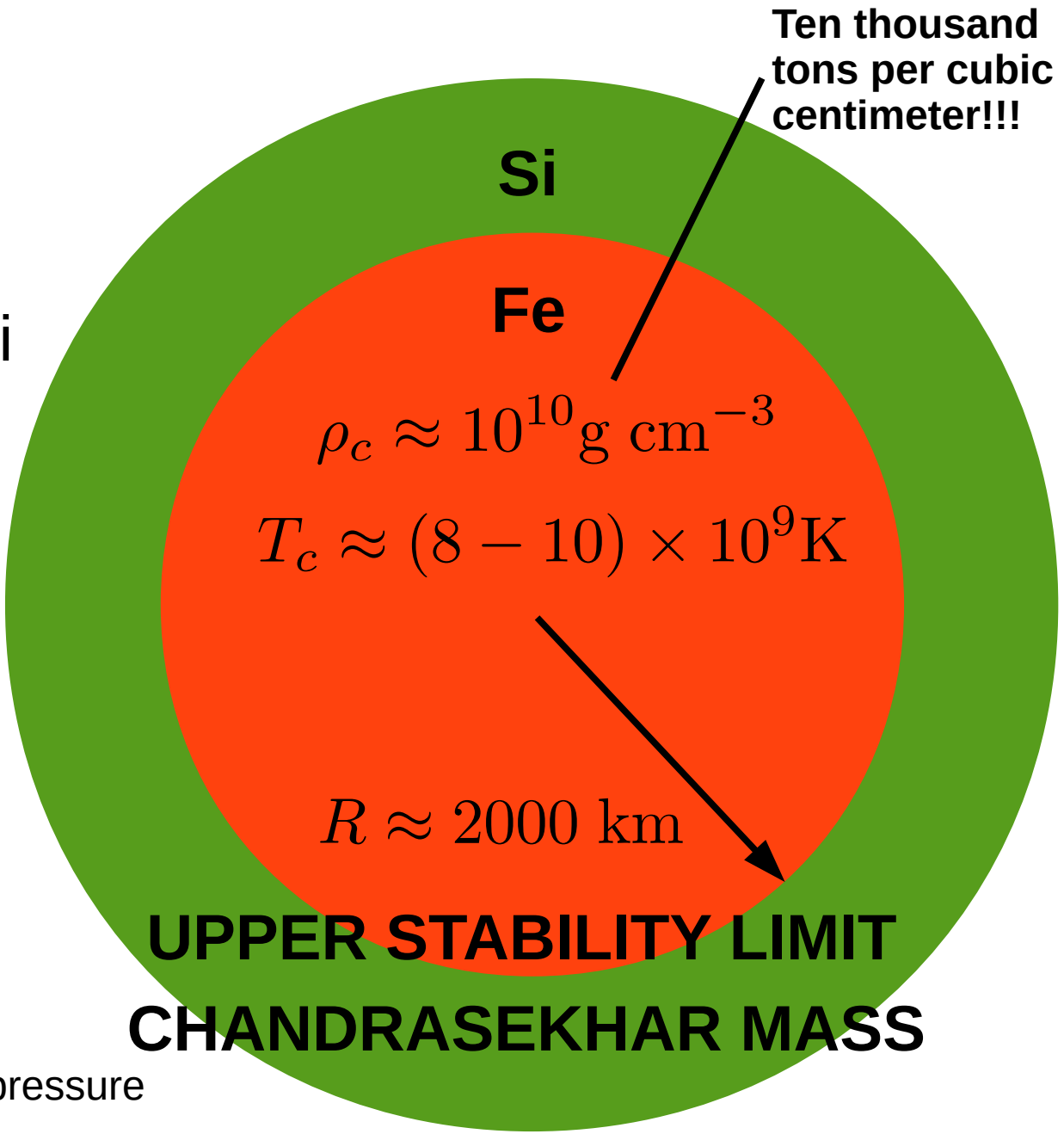


Collapse

Iron core collapse due:

1. Mass grows due to accreting Si burning ashes ultimately reaching

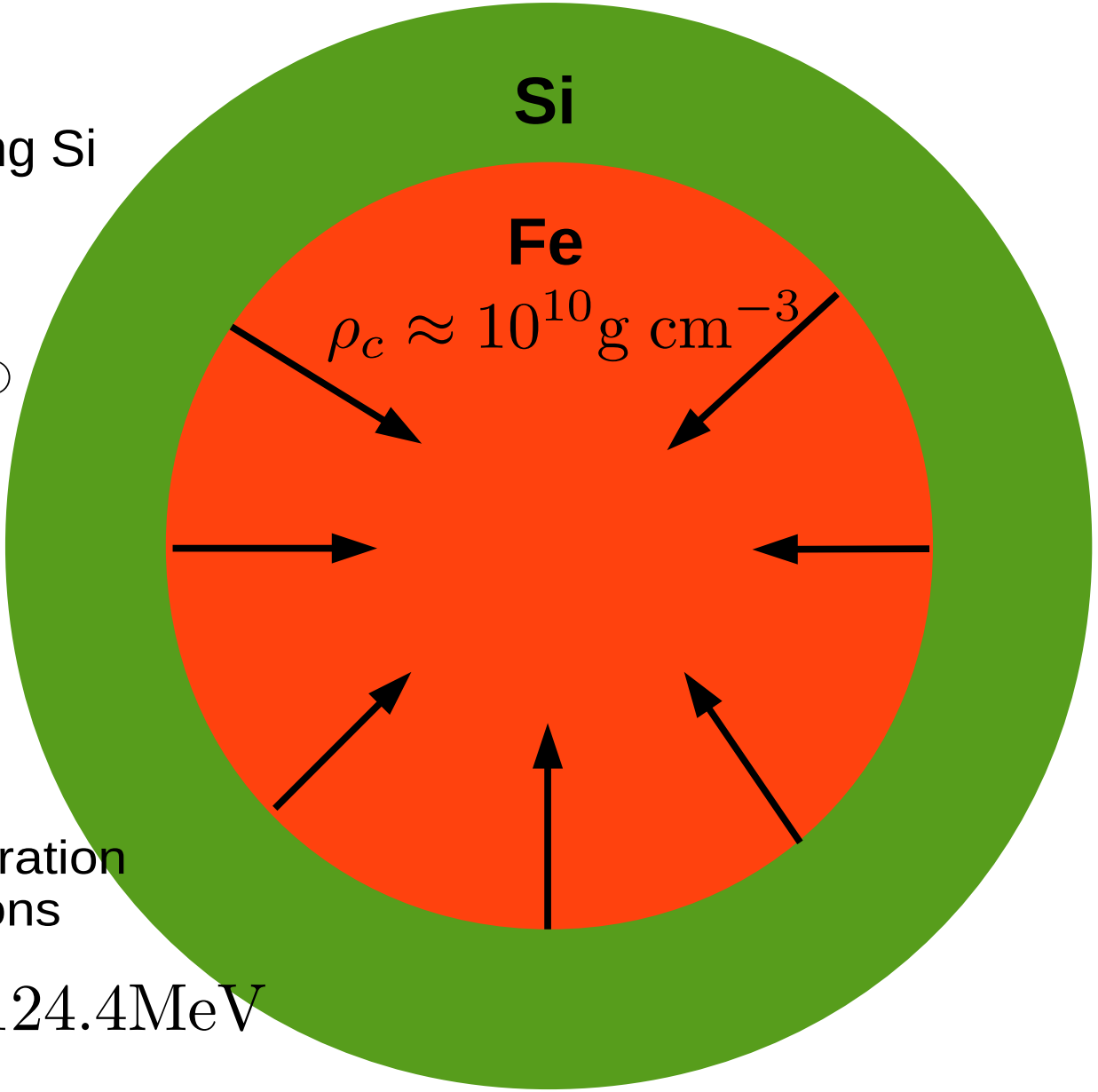
$$M_{Ch} = 1.457(2Y_e)^2 M_\odot$$

2. Electron captures reduce lepton number and neutrinos escape freely

$$^{56}Fe + e^- \rightarrow {}^{56}Mn + \nu_e$$

3. Pressure reduced due to endothermic photo-disintegration of nuclei by energetic photons

$$\gamma + {}^{56}Fe \rightarrow 13\alpha + 4n \; - 124.4\text{MeV}$$



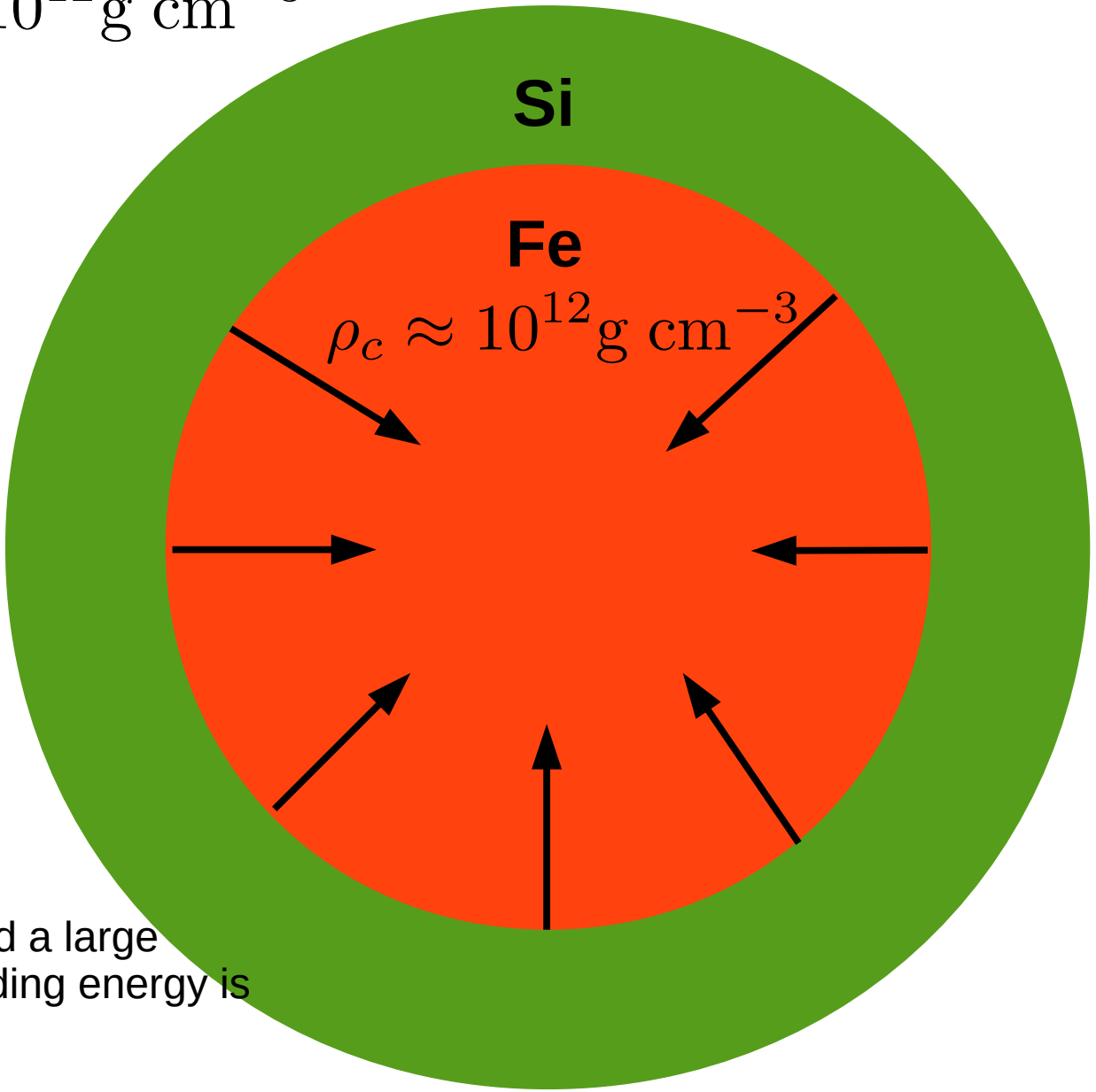
Collapse: trapping

Neutrino trapping: $\rho \geq 10^{12} \mathrm{g\ cm^{-3}}$

$$\tau_{diff} > \tau_{dyn}$$

The outward “diffusion speed” equals or is less than the infall speed

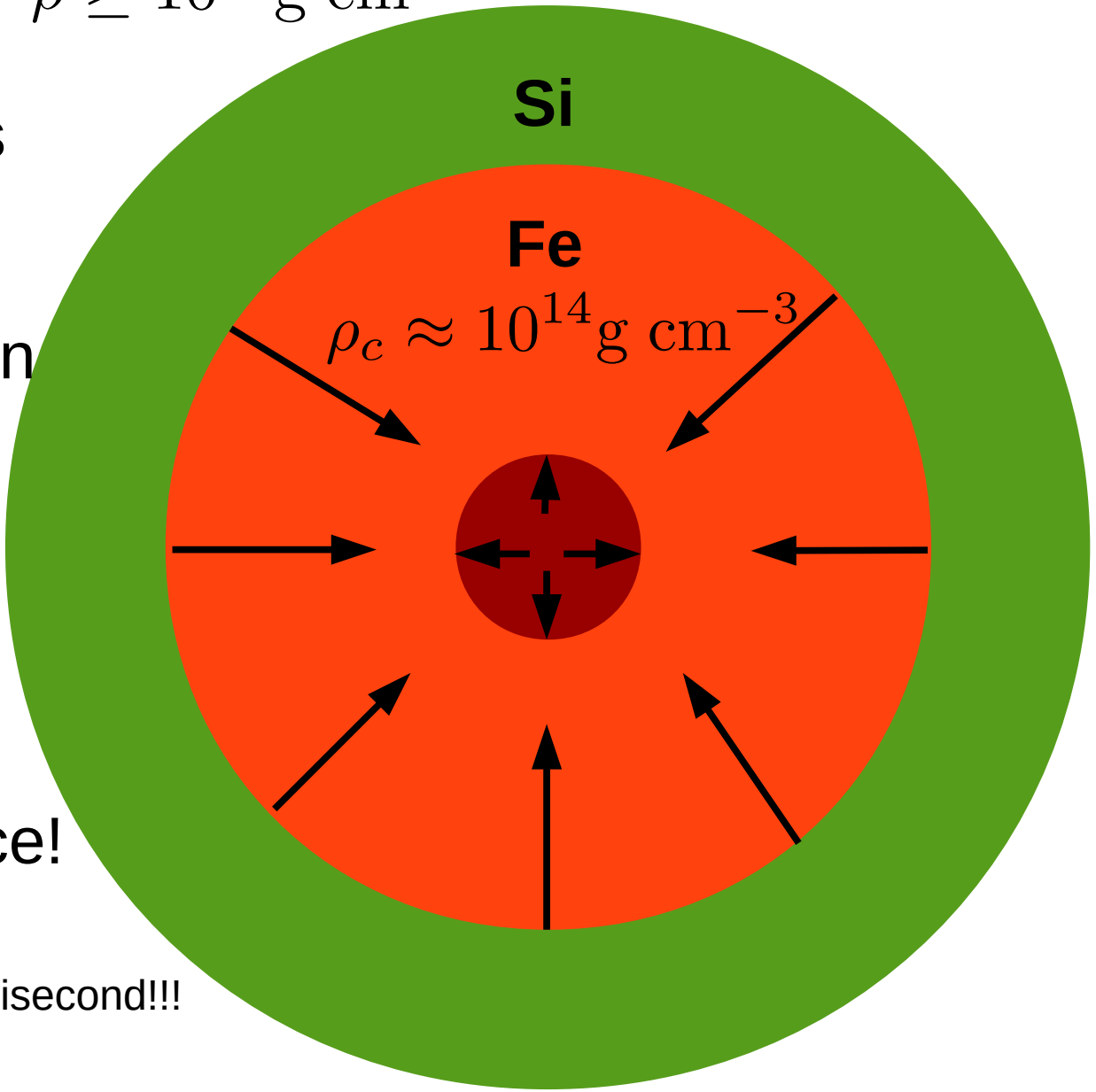
The collapse proceeds adiabatically



Neutrinos as still copiously produced and a large fraction of the liberated gravitational binding energy is accumulated in the neutrinos

Bounce

At densities exceeding $\rho \geq 10^{14}\mathrm{g\ cm^{-3}}$ the nuclei phase transition into nucleons that at such high densities experience strong nuclear repulsion



This halts and reverses the collapse: bounce!

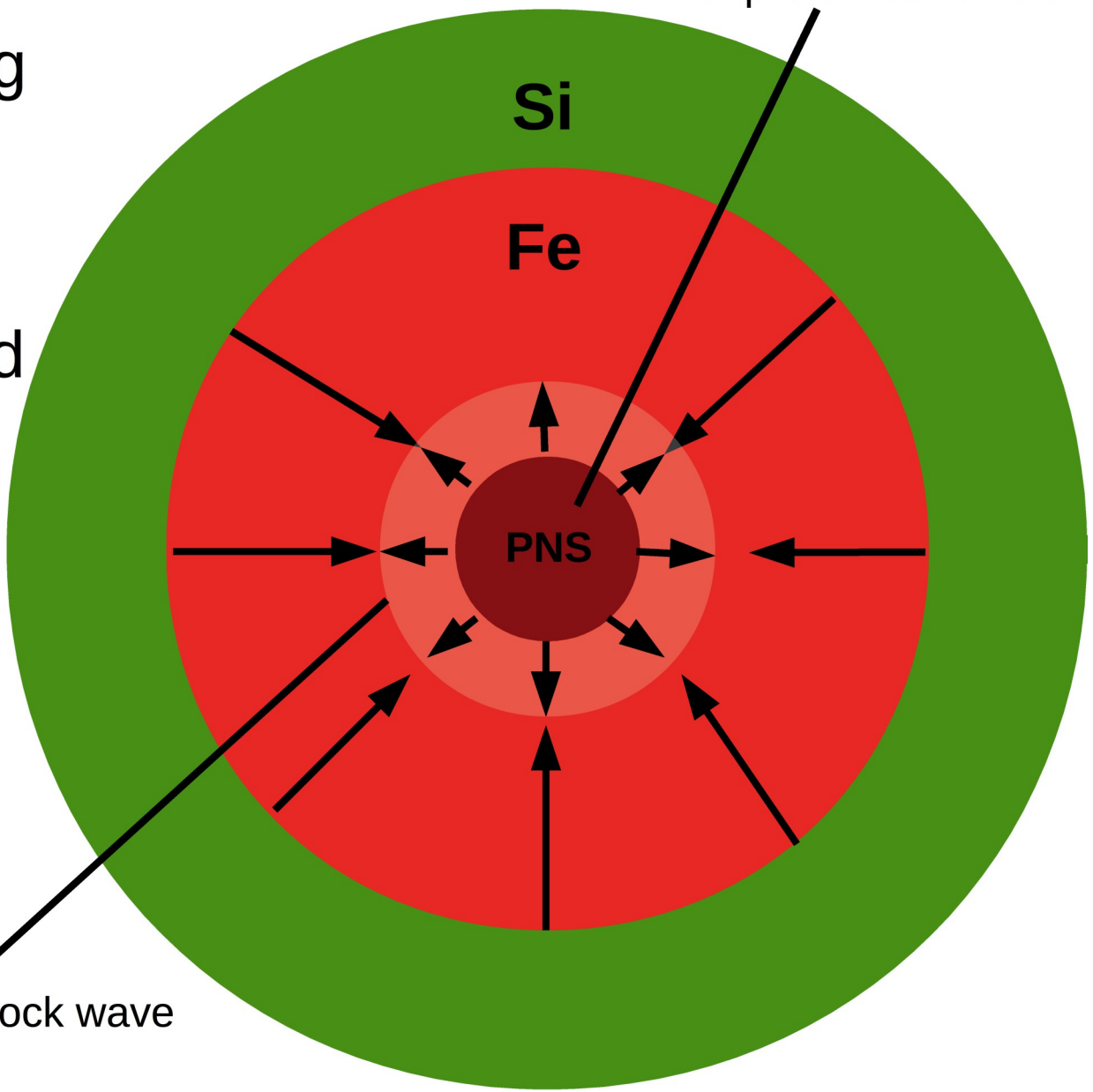
This happens within less than a millisecond!!!

Shock

The bounce launches an outward propagating strong shock wave

The mechanical energy transferred to the shock during bounce is enough to blow away the rest of the star!!!

