Diploma Project/Diplomarbeit

(Mathematics/Computational Science & Engineering)

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Least Squares Based A-posteriori Error Estimators

Field. Numerical analysis, finite elements, first order least squares, error estimation, mesh refinement, *coding*

Problem. We consider a second order elliptic boundary value problem

$$\operatorname{div}(\alpha \,\mathbf{grad}\, u) = f \quad , \quad u = 0 \;\text{ on } \Gamma_D \quad , \quad \langle \alpha \,\mathbf{grad}\, u, \mathbf{n} \rangle = 0 \;\text{ on } \Gamma_N \;, \tag{1}$$

on $\Omega \subset \mathbb{R}^2$, whose boundary has been partitioned into Γ_D and Γ_N. The related least squares problem is: Seek $\mathbf{j} \in \boldsymbol{H}_{\Gamma_N}(\operatorname{div};\Omega)$, $u \in H^1_{\Gamma_D}(\Omega)$, such that

$$\left\| \alpha^{-1}\mathbf{j} - u \right\|^2_{\boldsymbol{L}^2(\Omega)} + \left\| \operatorname{div} \mathbf{j} - f \right\|^2_{L^2(\Omega)} \;\rightarrow\; \min \;. \tag{2}$$

Given approximations $\mathbf{j}_h$ and u_h it turns out that merely plugging them into the functional from (2) provides an efficient and reliable estimate for the total error in the least squares energy norm [2].

Issues. The main obstacle to using the above result is that common finite element schemes for (1) fail to provide good approximations for both $\mathbf{j}$ and u: In the primal method $\mathbf{j}_h := \alpha u_h$ fails to belong to $\boldsymbol{H}(\operatorname{div};\Omega)$ and u_h obtained from a mixed method is discontinuous [3].

Thus, we have to employ *post-processing* to recover suitable approximations for missing quantities. For the primal finite element schemes this can be done by suitable averaging [1, 5], whereas mixed-hybrid techniques seem attractive in a mixed context. Of course, there has to be a theoretical guarantee that post-processing leads to error estimators with the usual desirable properties. It also has to be computationally efficient, i.e. should involve only local computations.

Approaches. The focus is both on theory and numerical experiments. First appropriate post-processing schemes have to be devised and examined. They should be implemented and studied empirically for a couple of model problems in 2D. A comparison of the resulting least squares error estimators with established techniques [4, 6] will shed light on their actual performance.

Coding. Implementation can rely on existing adaptive finite element codes (either C++ or MATLAB).

References

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