SOLVED THE CCSN PROBLEM



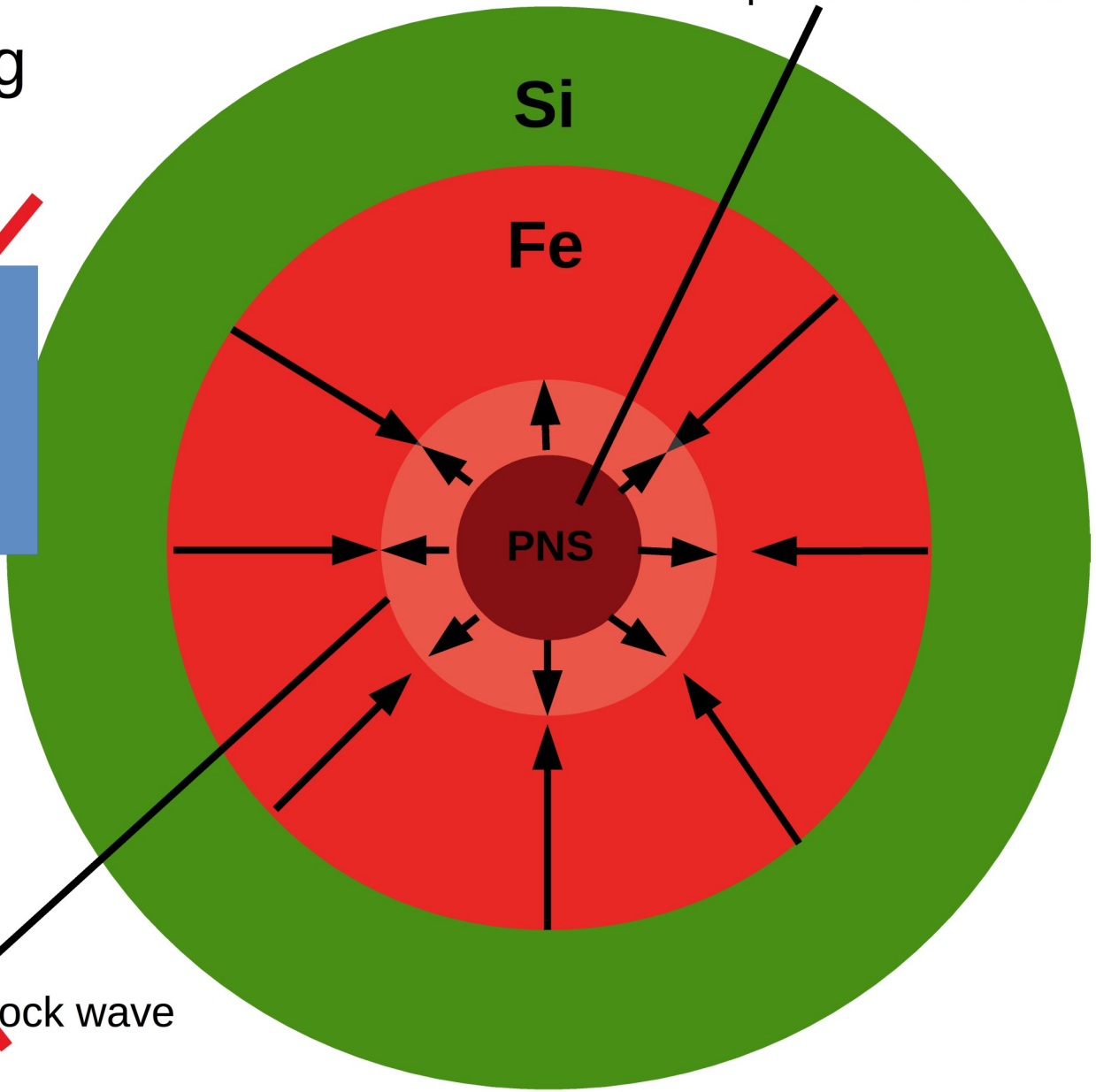
Shock

The bounce launches an outward propagating strong shock wave

The so-called prompt mechanism fails...

enough to blow away the rest of the star!!!

SOLVED THE CCSN PROBLEM



Shock stalling

The shock stalls due to:

1. Shock looses energy due to dissociation of iron nuclei

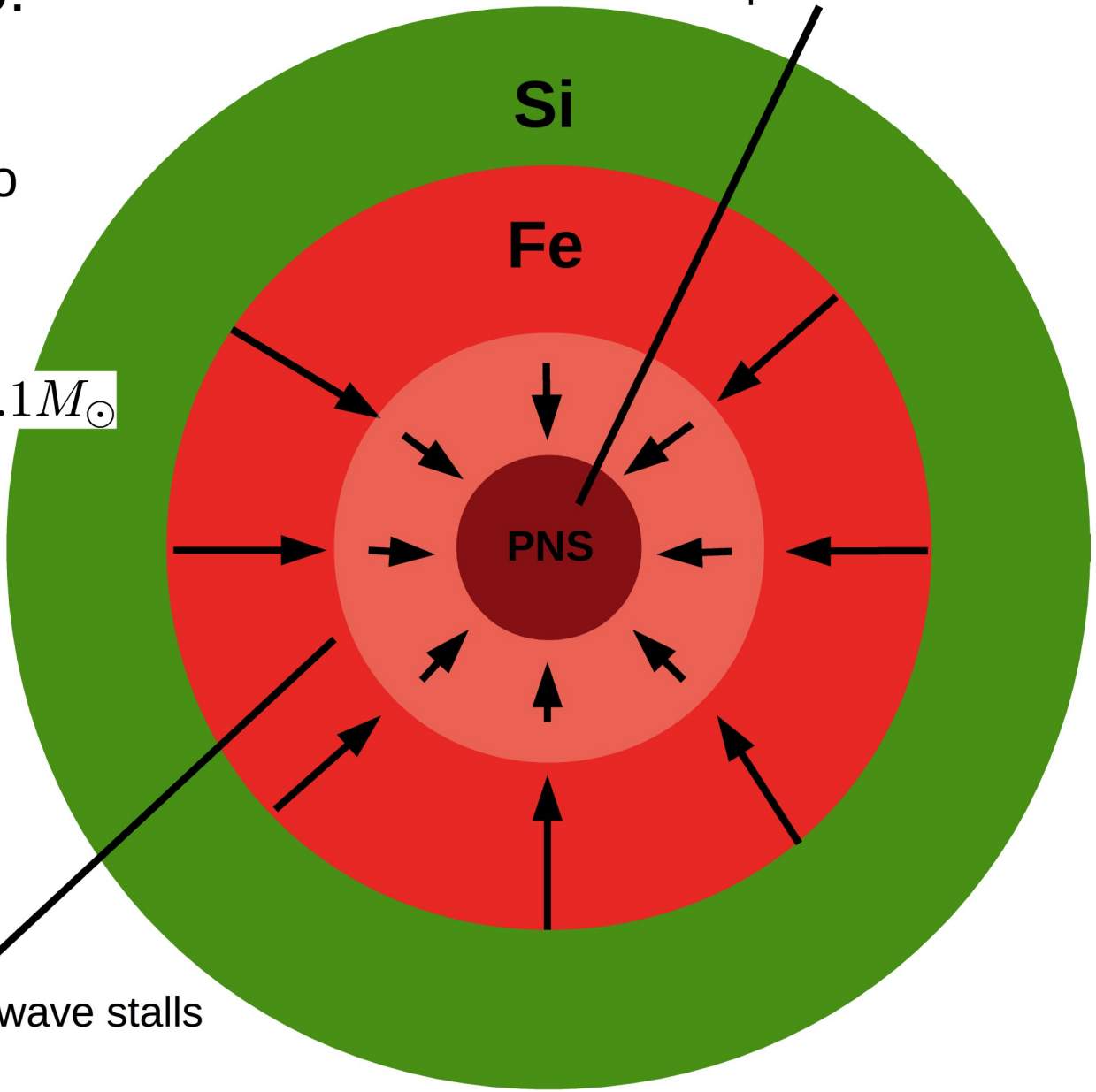
   ~8.8 MeV/nucleon

   $\sim (1.6 - 1.8) \times 10^{51} \mathrm{erg}/0.1 M_{\odot}$

2. Neutrino losses due to electron captures on free protons

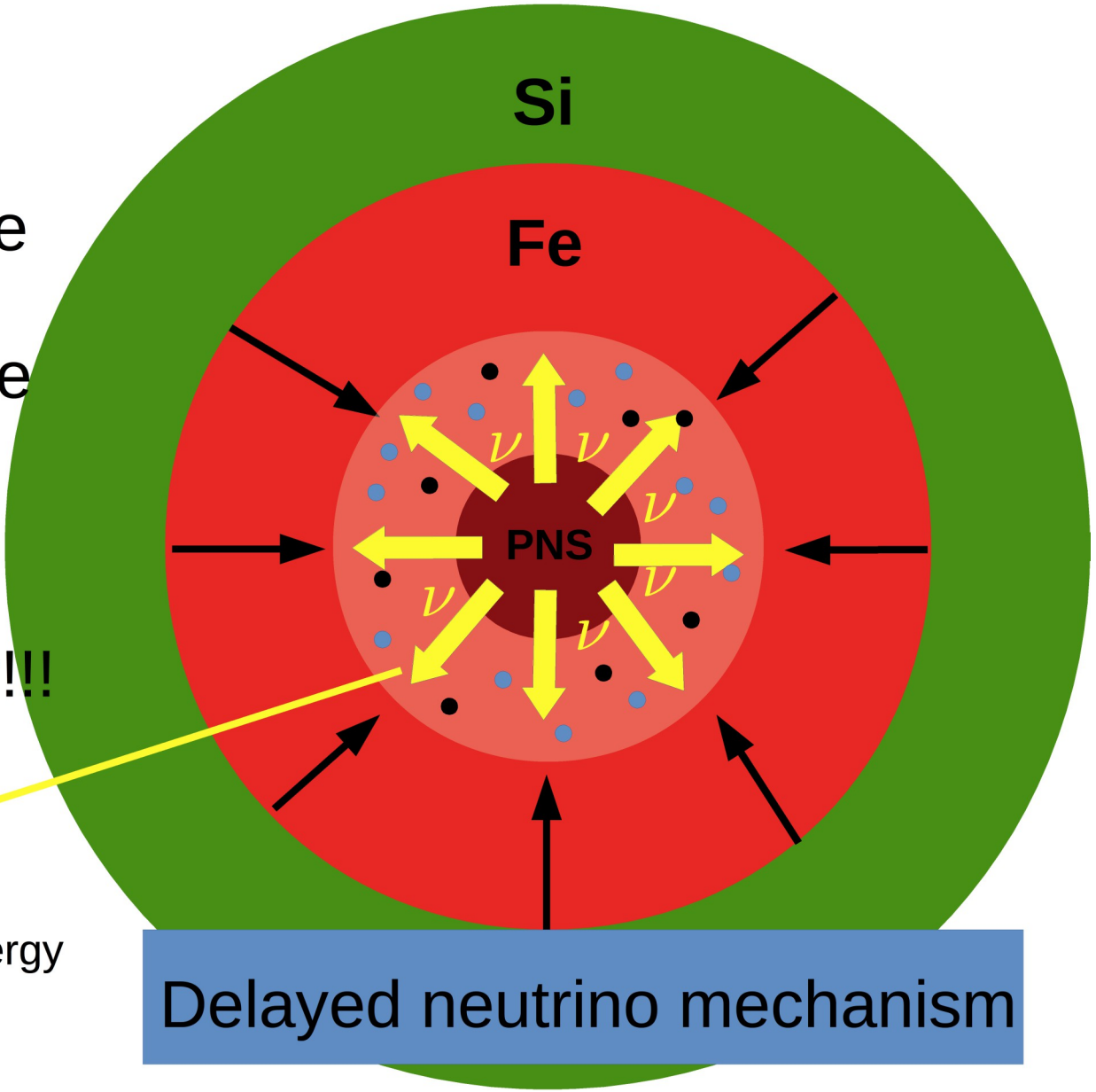
$$e^- + p \rightarrow n + \nu_e$$

Neutrino burst!!!



Shock “revival”

The stalled shock wave must regain energy to start the expansion against the ram pressure of the in-falling matter and finally blow away the outer mantle!



Tap the neutrino “energy” reservoir!!!

$$n + \nu_e \rightarrow e^- + p$$

$$p + \overline{\nu}_e \rightarrow e^+ + n$$

From the gravitational binding energy

- Neutrons
- Protons

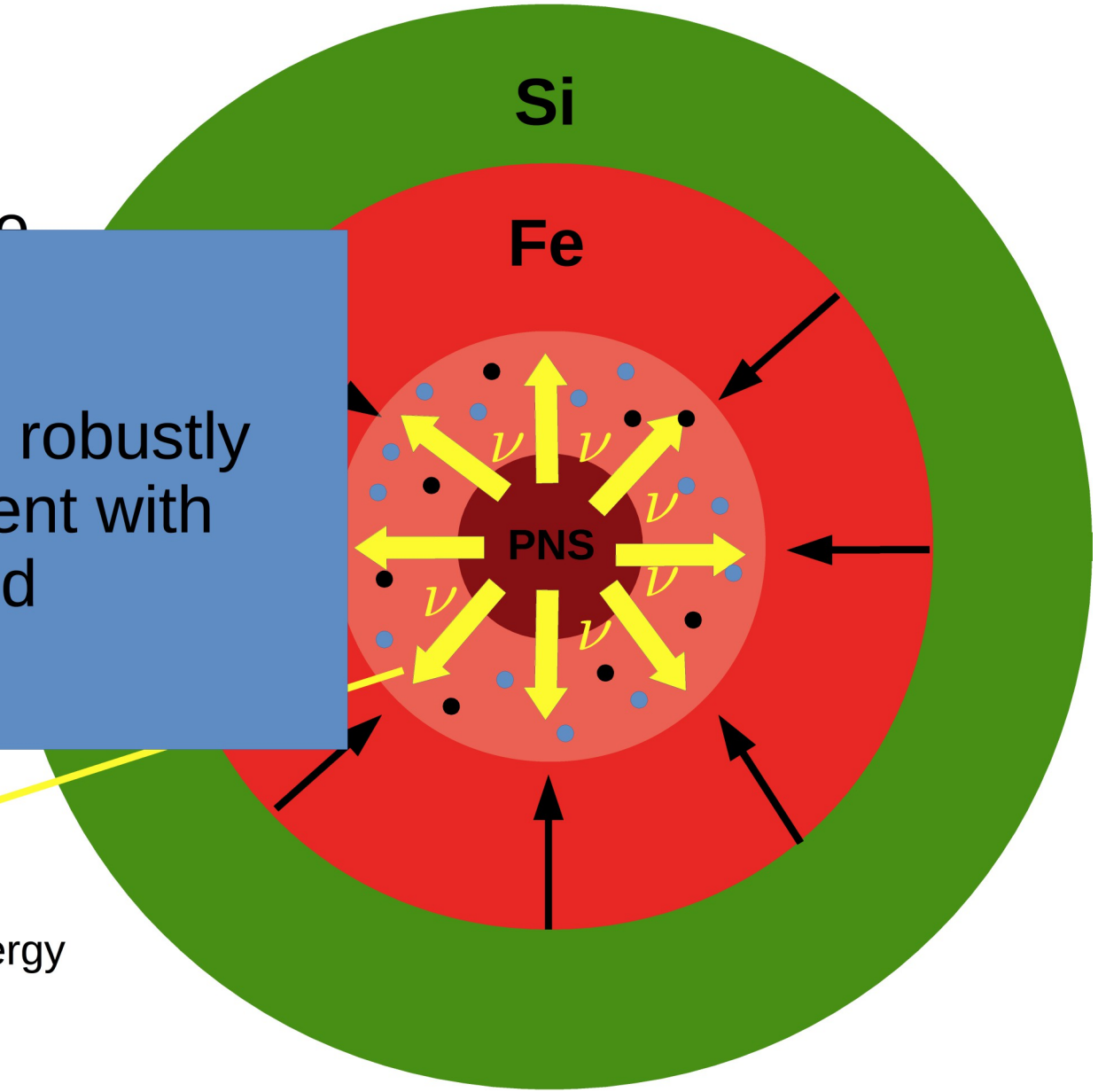
Delayed neutrino mechanism

Shock “revival”

The stalled shock wave must regain energy to start the expansion against the ram pressure

The problem:

Find a way to explode robustly massive stars consistent with the available direct and indirect observables



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From the gravitational binding energy

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CCSN Explosion Mechanism?

- Discussed explosion mechanisms:
  - "Enhanced" neutrino-driven explosion mechanism

    Hydro. instabilities: convection, Standing Accretion Shock Instabilities (SASI) e.g. Blondin et al. 2003, Blondin & Shaw 2007, Foglizzo et al. 2008, Iwakami et al. 2008, Marek & Janka 2009, Suwa et al. 2010, 2012, Takiwaki et al. 2013, Bruenn et al. 2013...
  - MHD mechanism

    Rapid rotation + Magnetic field amplification (Flux compression, winding, MRI, dynamos) e.g. Akiyama et al. 2003, Wilson et al. 2005, Kotake et al. 2006, Burrows et al. 2007, Winteler et al. 2012, ...
  - Acoustic mechanism

    Excitation of ProtoNeutron Star (PNS) oscillations by accretion/SASI generating acoustic power to reheat the stalled shock Burrows et al. 2006,2007
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    Additional compactification of PNS due to phase transition from hadronic matter to quark matter Migdal et al. 1971, … Sagert, Fischer et al. 2009, Fischer et al. 2011, ...

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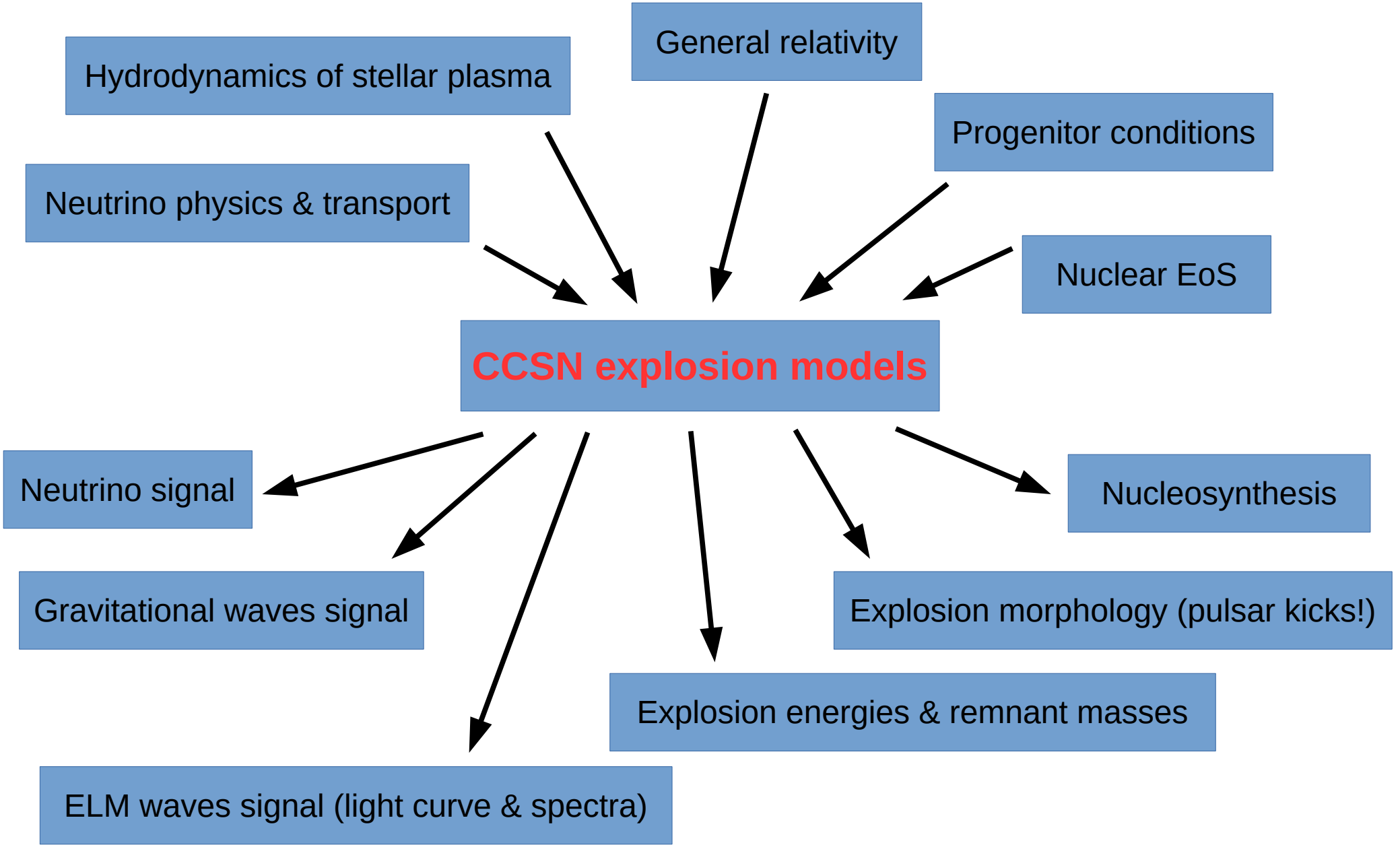
The overall challenge



CCSN model

Model's ingredients wish list:

1) Multi-D hydro. — It's a 3D problem
2) Plasma physics — Stars have magnetic fields, e.g. Sun, ... pulsars & magnetars!
3) Weak interactions — Most of the released gravitational binding energy “available” in form of neutrinos!
4) Neutrino transport

5) Nuclear physics — Equation of state describing matter at extreme conditions
6) General relativity — Very compact and very massive objects!

7) “Accurate” initial conditions

DOABLE

DIFFICULT

VERY DIFFICULT

CCSN model: 1D (spherical symm.)

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NO explosions for $M > 11 M_{\odot}$

Thompson et al. 2003, Rampp & Janka 2002, Liebendörfer et al. 2002/2005

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CCSN model: 2D (axisymmetry)

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7) “Accurate” initial conditions

Some explosions...

DOABLE

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VERY DIFFICULT

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Enhanced-nu-CCSN model 3D

Actual model's ingredients list:

1) Multi-D hydro.
2) Plasma physics

→ Parallel 3D ideal MHD code (Assume infinite conductivity)

3) Weak interactions
4) Neutrino transport

→ IDSA Liebendörfer et al. 2009

5) Nuclear physics

→ EoS e.g. Lattimer & Swesty 1991, Shen et al. 1998, Hempel et al. 2011

6) General relativity

→ Spherical effective GR potential Marek et al. 2006
+
2D axisymmetric Newton potential

7) “Accurate” initial conditions

The Radiation-MHD equations (2)

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_i}{\partial x_i} = 0$$ Mass

$$\frac{\partial (\rho v_i)}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho v_i v_j + P_* I_{ij} - b_i b_j\right) = -\rho \frac{\partial \phi}{\partial x_i} + \left(\dot{\rho v_i}\right)_\nu$$ Momentum

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_j}\left[\left(E + P_*\right) v_j - v_i b_i b_j\right] = -\rho v_i \frac{\partial \phi}{\partial x_i} + \left(\dot{\rho e}\right)_\nu$$ Energy

$$\frac{\partial \rho Y_e}{\partial t} + \frac{\partial Y_e \rho v_i}{\partial x_i} = \left(\rho \dot{Y}_e\right)_\nu$$ Electron #

$$\left(\boldsymbol{b} = \frac{\boldsymbol{B}}{\sqrt{4\pi}}\right)$$

Magnetic flux $$\frac{\partial b_i}{\partial t} = \frac{\partial}{\partial x_j}\left(v_i b_j - v_j b_i\right)$$

No monopoles $$\boldsymbol{\nabla} \cdot \boldsymbol{b} = 0$$

$$E = \frac{1}{2}\rho v^2 + \rho e + \frac{b^2}{2}$$

$$P_* = p + \frac{b^2}{2}$$

EoS: $p = p(\rho, e, ...)$

Solution Algorithm: MHD

- MHD (FISH) Käppeli et al. 2011
  - Split hydro. and magnetic variables update
  - Dimensional splitting: solves eqs in 1D
  - Uses dim.-split constrained transport for $\boldsymbol{\nabla} \cdot \boldsymbol{b} = 0$
  - Well-balanced scheme for hydrostatic equilibrium

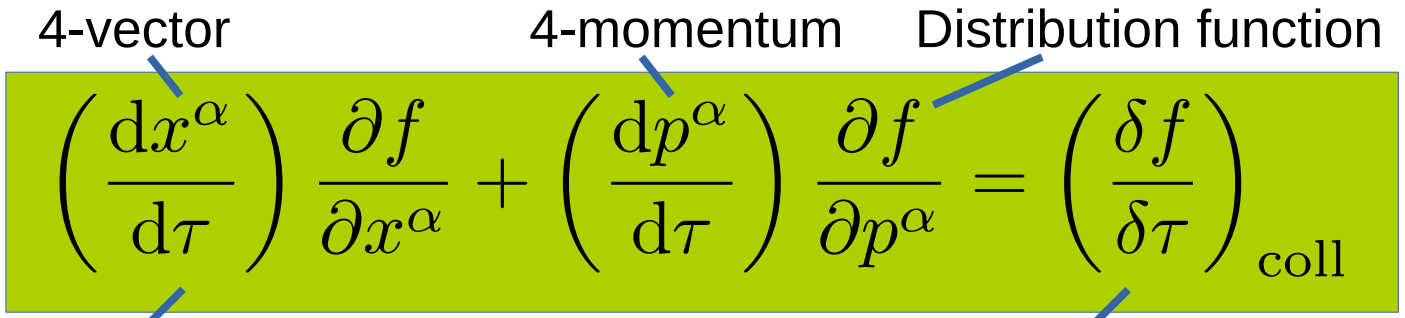
    $\Rightarrow$ Preserves discrete HSE exactly by reconstruction in equilibrium variables

    Käppeli & Mishra, 2014

  - Hybrid MPI/OpenMP parallelisation for distributed/shared memory architectures

Solution Algorithm: Neutrinos

- In principle, should solve the relativistic Boltzmann eq



e.g. Cercignani & Kremer 2002

Full transfer NOT feasible in 3D (3+3+1=7 dim. problem!)

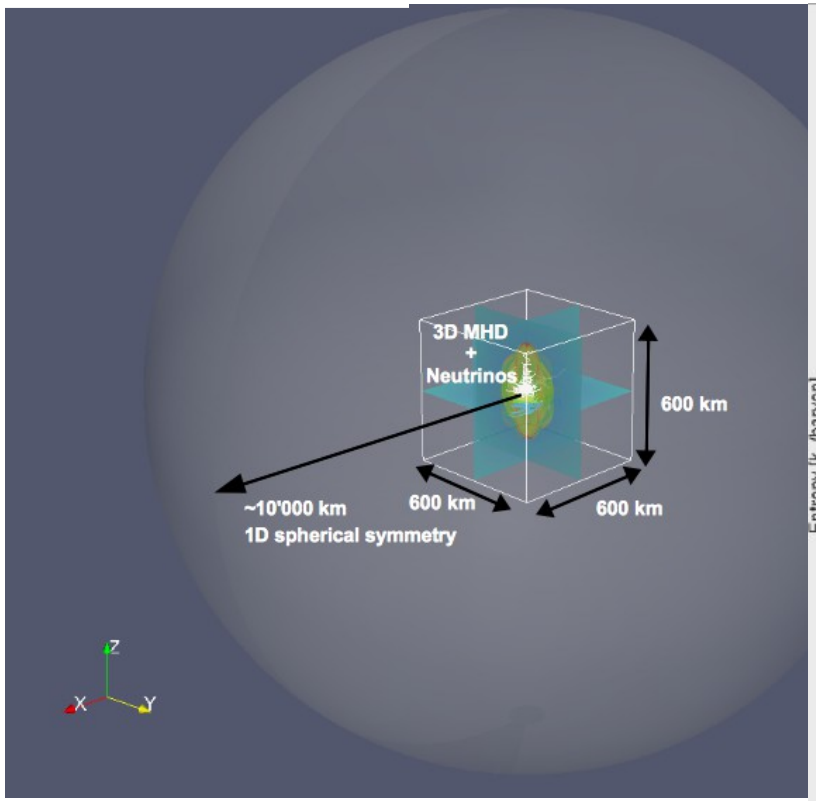
- Approximations:
  - Parametrisation scheme: only collapse phase Liebendörfer 2005
  - Spectral Leakage scheme: only cooling Perego et al. in prep.

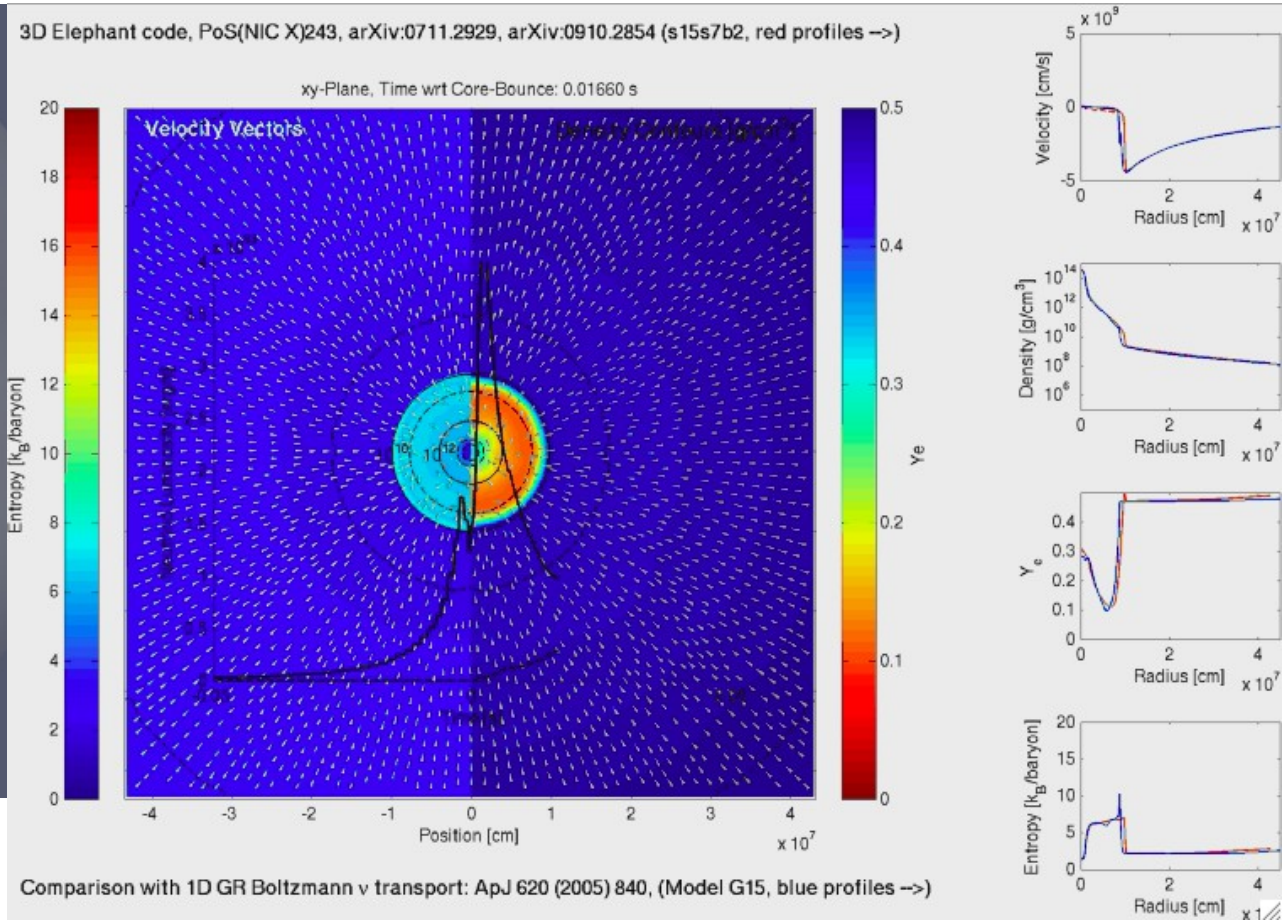
    Epstein & Pethick 1981, van Riper & Lattimer 1981, ..., Ruffert et al. 1996, Rosswog & Liebendörfer 2003
  - Isotropic Diffusion Source Approx. (IDSA) Liebendörfer et al. 2009

Enhanced-nu-CCSN model 3D

Computations @ CSCS Swiss National Supercomputing Centre

By M. Liebendoerfer





Implementation details

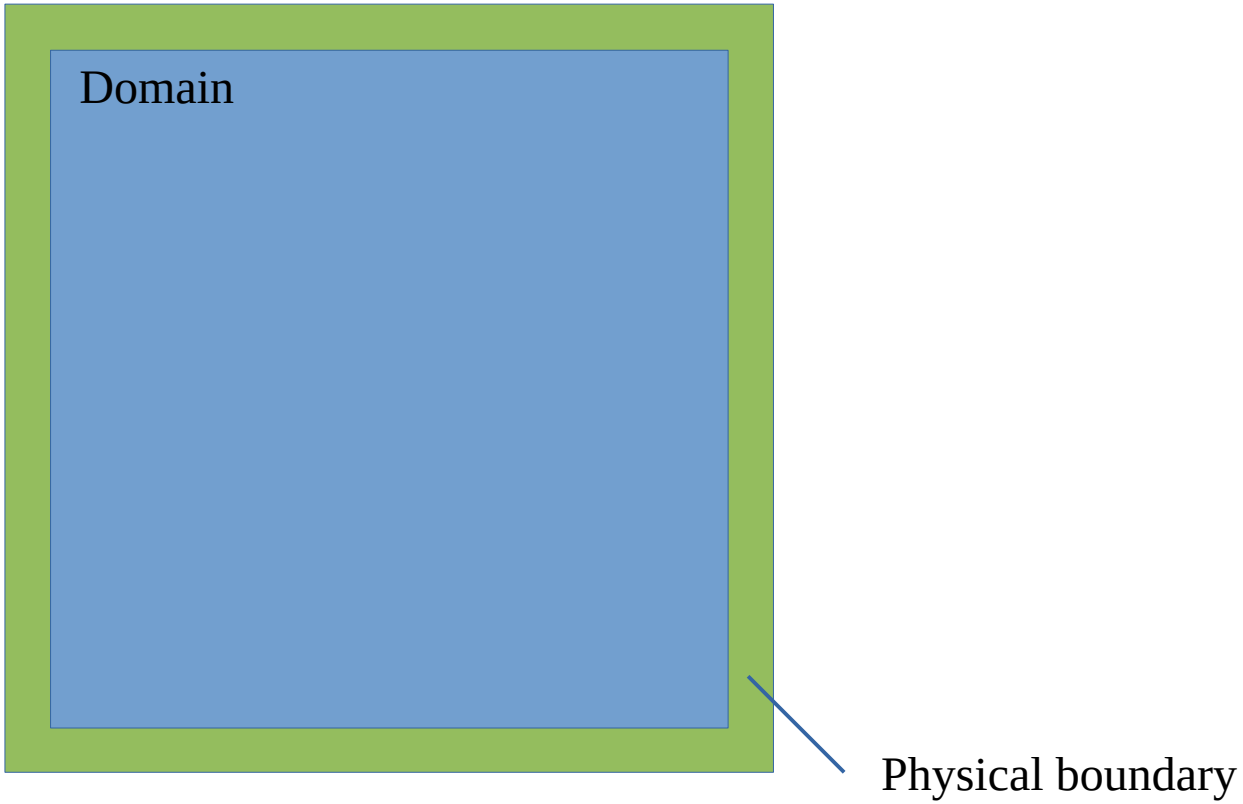
- Directional operator splitting

  $\Longrightarrow$ Write only 1D update routines

- Data rotated so that stencil OPs along contiguous memory direction
- Distributed memory parallelisation with MPI
- Overlap communication/computation with non-blocking communication
- Shared memory parallelisation with OpenMP

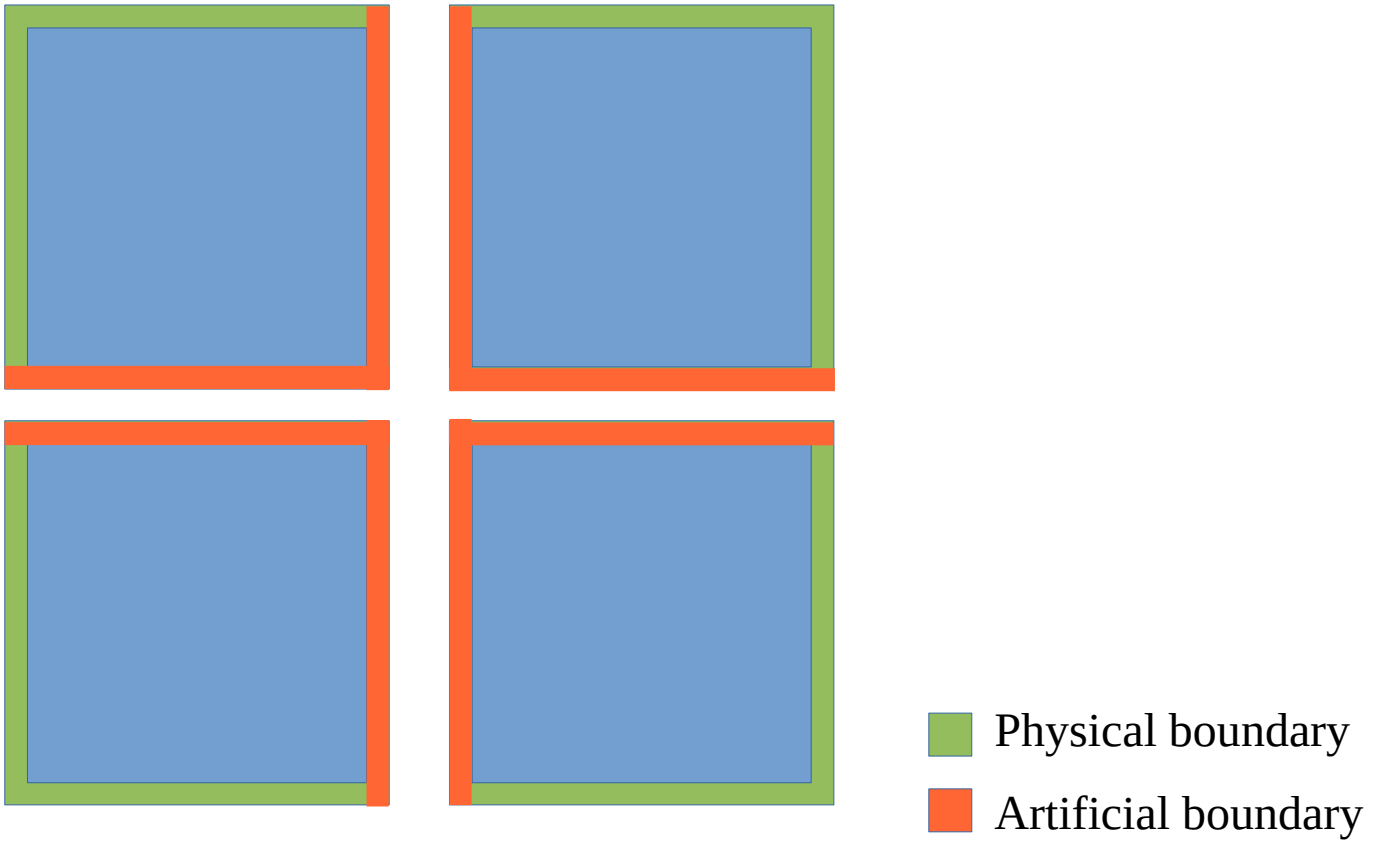
Parallelisation: MPI

- Domain decomposition



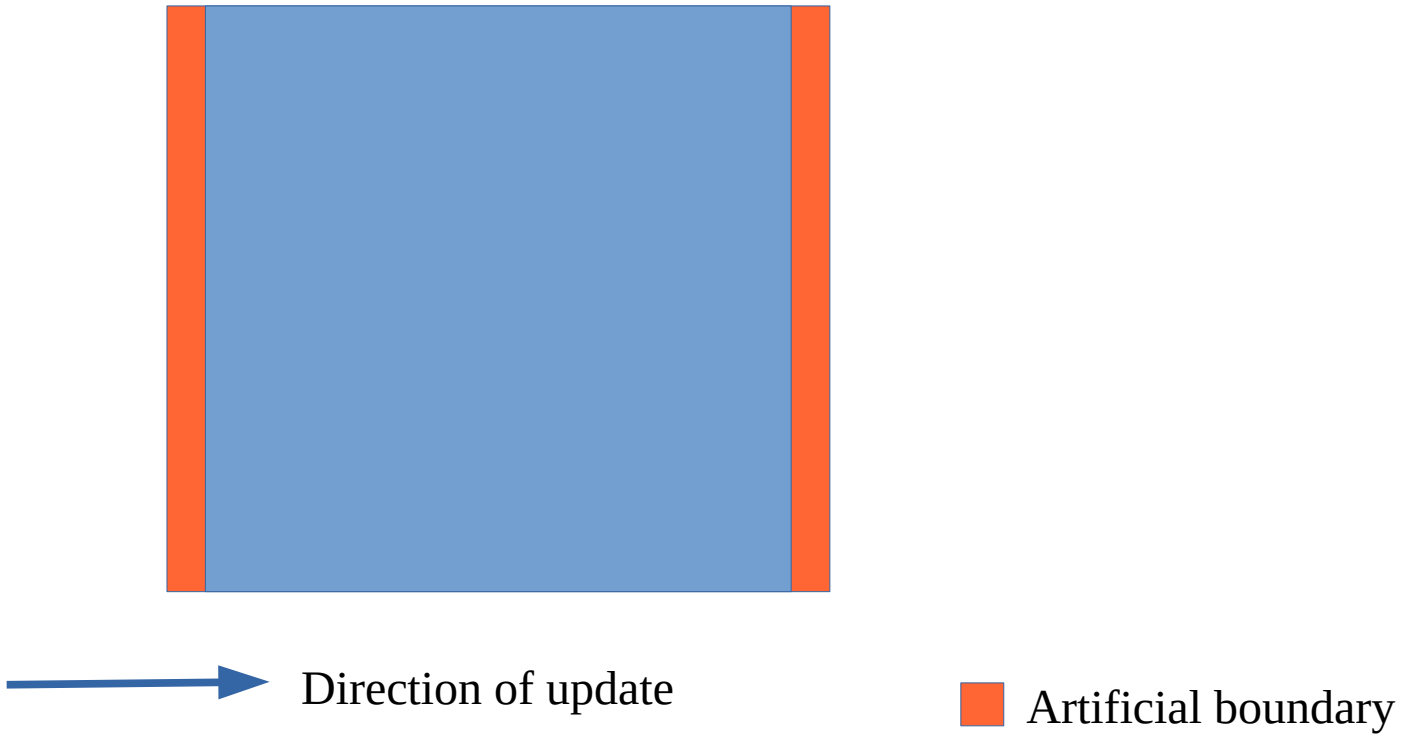
Parallelisation: MPI

- Domain decomposition



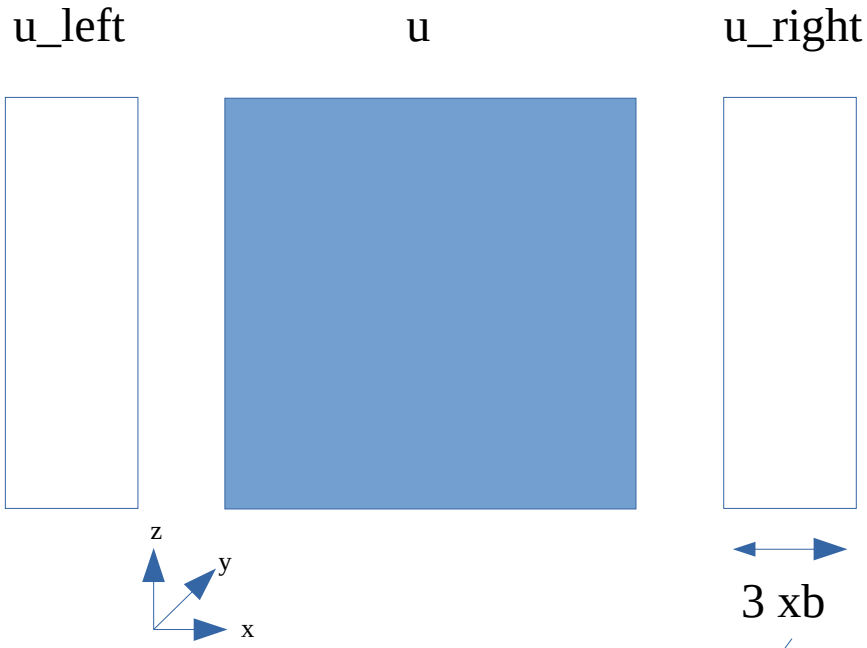
Parallelisation: MPI

- Domain decomposition



Parallelisation: MPI

- Overlapping communication/computation




```
!sweep in x-direction

!copy data from center to left and right data
u_left = u(...)
u_right = u(...)

!copy data into sending buffer and initiate comm
call comm_mpistartall(nx%requests)

!perform computation on center
call update(u,nx,ny,nz)

!wait for communication to complete
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!copy received data and perform update on boundary
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!copy left and right data to center
u(...) = u_left
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```
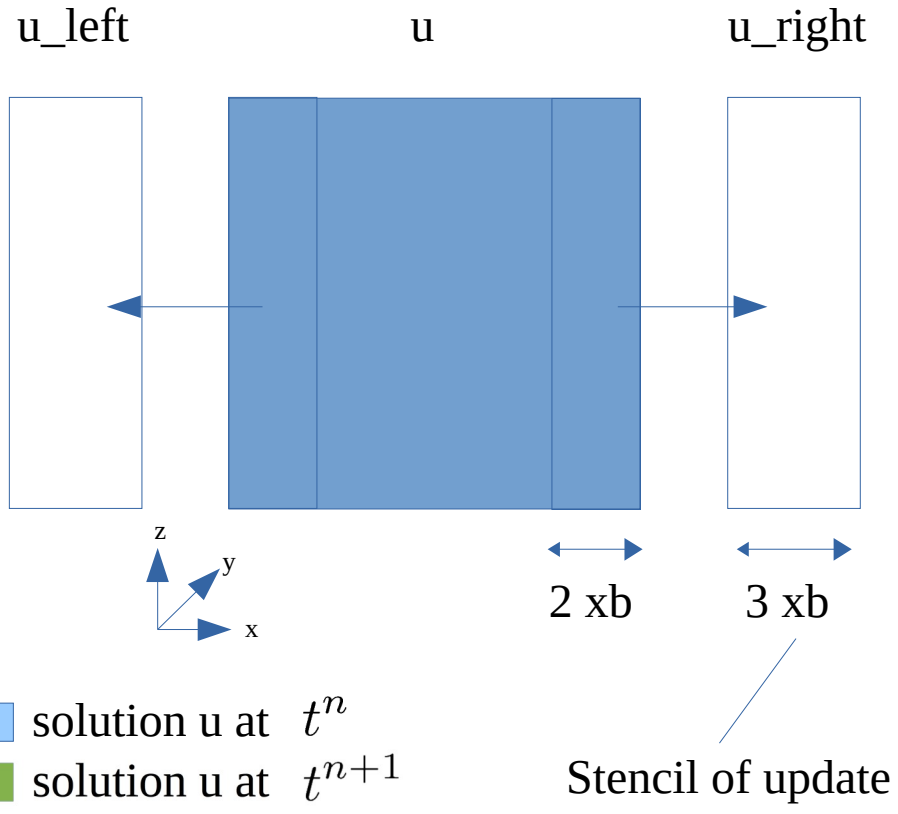

Parallelisation: MPI

- Overlapping communication/computation

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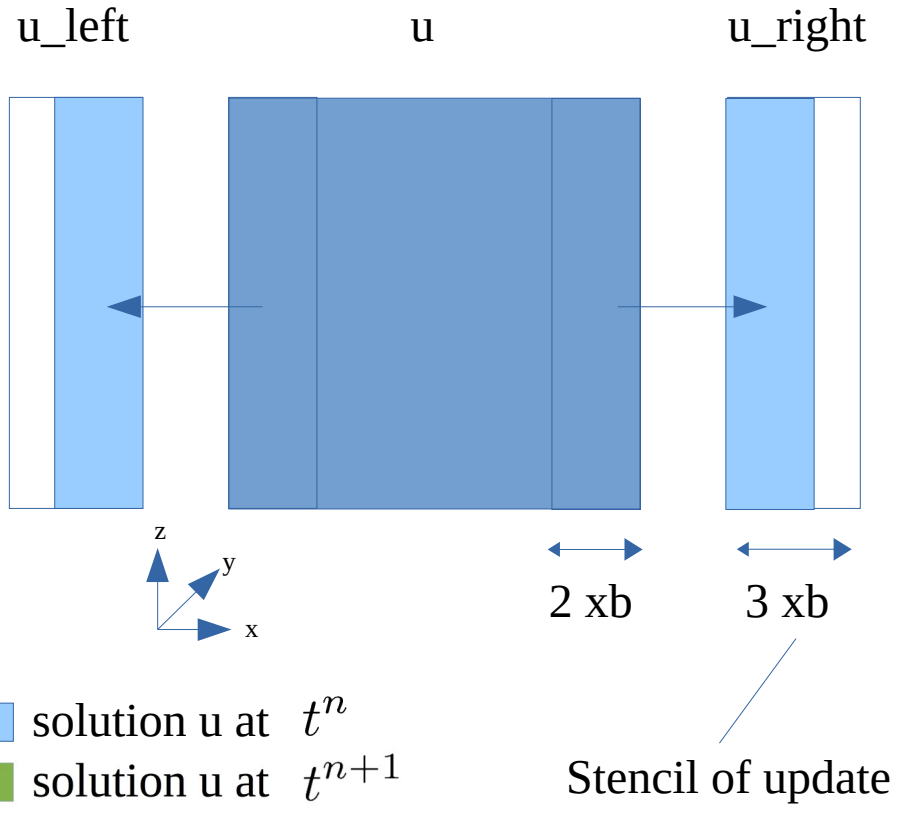
Parallelisation: MPI

- Overlapping communication/computation

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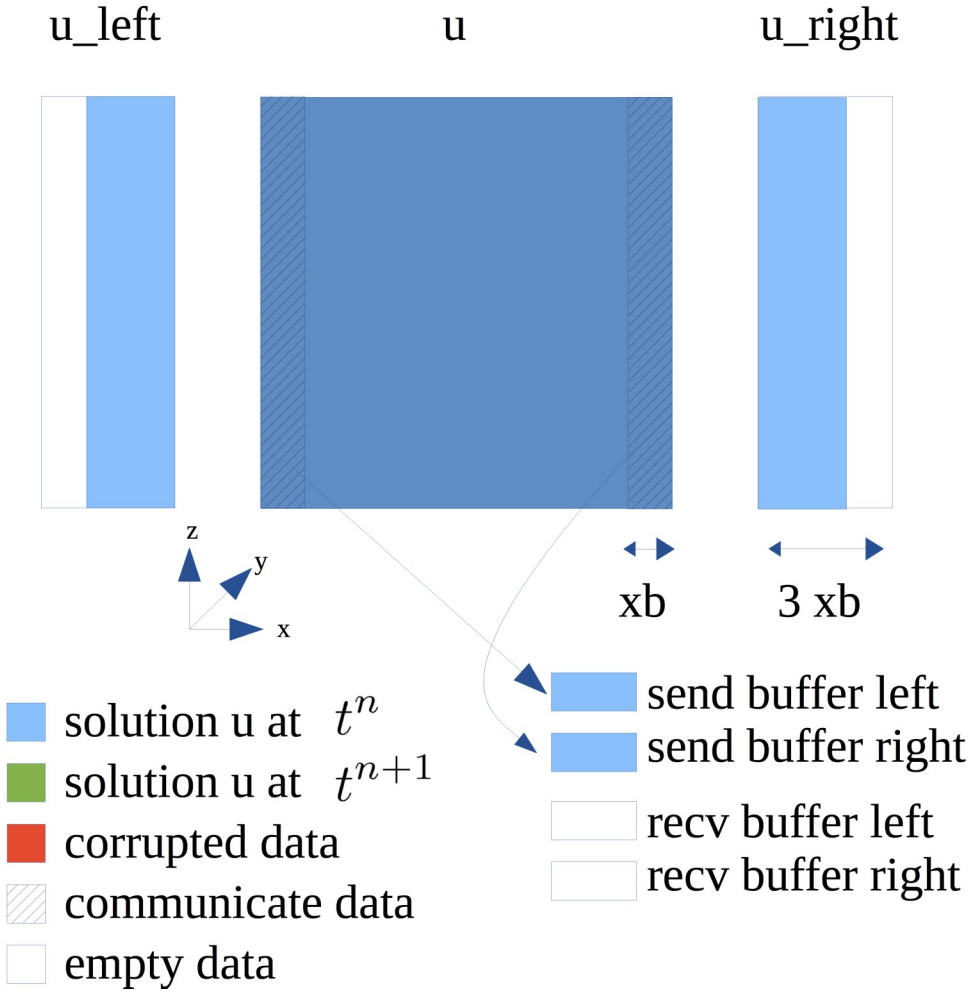
Parallelisation: MPI

- Overlapping communication/computation

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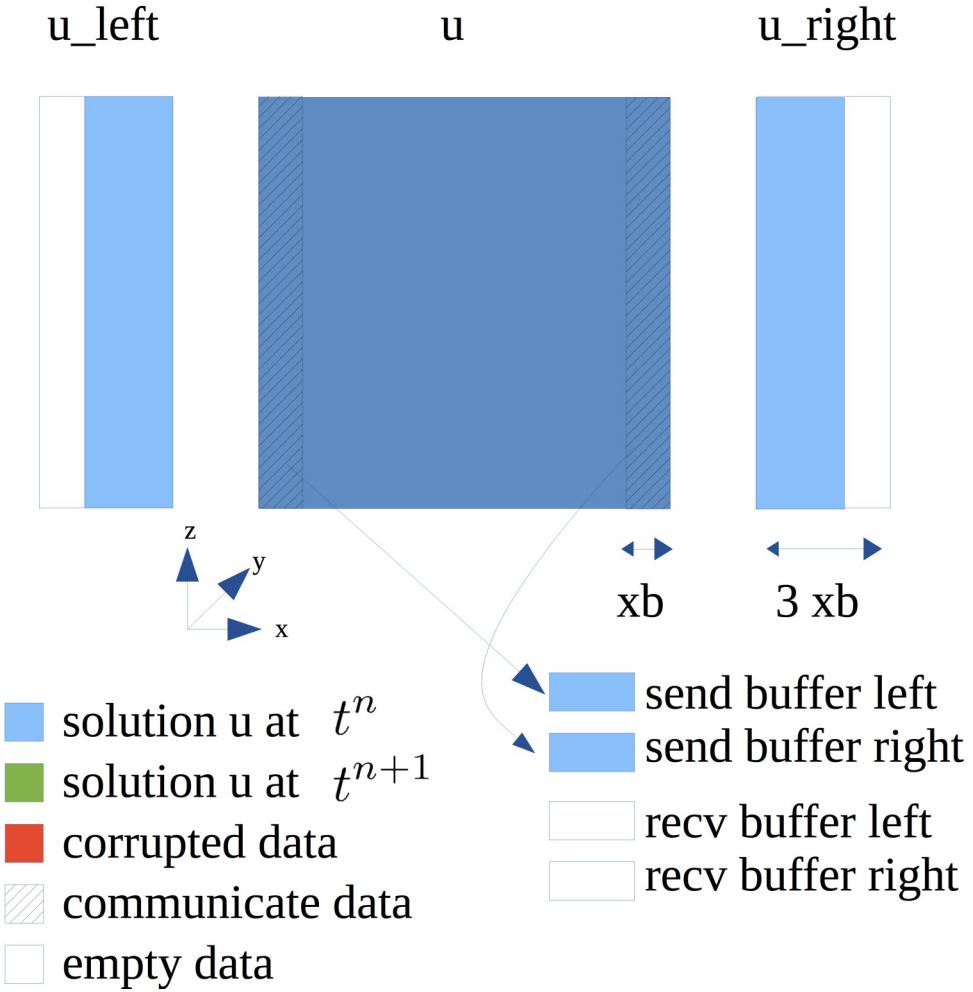
Parallelisation: MPI

- Overlapping communication/computation

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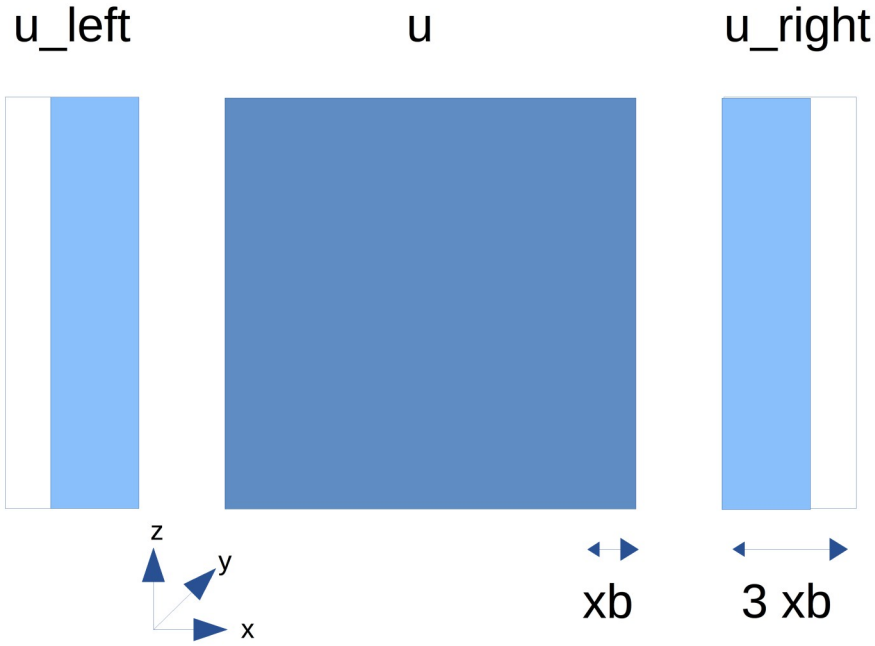
Parallelisation: MPI

- Overlapping communication/computation



solution u at t^n
solution u at t^{n+1}
corrupted data
communicate data
empty data

send buffer left
send buffer right
recv buffer left
recv buffer right

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!sweep in x-direction

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Parallelisation: MPI

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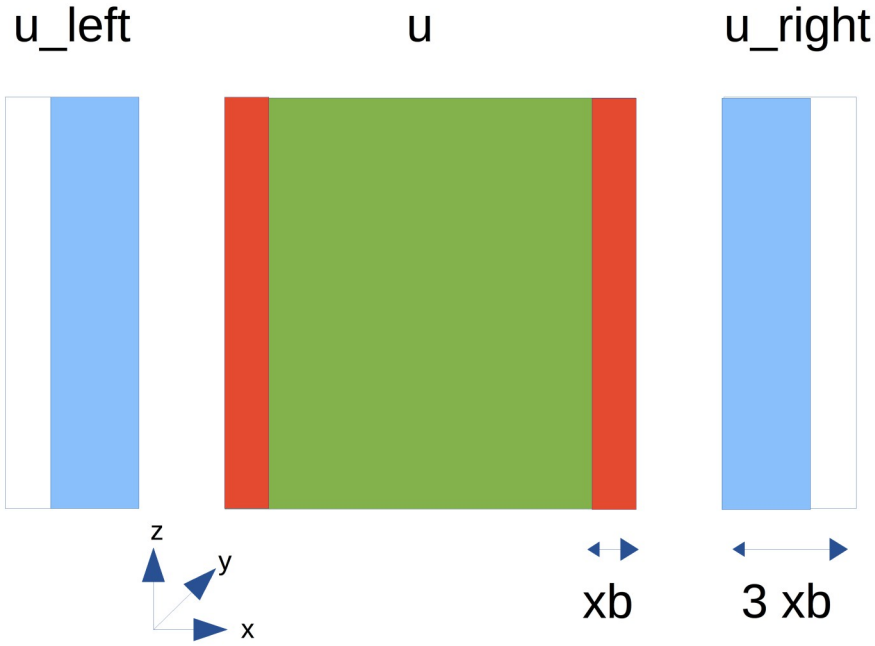

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- empty data

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Parallelisation: MPI

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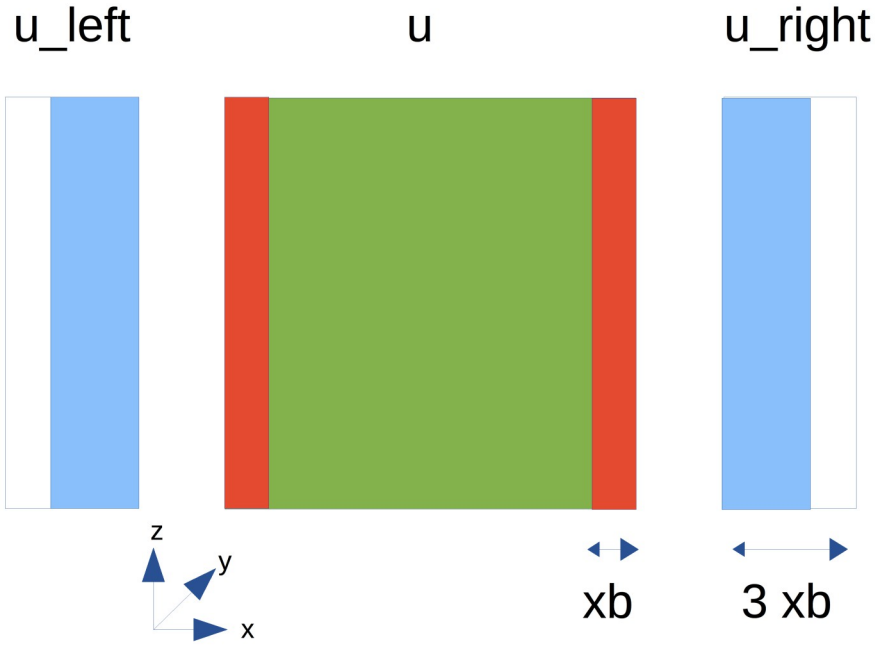

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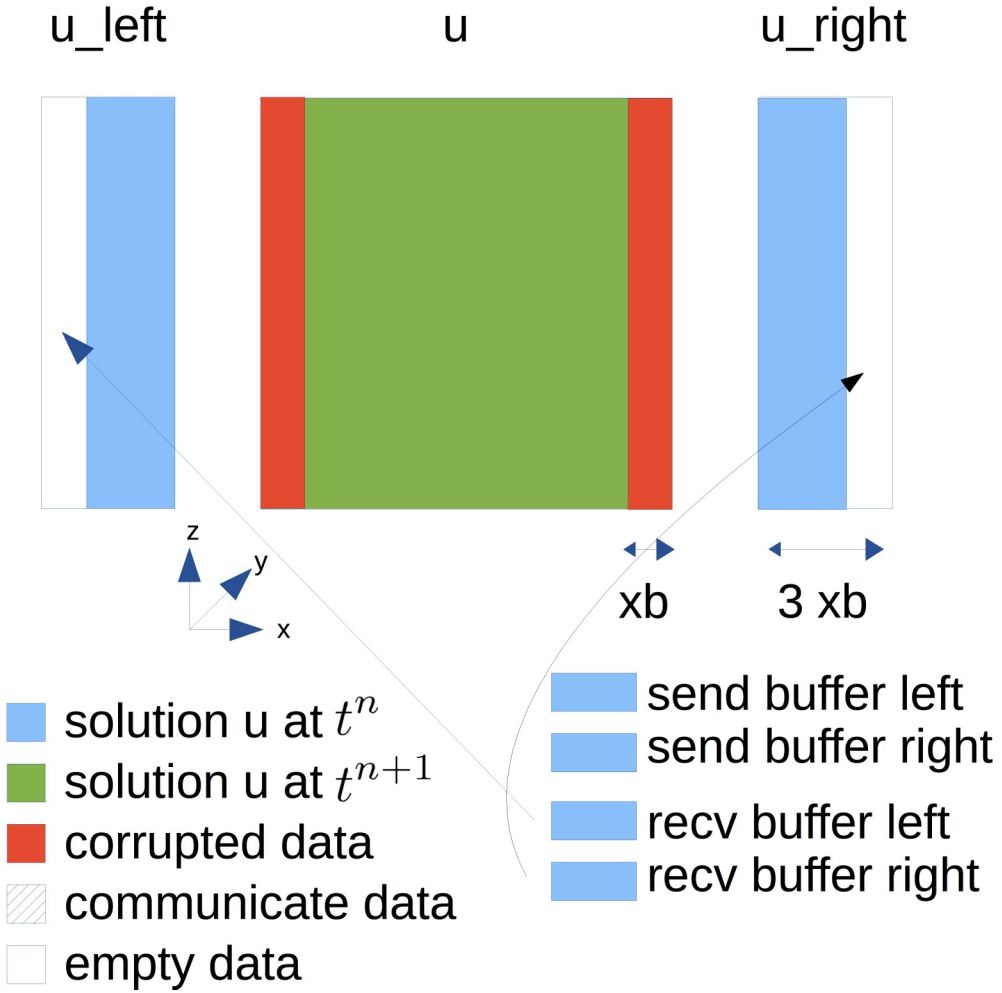


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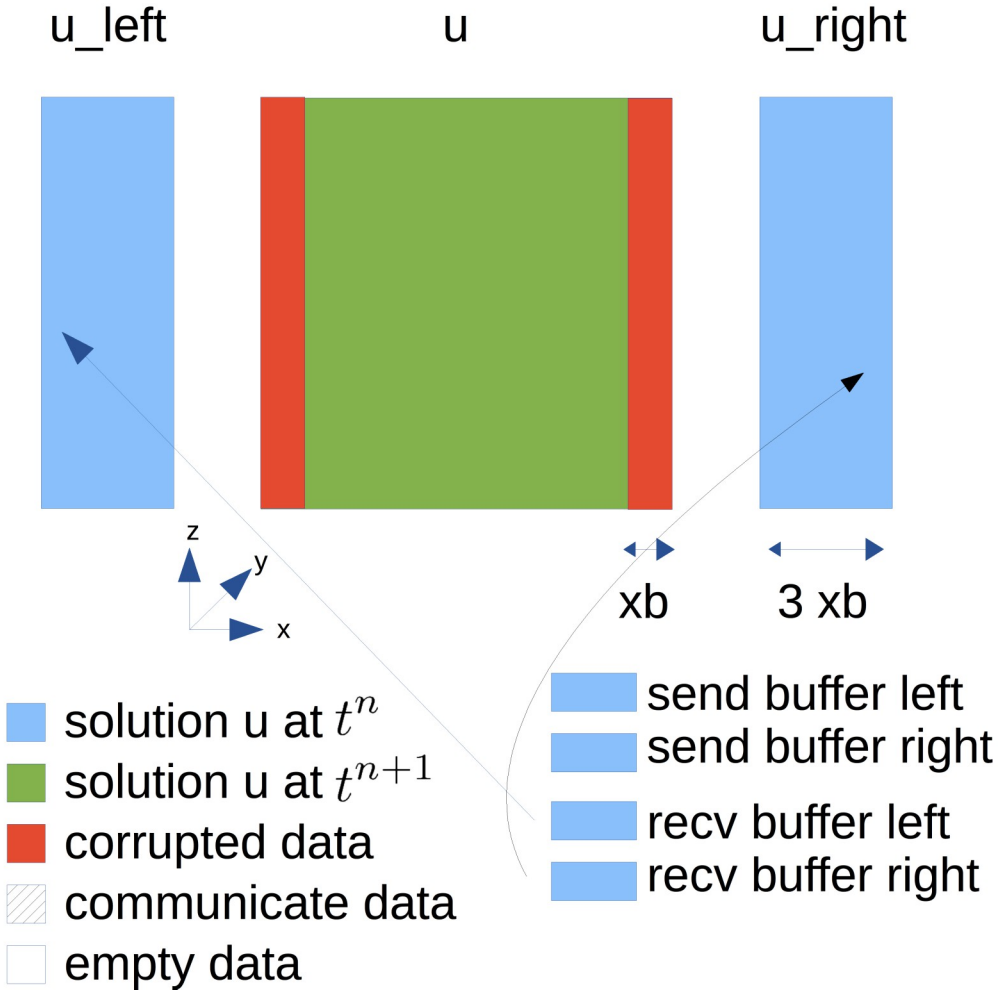


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Parallelisation: MPI

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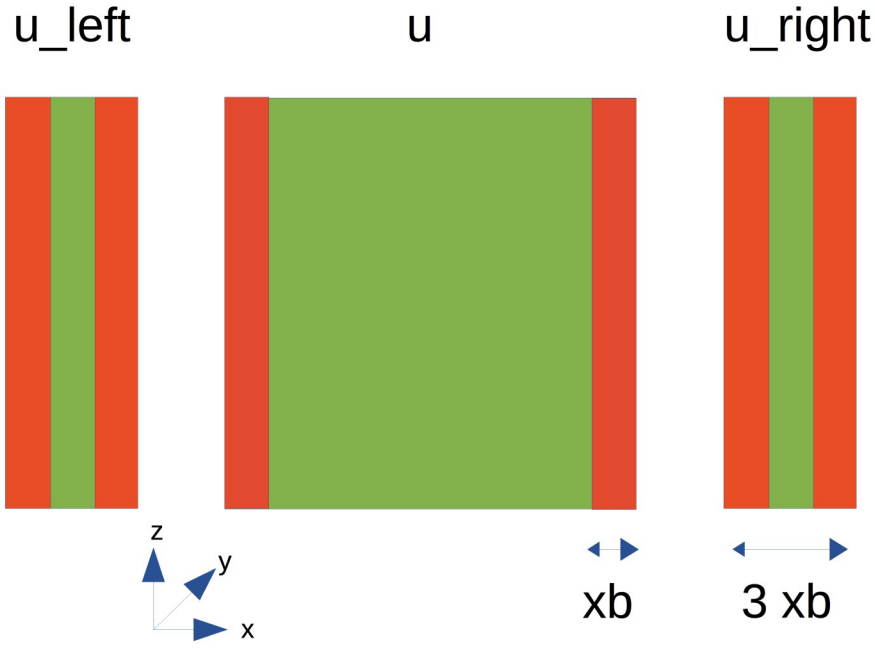


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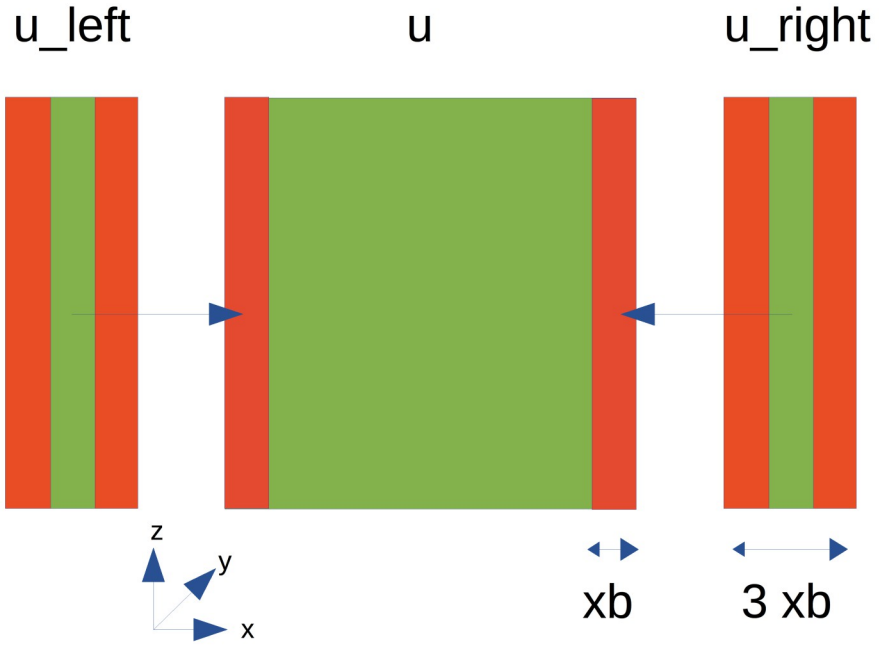
Parallelisation: MPI

- Overlapping communication/computation



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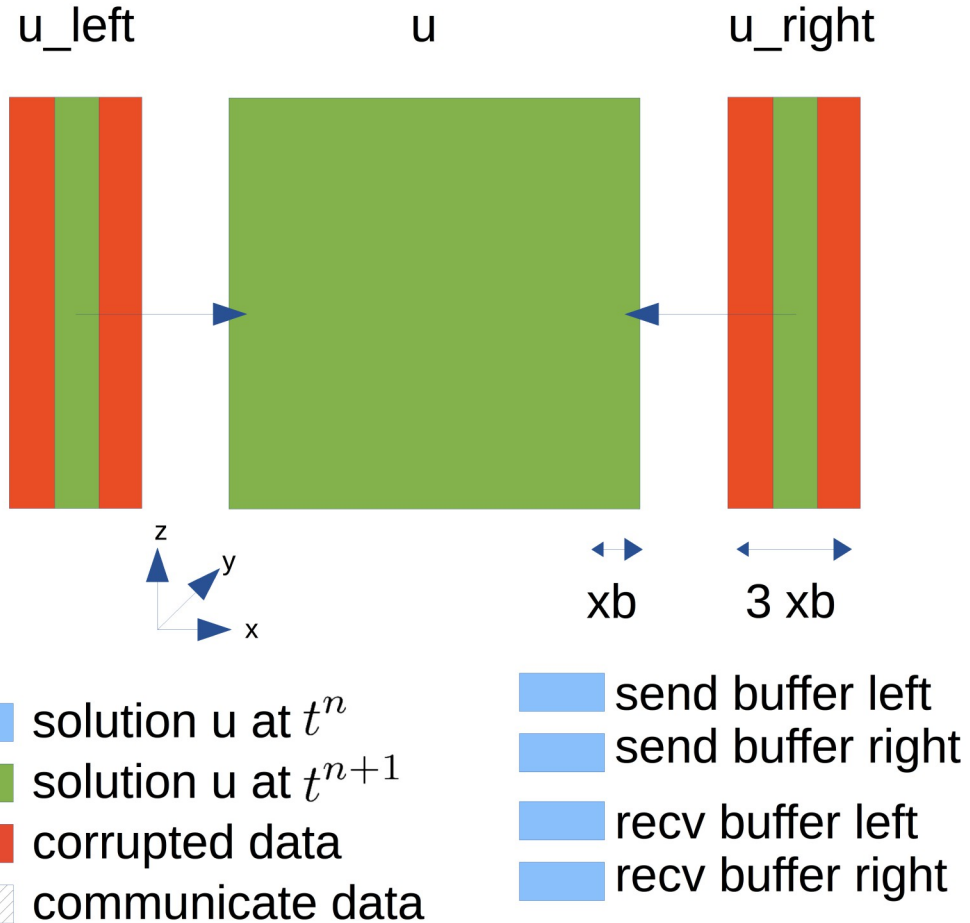
Parallelisation: MPI

- Overlapping communication/computation

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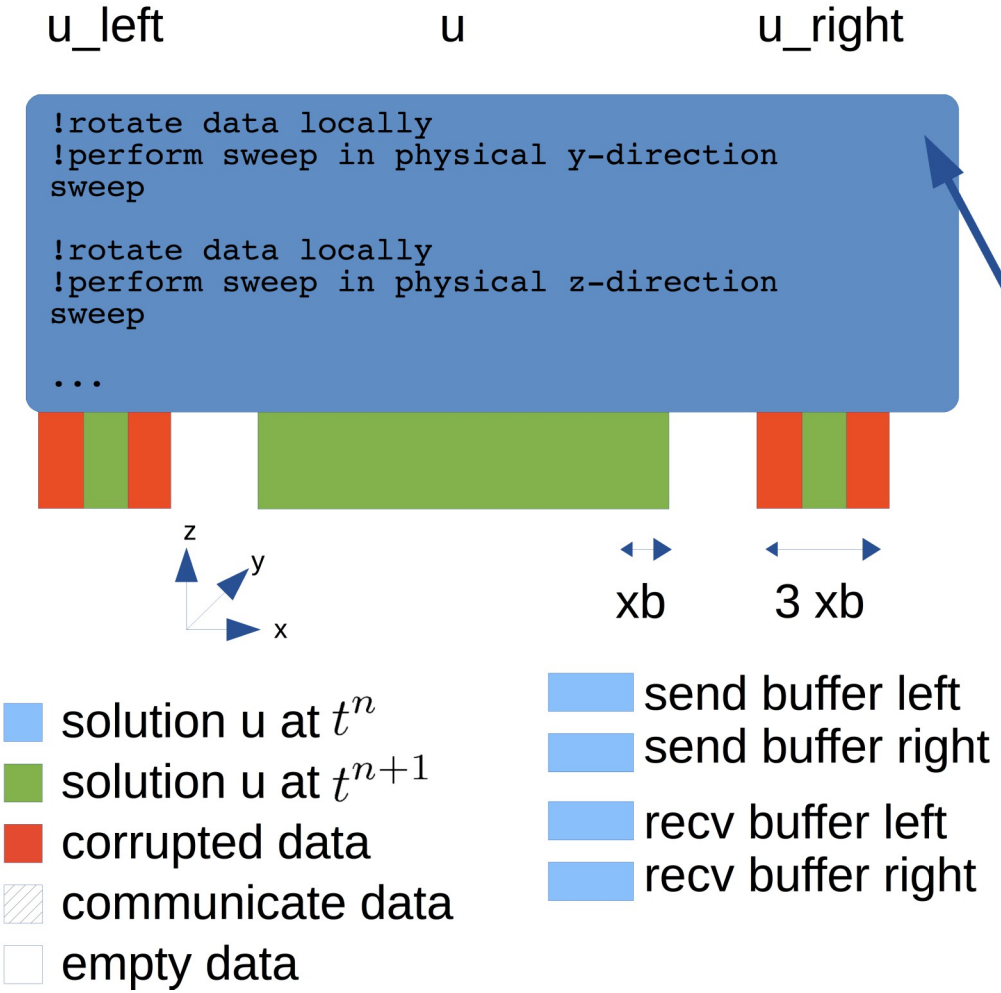
Parallelisation: MPI

- Overlapping communication/computation

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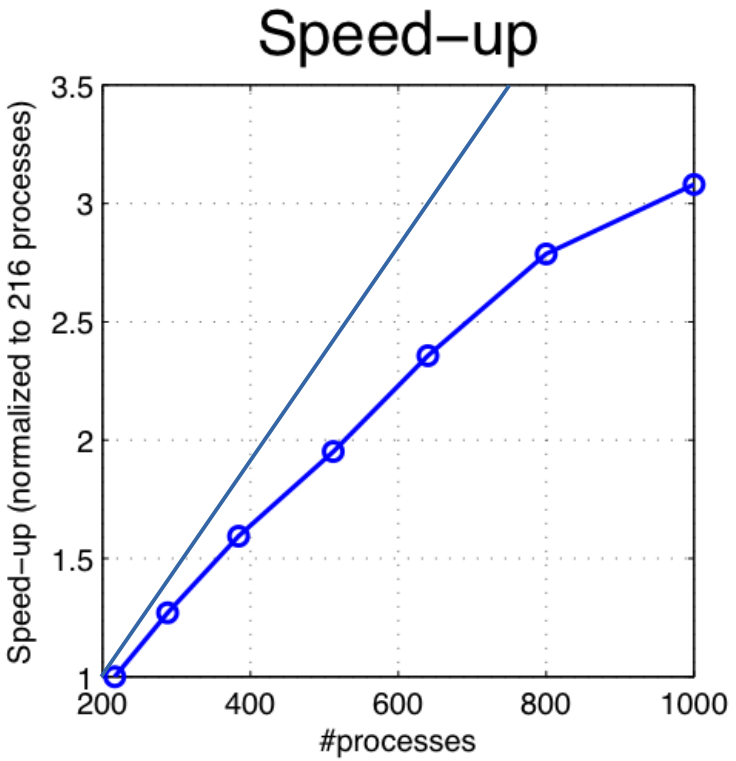
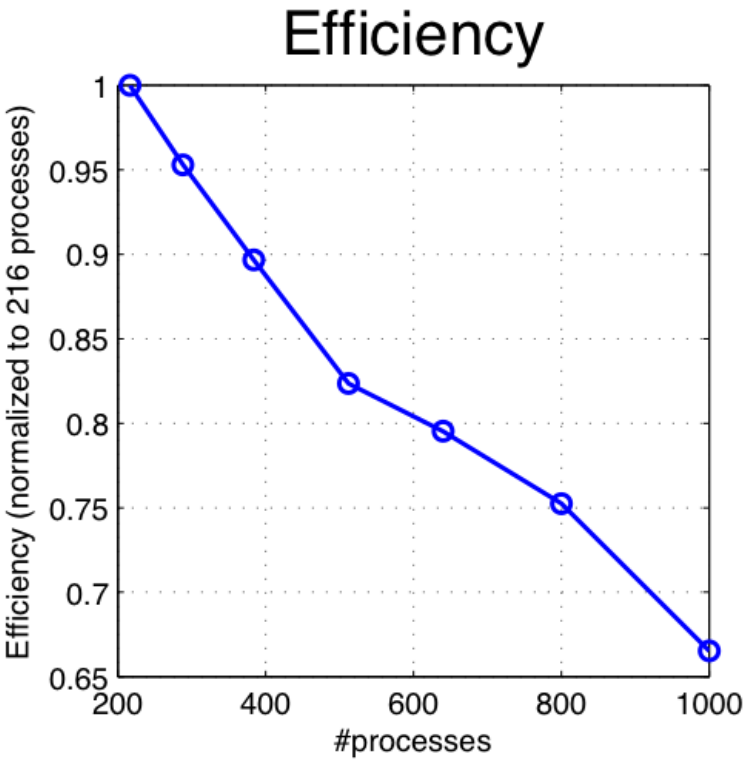
```

(Strong) Scaling: MPI (2)

FISH+ELEPHANT (Radiation+MHD): 600x600x600 zones

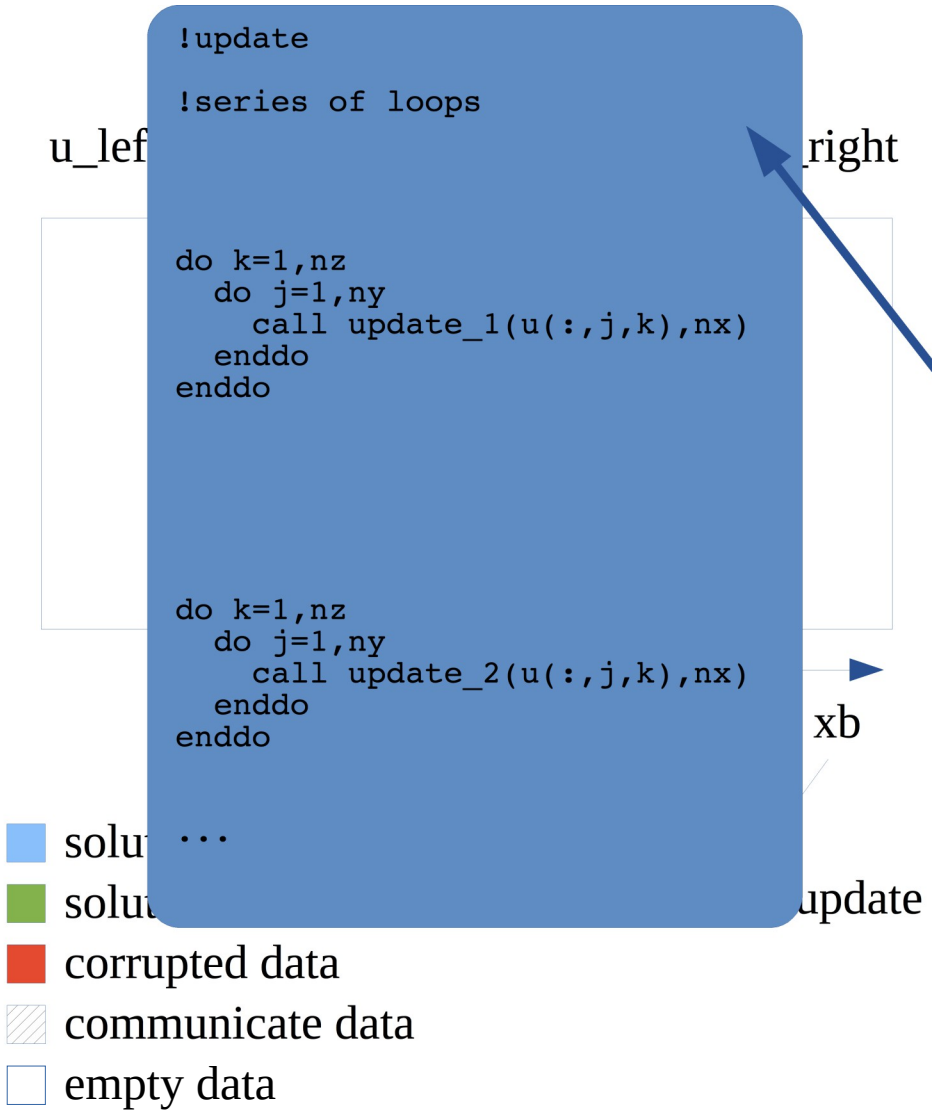




Performed on Cray XT5 @ CSCS

Parallelisation: MPI+OpenMP

- Overlapping communication/computation




```
!update

!series of loops



do k=1,nz
  do j=1,ny
    call update_1(u(:,j,k),nx)
  enddo
enddo




do k=1,nz
  do j=1,ny
    call update_2(u(:,j,k),nx)
  enddo
enddo


...
```

```
!sweep in x-direction

!copy data from center to left and right data
u_left = u(...)
u_right = u(...)

!copy data into sending buffer and initiate comm
call comm_mpistartall(nx%requests)

!perform computation on center
call update(u,nx,ny,nz)

!wait for communication to complete
call comm_waitall(nx%requests)

!copy received data and perform update on boundary
!left
call update(u_left,nx_left,ny,nz)
!right
call update(u_right,nx_right,ny,nz)

!copy left and right data to center
u(...) = u_left
u(...) = u_right
```

Parallelisation: MPI+OpenMP

- Overlapping communication/computation

```
!update

!series of loops
!$OMP PARALLEL DO
!$OMP SHARED(u)
!$OMP PRIVATE(j,k)
!$OMP FIRSTPRIVATE(nx,ny,nz)
do k=1,nz
  do j=1,ny
    call update_1(u(:,j,k),nx)
  enddo
enddo
!$OMP END PARALLEL DO

!$OMP PARALLEL DO
!$OMP SHARED(u)
!$OMP PRIVATE(j,k)
!$OMP FIRSTPRIVATE(nx,ny,nz)
do k=1,nz
  do j=1,ny
    call update_2(u(:,j,k),nx)
  enddo
enddo
!$OMP END PARALLEL DO

...
```

```
!sweep in x-direction

!copy data from center to left and right data
u_left = u(...)
u_right = u(...)

!copy data into sending buffer and initiate comm
call comm_mpistartall(nx%requests)

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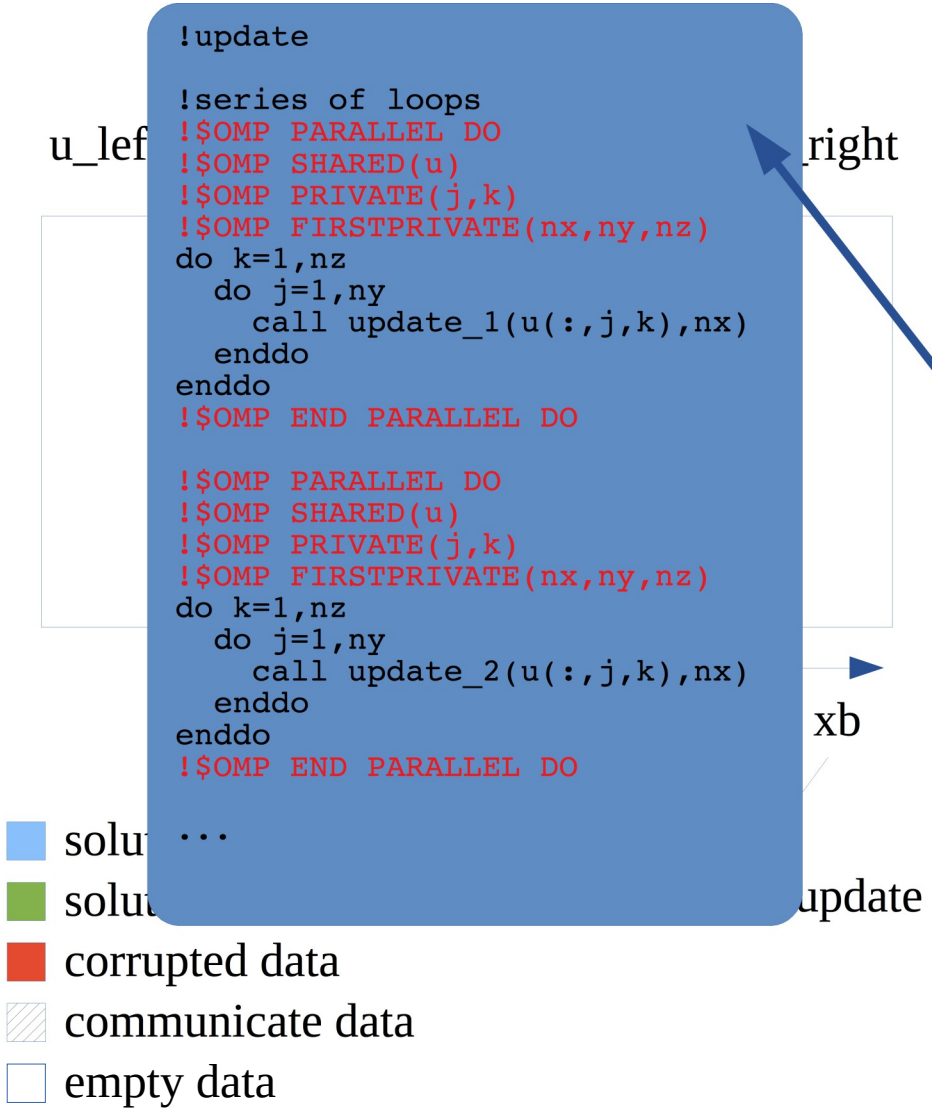

Parallelisation: MPI+OpenMP

- Overlapping communication/computation

```
!update

!series of loops
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!$OMP FIRSTPRIVATE(nx,ny,nz)
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  enddo
enddo
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...
```

```
!sweep in x-direction

!copy data from center to left and right data
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u_right = u(...)

!copy data into sending buffer and initiate comm
call comm_mpistartall(nx%requests)

!perform computation on center
call update(u_nx,ny,nz)

call update(u_right,nx_right,ny,nz)

!copy left and right data to center
u(...) = u_left
u(...) = u_right
```

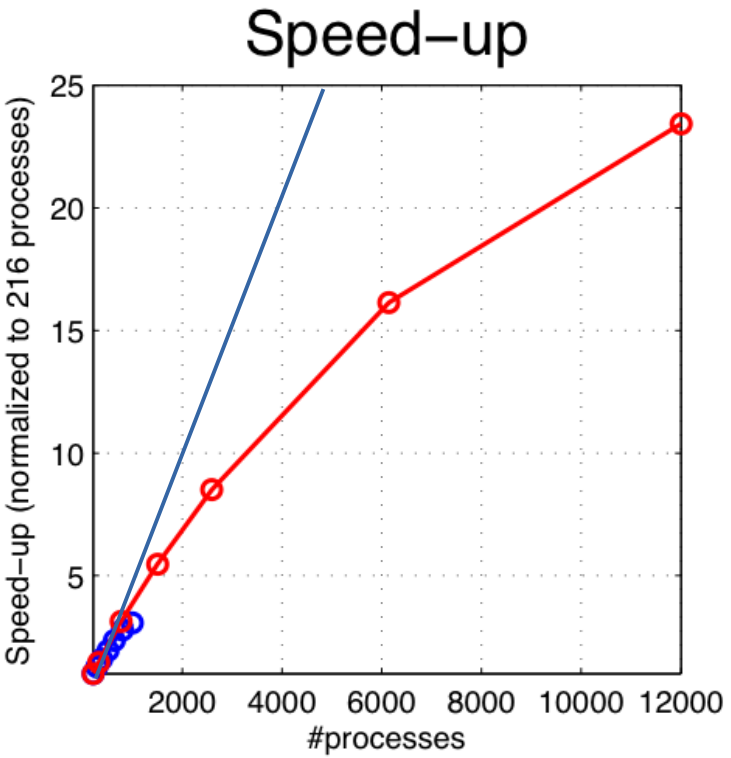
OMP parallelised all other (computationally intensive) loops...

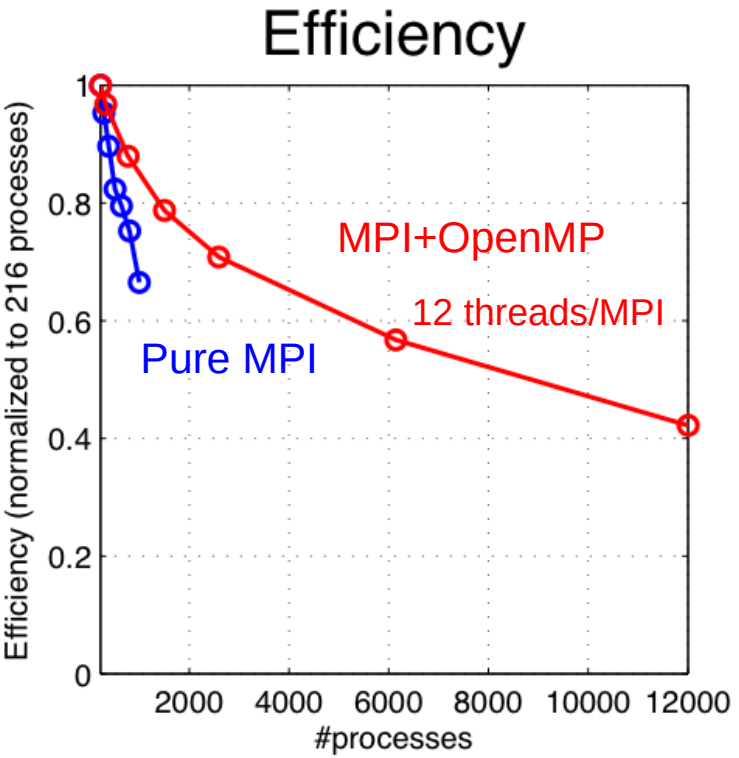
u_lef … _right … update

- solu…
- solu…
- corrupted data
- communicate data
- empty data

(Strong) Scaling: MPI+OpenMP (2)

FISH+ELEPHANT (Radiation+MHD): 600x600x600 zones





Performed on Cray XT5 @ CSCS

Outline

- **Introduction**
  - A brief introduction of the problem
- **Physical model, numerical & computational aspects**
  - Physics ingredients & mathematical model
- **Simulation of magneto-rotational core-collapse**
  - MHD CCSN mechanism
  - Magnetic field amplification, formation and driving mechanism of bipolar outflow
  - Explosion energy, ejected mass and its composition
- **Conclusion**

MHD-CCSN model

Actual model's ingredients list:

1) Multi-D hydro.
2) Plasma physics

→ Parallel 3D ideal MHD code (Assume infinite conductivity)

3) Weak interactions
4) Neutrino transport

→ Spectral leakage scheme
developed by A. Perego
Rosswog & Liebendörfer 2003
"Not so bad"... 2D simulations shown that ν contribute only 10-25% to explosion energy

5) Nuclear physics

→ EoS e.g. Lattimer & Swesty 1991, Shen et al. 1998, Hempel et al. 2011

6) General relativity

→ Spherical effective GR potential Marek et al. 2006
+
2D axisymmetric Newton potential

7) "Accurate" initial conditions

The Radiation-MHD equations (2)

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_i}{\partial x_i} = 0$$ Mass

$$\frac{\partial (\rho v_i)}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho v_i v_j + P_* I_{ij} - b_i b_j\right) = -\rho \frac{\partial \phi}{\partial x_i} + \left(\dot{\rho v_i}\right)_\nu$$ Momentum

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_j}\left[\left(E + P_*\right) v_j - v_i b_i b_j\right] = -\rho v_i \frac{\partial \phi}{\partial x_i} + \left(\dot{\rho e}\right)_\nu$$ Energy

$$\frac{\partial \rho Y_e}{\partial t} + \frac{\partial Y_e \rho v_i}{\partial x_i} = \left(\rho \dot{Y}_e\right)_\nu$$ Electron #

$\left(\boldsymbol{b} = \frac{\boldsymbol{B}}{\sqrt{4\pi}}\right)$

Magnetic flux $\frac{\partial b_i}{\partial t} = \frac{\partial}{\partial x_j}\left(v_i b_j - v_j b_i\right)$

No monopoles $\boldsymbol{\nabla} \cdot \boldsymbol{b} = 0$

$$E = \frac{1}{2}\rho v^2 + \rho e + \frac{b^2}{2}$$

$$P_* = p + \frac{b^2}{2}$$

EoS: $p = p(\rho, e, ...)$

Solution Algorithm: MHD

- MHD (FISH) Käppeli et al. 2011
  - Split hydro. and magnetic variables update
  - Dimensional splitting: solves eqs in 1D
  - Uses dim.-split constrained transport for $\boldsymbol{\nabla} \cdot \boldsymbol{b} = 0$
  - Well-balanced scheme for hydrostatic equilibrium

    $\Rightarrow$ Preserves discrete HSE exactly by reconstruction in equilibrium variables

    Käppeli & Mishra, 2014

  - Hybrid MPI/OpenMP parallelisation for distributed/shared memory architectures

Solution Algorithm: Neutrinos

- In principle, should solve the relativistic Boltzmann eq

4-vector 4-momentum Distribution function

$$\left(\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\right)\frac{\partial f}{\partial x^\alpha}+\left(\frac{\mathrm{d}p^\alpha}{\mathrm{d}\tau}\right)\frac{\partial f}{\partial p^\alpha}=\left(\frac{\delta f}{\delta\tau}\right)_{\mathrm{coll}}$$

e.g. Cercignani & Kremer 2002

Proper time

Absorption, emission, scattering in medium

Full transfer NOT feasible in 3D (3+3+1=7 dim. problem!)

- Approximations:
  - Parametrisation scheme: only collapse phase Liebendörfer 2005
  - Spectral Leakage scheme: only cooling Perego et al. in prep.

    Epstein & Pethick 1981, van Riper & Lattimer 1981, ..., Ruffert et al. 1996, Rosswog & Liebendörfer 2003
  - Isotropic Diffusion Source Approx. (IDSA) Liebendörfer et al. 2009

Computational cost

Role of Rotation & Magnetic Field

Pre-collapse

- Rotation (???)
- B (???)

Distribution in Fe core ???

Observations: e.g.
Thompson et al. 2003
Donati & Landstreet 2009

Stellar evolution models:
Heger et al. 2005
Hirschi et al. 2004 & 2005

Successful Explosion...

Post-collapse

- Pulsar
  Magnetar
  Rotation (???)
  B (???)
  Taylor et al. 1993
  Kouveliotou et al. 1998
  Mereghetti 2008
- Observable Asymmetries
  Wang & Wheeler 2008
  Kjaer et al. 2010

Role of Rotation & Magnetic Field

Pre-colla

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- B (

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on (???)

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t al. 1993
u et al. 1998
Meregnett 008

- Observable Asymmetries

Wang & Wheeler 2008
Kjaer et al. 2010

Rotation & Magnetic fields present before and after explosion!

Influence of Rotation & B on explosion???

If strong effects, is it common or only (very) rare?

MHD CCSN Mechanism

- Rotational energy of Proto-Neutron Star (PNS)

$$T_{\rm rot} = \frac{1}{2} I_{\rm PNS} \Omega_{\rm PNS}^2$$
$$\approx 1 \times 10^{51}\ {\rm ergs} \times \left(\frac{M}{1.5 M_\odot}\right) \left(\frac{P}{2\ {\rm ms}}\right)^{-2} \left(\frac{R}{10\ {\rm km}}\right)^2$$

$\Longrightarrow$ Requires fast rotation!

- Idea: Extract "free" energy stored in differential rotation with
  - Viscosity Thompson et al. 2005
  - Magnetic Field

Typical CCSN explosion energy $E_{\rm expl} \sim 10^{51}$ erg

MHD CCSN Mechanism

- Rotational energy of Proto-Neutron Star (PNS)

“Free” energy in differential rotation?

$$E_{\mathrm{rot,free}} = T_{\mathrm{rot}}\left(L\right) - T_{\mathrm{rot,solid}}\left(L\right)$$

Angular momentum

Appreciable fraction of energy can be extracted by magnetic field and maybe trigger an explosion

Note: Structure assumed constant for the two realisations $\Longrightarrow$ Only approx.!

- I … al

Typical CCSN explosion energy $E_{\mathrm{expl}} \sim 10^{51}\ \mathrm{erg}$

Simulation of MHD CCSN

- Simulation parameters
  - L&S EoS K=180 MeV
  - Rotation laws:

1) Solid body $\Omega = \Omega_{0,c}$

2) Shellular $\Omega = \Omega_{0,c} \frac{R_0^2}{r^2 + R_0^2}$

Popular in axisym. → 3) Cylindrical I $\Omega = \Omega_{0,c} \frac{X_0^2}{r^2 \sin^2(\theta) + X_0^2}$

Popular in Japan → 4) Cylindrical II $\Omega = \Omega_{0,c} \frac{X_0^2}{r^2 \sin^2(\theta) + X_0^2} \frac{Z_0^4}{r^4 \cos^4(\theta) + Z_0^4}$

$r = \sqrt{x^2 + y^2 + z^2}$

$\cos(\theta) = z/r$

R_0, X_0, Z_0 Degree of diff. rotation

  - Magnetic field:

1) Uniform poloidal

2) Dipole-like poloidal

Simulation of MHD CCSN

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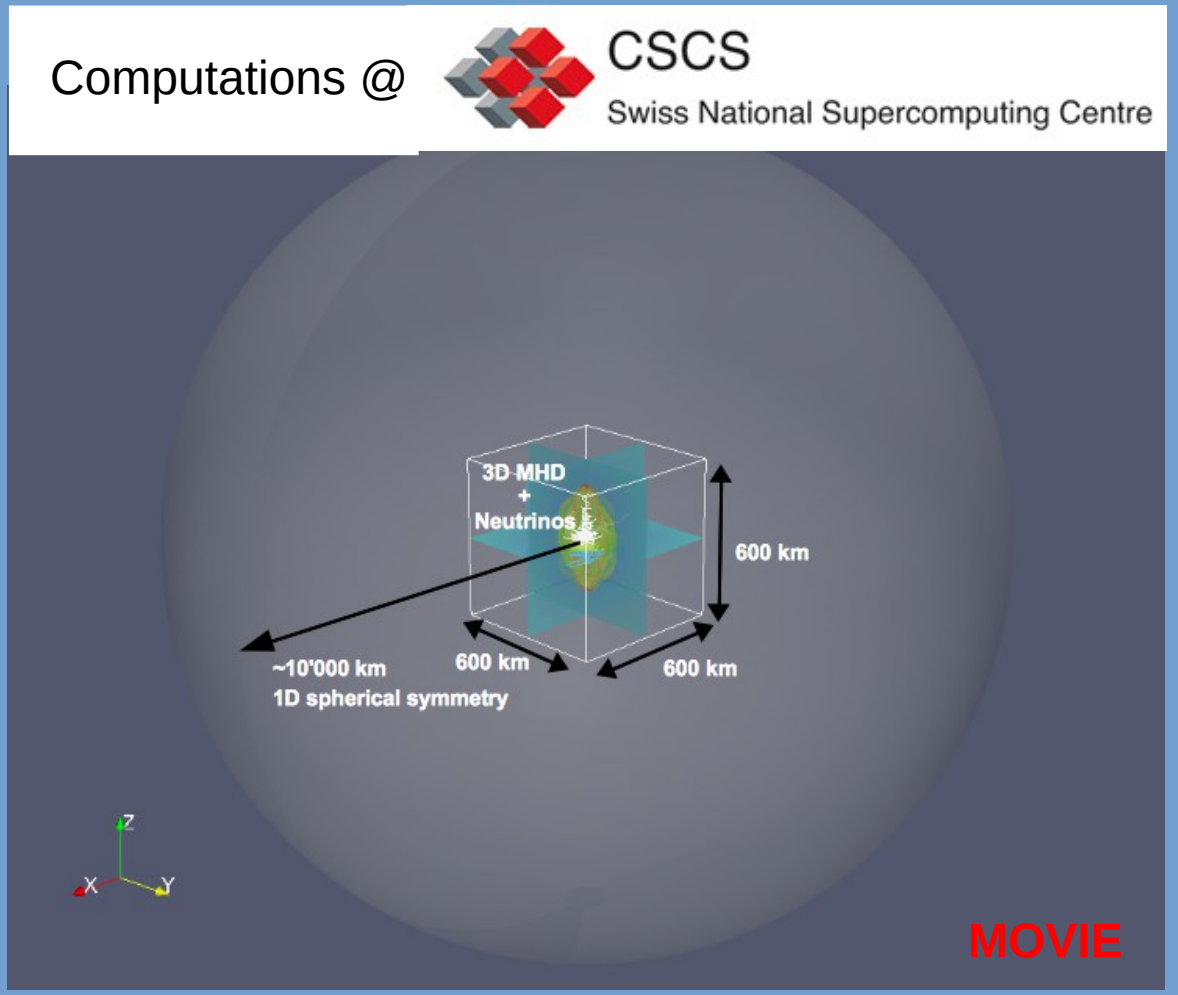
Simulation of MHD CCSN

Initial conditions:

$$\Omega_{0,c} = \pi \text{ rad/s}$$

$$B_{\text{pol}} = 5 \times 10^{12}$$

Constant 1 km resolution!



Magnetic field amplification

- Flux compression $B \propto \rho^{2/3}$ → Factor ≈ 1000

  Works well during collapse... Actually the main amplification

- Winding $\dot{B}_{\rm tor} \sim B_{\rm pol} \dfrac{\partial \Omega}{\partial \ln r}$

  With differential rotation, linear growth with time

- Magneto-Rotational Instability (MRI)

  With differential rotation, exponential growth with time, VERY small wavelengths...

- Dynamo?

Formation and driving mechanism of bipolar outflow

- Differential rotation winds poloidal magnetic field into toroidal field increasing the magnetic energy and pressure

$$P_{\mathrm{mag}} = \frac{B^2}{8\pi}$$

- Strongly magnetised regions appear along the rotational axis
- Magnetic pressure quickly reaches and exceeds the gas pressure
- Consider the equation of motion

$$\rho \frac{\boldsymbol{v}}{t} = -\nabla p - \rho \nabla \phi - \boldsymbol{b} \times (\nabla \times \boldsymbol{b})$$

Pressure force

Gravity force

Lorentz force

Formation and driving mechanism of bipolar outflow

- Differential rotation winds poloidal magnetic field into toroidal field increasing the magnetic energy and pressure
- S
- M ure
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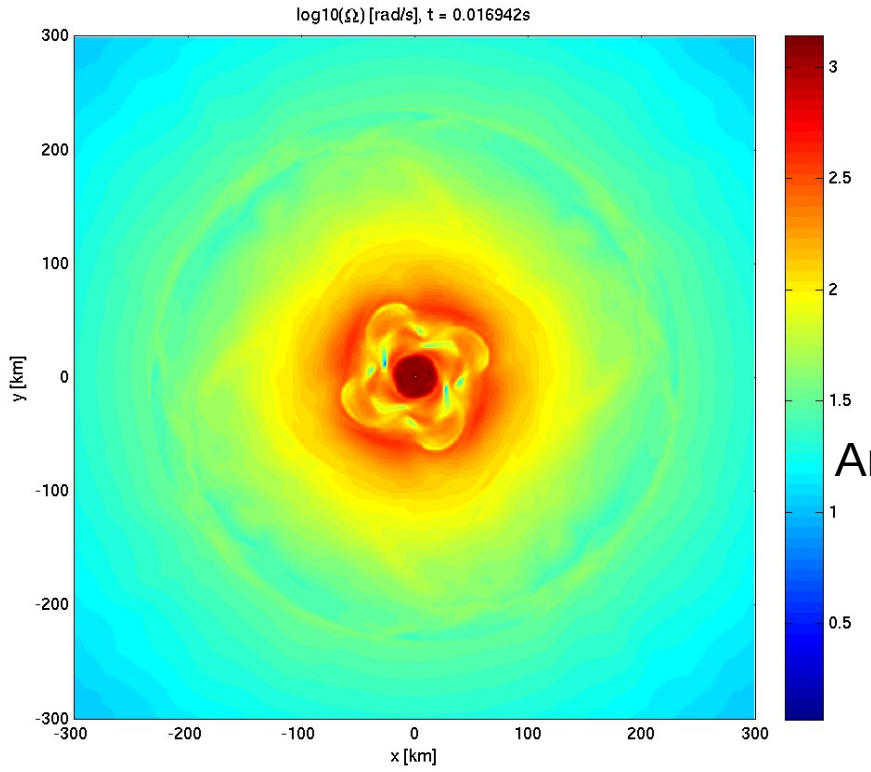

$\log \Omega$

Angular velocity [rad/s]

Gravity force

Formation and driving mechanism of bipolar outflow

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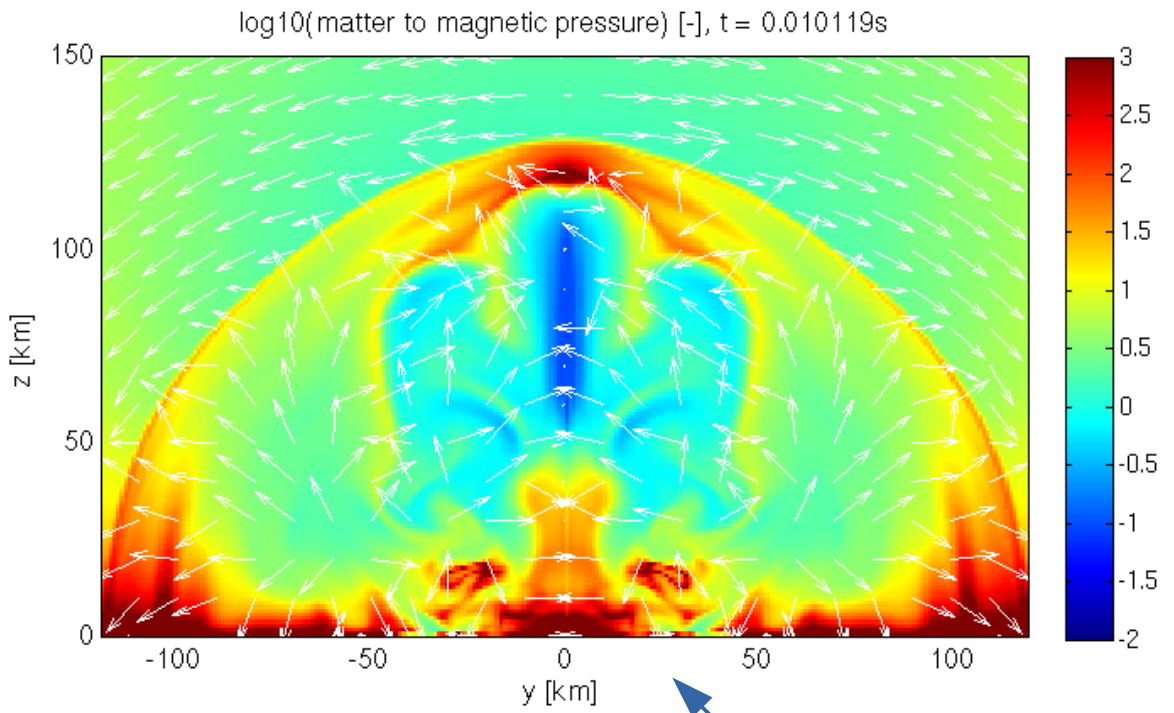
Pressure force

Gravity force

Lorentz force

Forma utflow

- Differ
  incre
- Stron
- Magr ure
- Cons



$$\log\left(P_{\mathrm{mat}}/P_{\mathrm{mag}}\right)$$

$$\rho\frac{\boldsymbol{v}}{t} = -\nabla p - \rho\nabla\phi - \boldsymbol{b}\times(\nabla\times\boldsymbol{b})$$

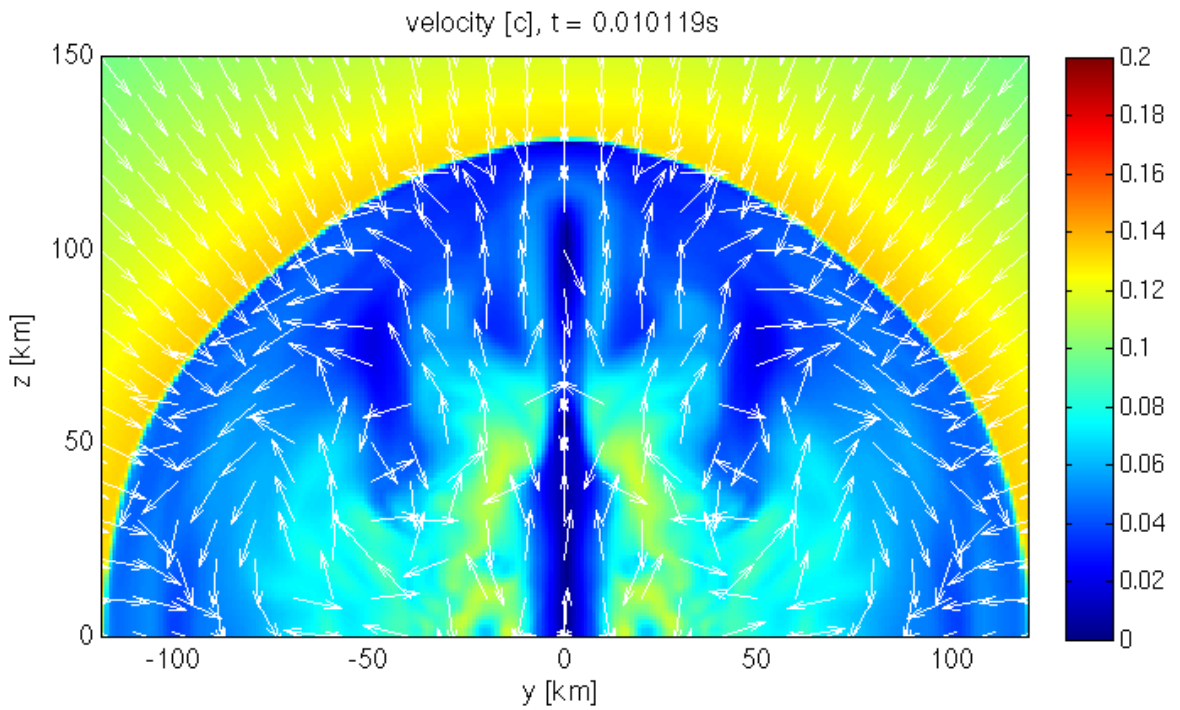
Pressure force

Gravity force

Lorentz force

Forma utflow

- Differ
  incre
- Stron
- Magn ure
- Cons



$$\rho \frac{\boldsymbol{v}}{t} = -\nabla p - \rho \nabla \phi - \boldsymbol{b} \times (\nabla \times \boldsymbol{b})$$

Pressure force

Gravity force

Lorentz force

Explosion energy, ejected mass and its composition

- Bipolar jets quickly expand & transport energy and stellar material outward against the gravitational attraction of the PNS
- Very neutron rich matter is lifted... r-process?
- Approximately determine explosion energy and ejected mass

Specific total energy $\epsilon = e_{\rm int} + \frac{v^2}{2} + \frac{b^2}{2\rho} + \phi > 0$

when shock reaches upper boundary of 3D domain 700 x 700 x 1400 km

$t_f = 33\mathrm{ms}$

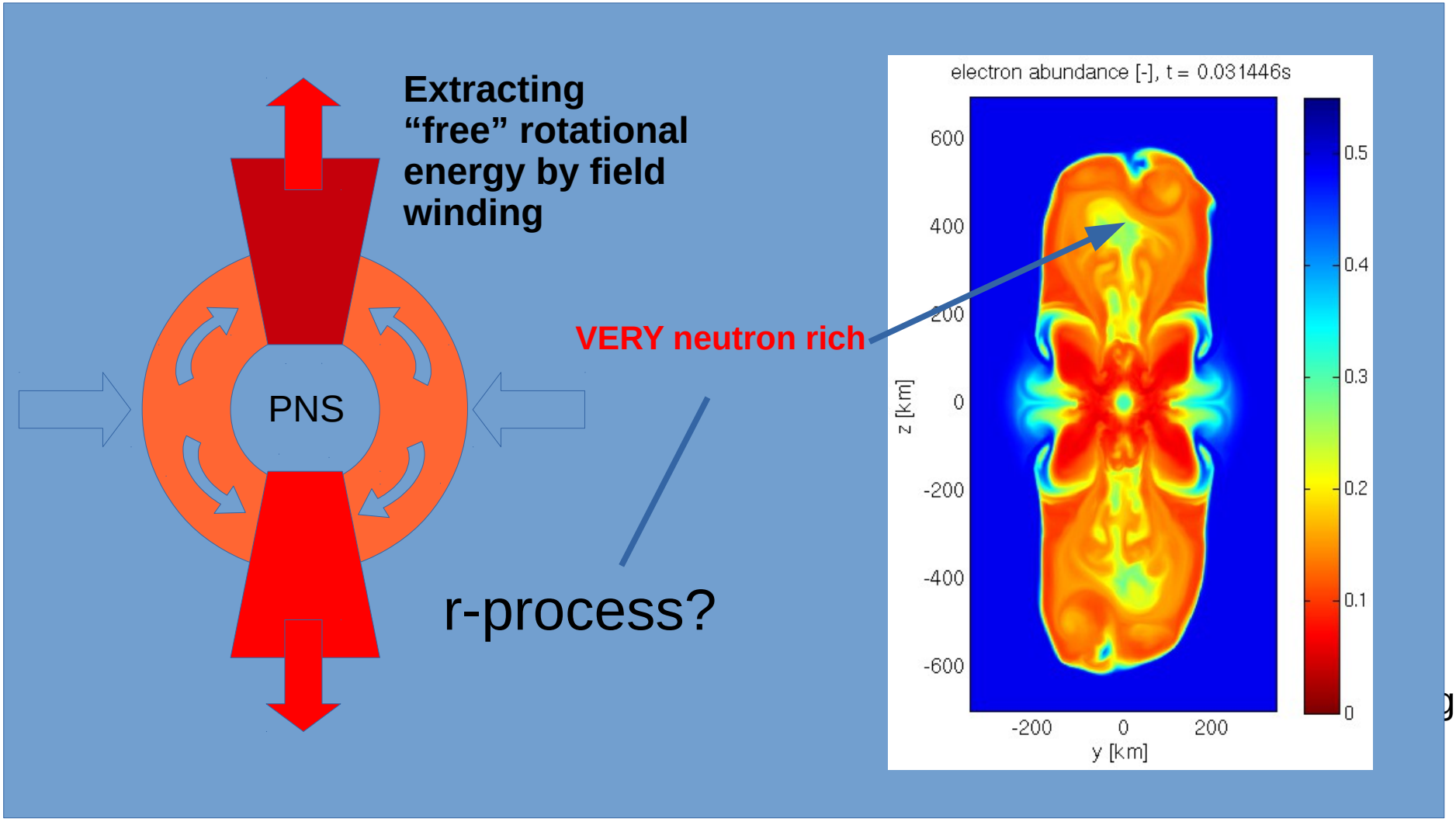
Prompt time scale...

$M_{\rm ej} = 6.72 \times 10^{-3} M_\odot$

$E_{\rm exp} = 8.45 \times 10^{+49}\mathrm{erg} \lesssim 10^{51}\mathrm{erg}$!

Still growing!

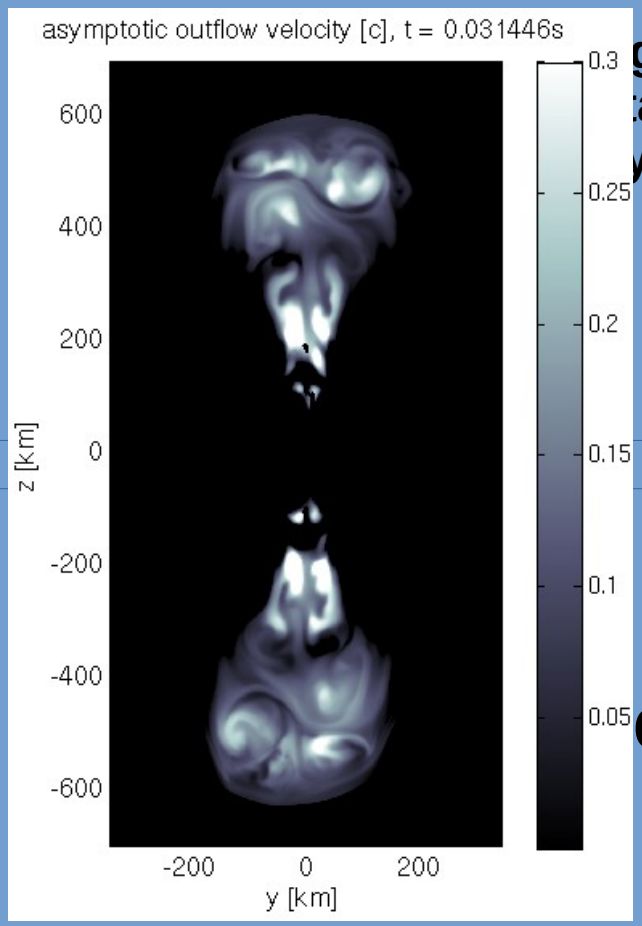
Explosion energy, ejected mass and its composition



km

g!

Explosion energy, ejected mass and its composition



g
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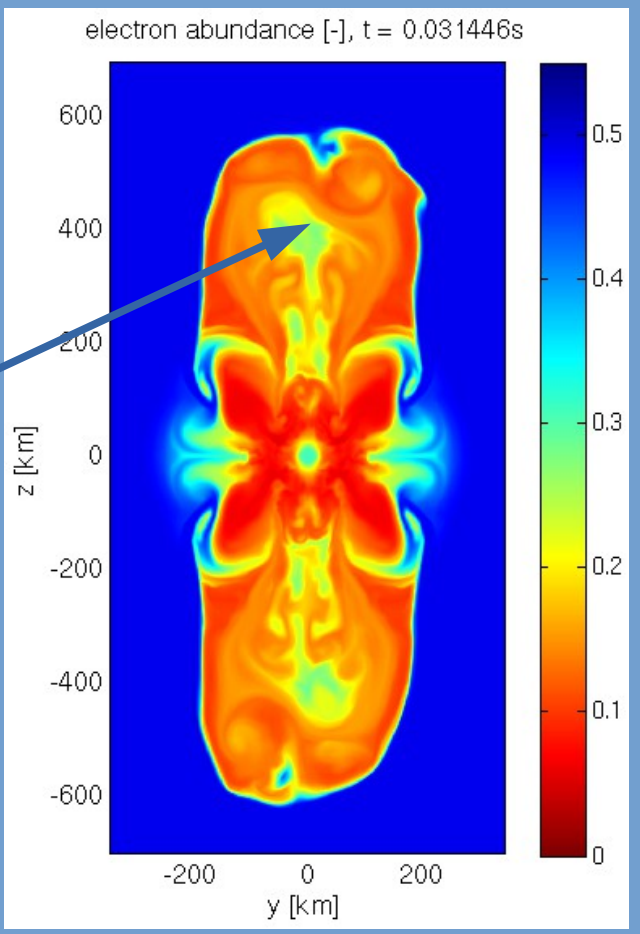
VERY neutron rich

cess?

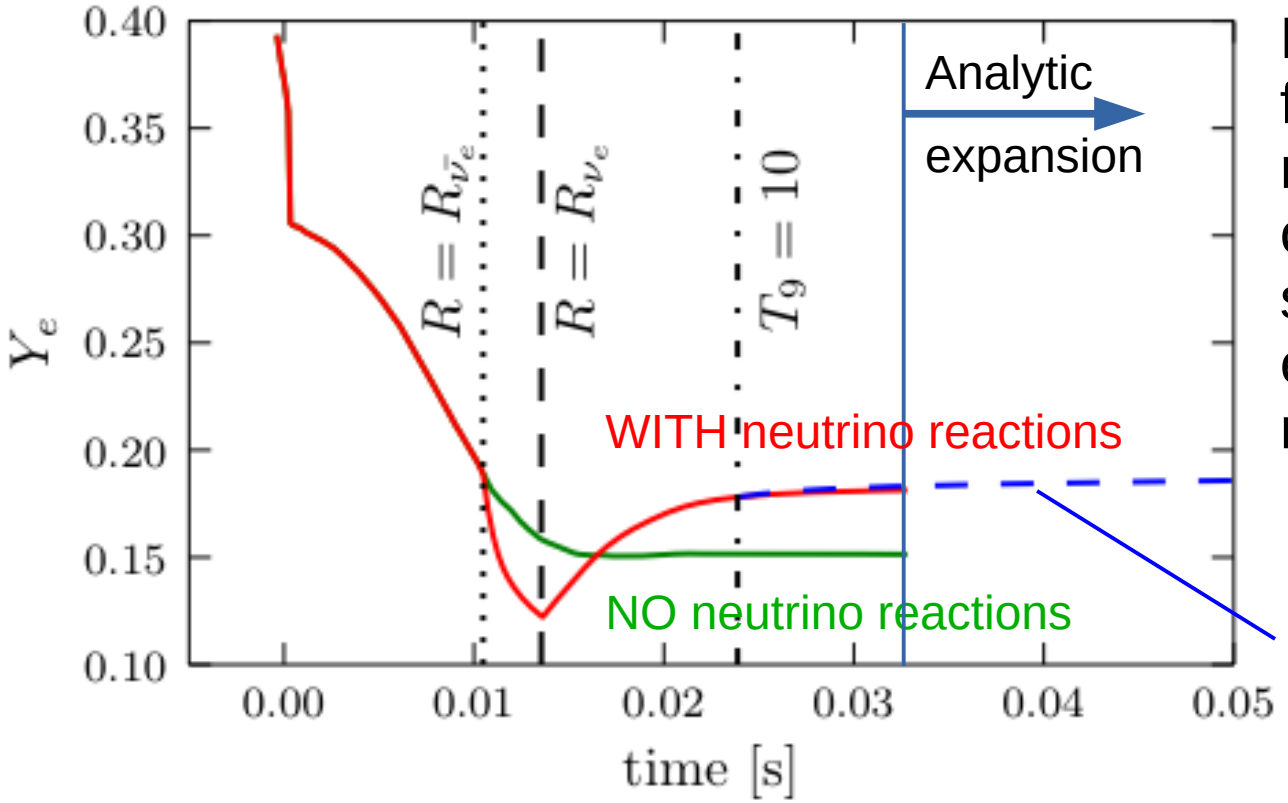


km

g!

Composition of the ejecta

- Included tracer particles to track the evolution of thermodynamic conditions in a Lagrangian manner
- Electron fraction is a key input for the nucleosynthesis and strongly depends on the challenging ν transport



Evolve the electron fraction with integrated neutrino luminosities outside of the neutrino spheres using approx. emission/absorption on nucleons Janka 2001

Winteler et al. 2012

!!! POST-PROCESSING !!!

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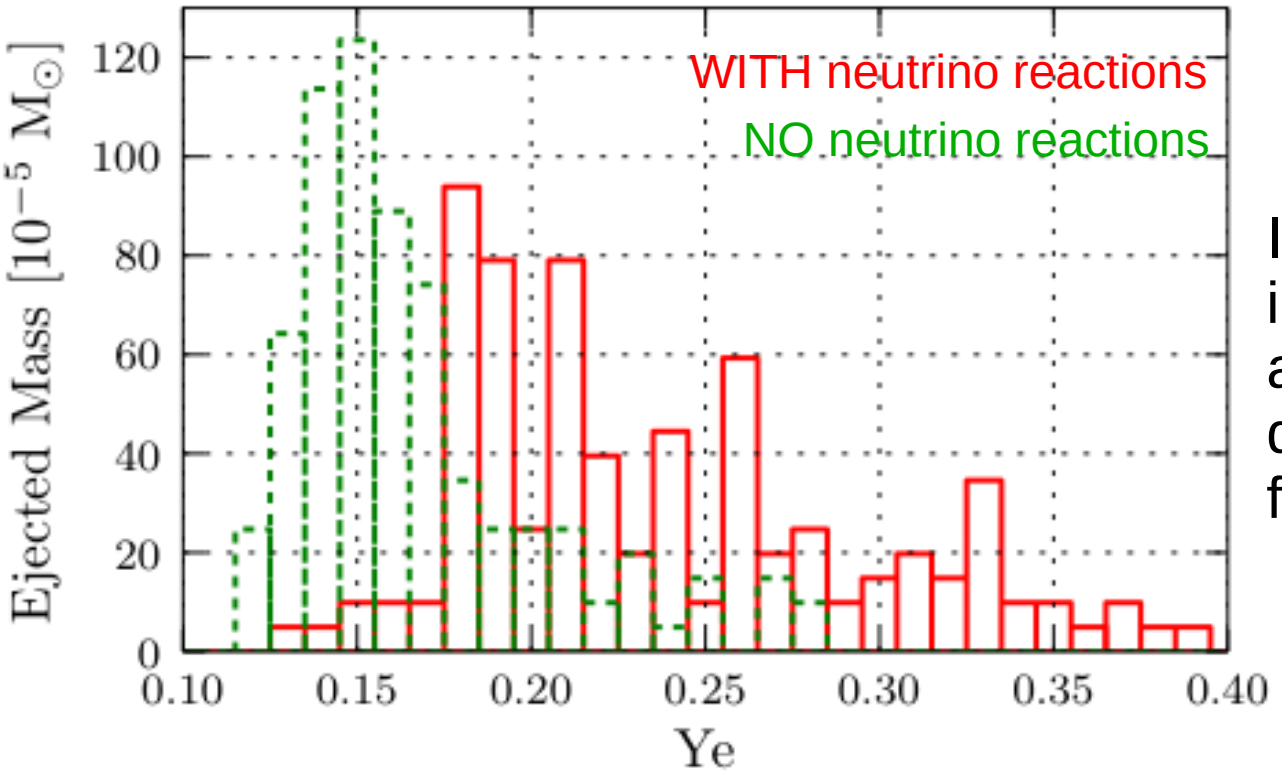

Including neutrino interactions shifts the peak and broadens the distribution of electron fraction to the right

!!! POST-PROCESSING !!!

Winteler et al. 2012

Composition of the ejecta

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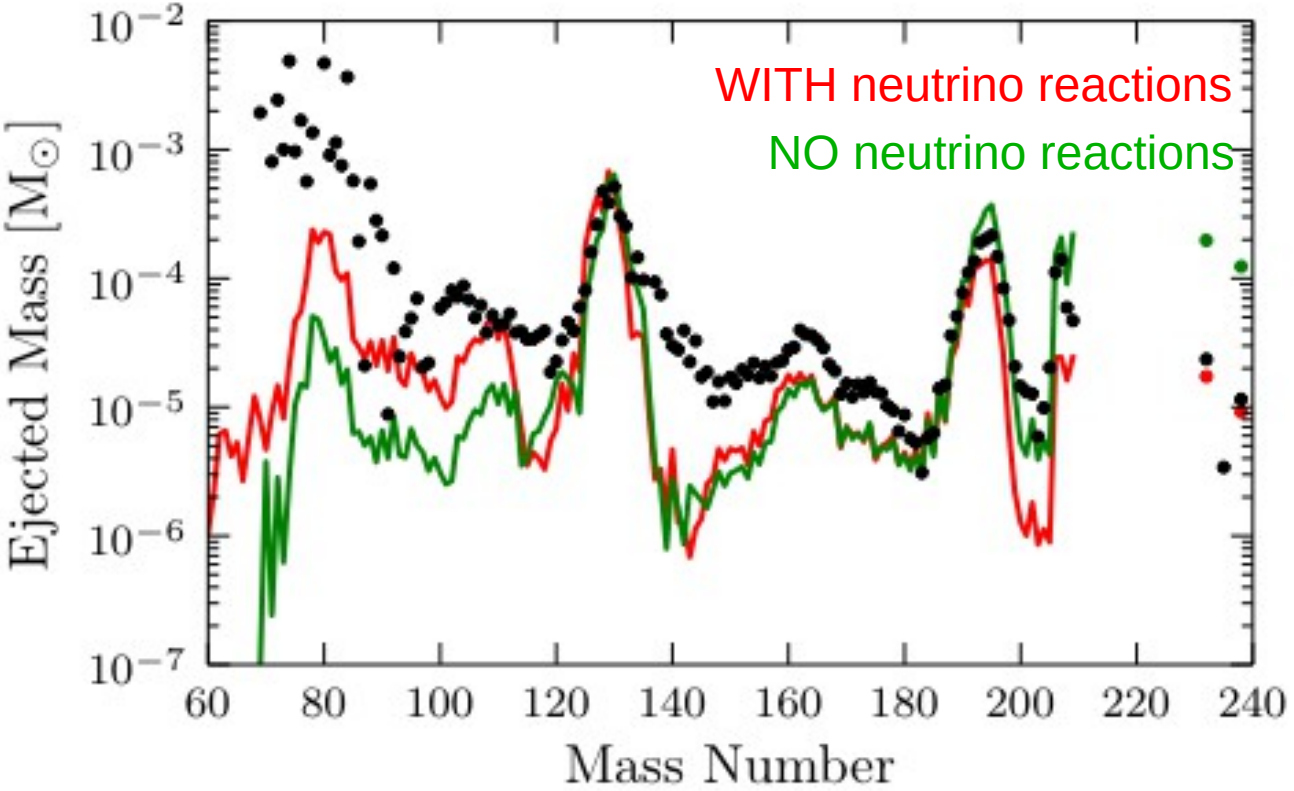


Mass integrated abundances

!!! POST-PROCESSING !!!

Winteler et al. 2012

Discussion of MHD-CCSN

- (Too) Fast initial rotation rate?

  Stellar evolution with magnetic fields $\Omega \sim 0.3\ \mathrm{rad/s}$
  Heger et al. 2005

  Stellar evolution without magnetic fields $\Omega \sim \pi\ \mathrm{rad/s}$
  e.g. Heger et al. 2000, Hirschi et al. 2004

- (Too) much r-process matter ejected?

  Eject $M_{\mathrm{r,ej}} = 5.64 \times 10^{-3} M_\odot$ of r-process material...

  But: if all CCSN exploded with the MHD mechanism,
  then r-process overproduced by factor $100 - 1000$

- (Too) strong initial magnetic field ~ 1000

  Some stars may have large field strengths... e.g. magnetars
  strong magnetic white dwarfs
  e.g. Wickramasinghe & Ferrario 2000

- (Very) Short simulation time?

  Only 33 ms !!!

MHD CCSN only rare event with special conditions

Woosley & Heger 2006

Consistent with large star-to-star scatter of r-process abundances at low metallicity

e.g. Cowan & Sneden 2006

Numerical issues...
Angular momentum conservation

Currently working on angular momentum preserving schemes

Resolution of MRI...

Conclusion

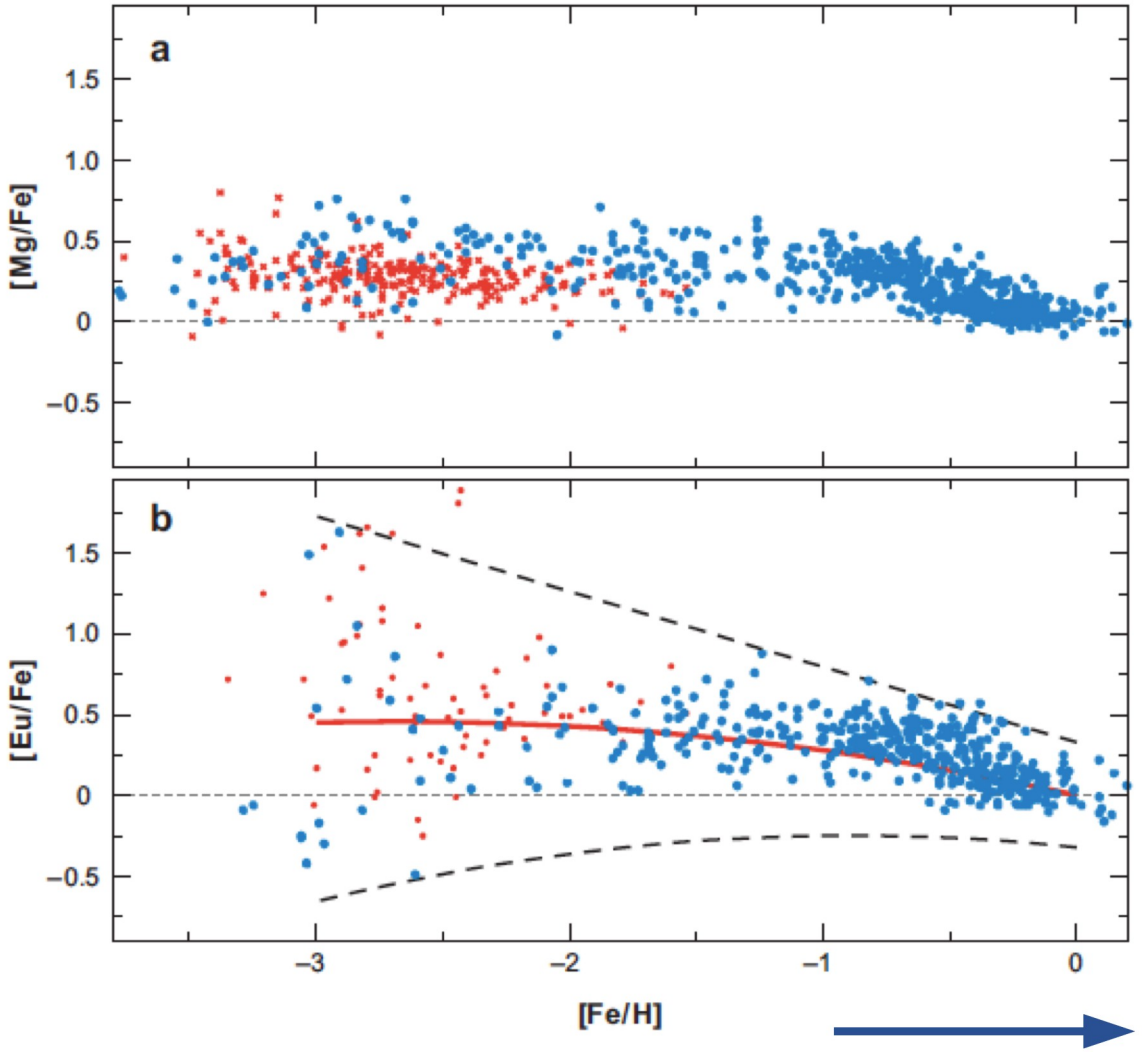
- A brief introduction to the problem of CCSN
- Simulation of CCSN a challenging field for physics/computational science/numerics
- Highly sophisticated 1D models do not explode 2D & ultimately 3D models with comparable sophistication are needed (what is important?)
- New physics?
- MHD mechanism works and performs a strong r-process nucleosynthesis
  BUT: it s probably not the standard explosion mechanism!

The End, Thanks!

Early galactic r-process nucleosythesis

α-element commonly produced in massive stars

Typical r-process element



SNIa

Figure 14

[Mg/Fe] and [Eu/Fe] abundances as a function of [Fe/H] metallicity for halo and disk stars. For this figure the data have been taken only from large-sample surveys: Fulbright (2000); Reddy et al. (2003); Cayrel et al. (2004); Cohen et al. (2004); Simmerer et al. (2004); Barklem et al. (2005, red points); Reddy, Lambert & Allende Prieto (2006); François et al. (2007). In both panels the dotted lines represent the Solar abundance ratios. In panel *b*, the solid red line is a least-square fit to the Eu data, and the two dashed black lines indicate the approximate extent of the Eu/Fe data (similar to Cowan & Thielemann 2004).

From Sneden, Cowan & Gallino 2008