

Term Project/Semesterarbeit

(Computational Science & Engineering)

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Angle conditions for edge and face elements

Problem. *Edge elements* are lowest order $\mathbf{H}(\mathbf{curl}; \Omega)$ -conforming finite elements (space $\mathcal{W}^1(\mathcal{M})$, \mathcal{M} a triangulation of $\Omega \subset \mathbb{R}^3$), whereas *face elements* provided finite element subspaces of $\mathbf{H}(\mathbf{div}; \Omega)$ (space $\mathcal{W}^2(\mathcal{M})$), see [1, Sect. 3.2]. There are *canonical local interpolation operators* $I^1 : (C^0(\Omega))^3 \mapsto \mathcal{W}^1(\mathcal{M})$ and $I^2 : (C^0(\Omega))^3 \mapsto \mathcal{W}^2(\mathcal{M})$. These are not defined on $\mathbf{H}(\mathbf{curl}; \Omega)$ or $\mathbf{H}(\mathbf{div}; \Omega)$, respectively, but on the spaces

$$\mathbf{H}^1(\mathbf{curl}, \Omega) := \{\mathbf{u} \in \mathbf{H}^1(\Omega) : \mathbf{curl} \mathbf{u} \in \mathbf{H}^1(\Omega)\},$$

and $\mathbf{H}^1(\Omega)$ respectively.

Task. The best constants $C_1, C_2 > 0$ in the local interpolation estimates

$$\begin{aligned} \|\mathbf{u} - I^1 \mathbf{u}\|_{\mathbf{L}^2(K)} &\leq C_1 \left(\|u\|_{\mathbf{H}^1(K)}^2 + \|\mathbf{curl} \mathbf{u}\|_{\mathbf{H}^1(K)}^2 \right)^{\frac{1}{2}}, \\ \|\mathbf{u} - I^2 \mathbf{u}\|_{\mathbf{L}^2(K)} &\leq C_2 \|u\|_{\mathbf{H}^1(K)}, \end{aligned}$$

are to be computed for an arbitrary tetrahedron K . This should be done by means of a polynomial spectral method, see <http://www.sam.math.ethz.ch/~hiptmair/tmp/NPDE.pdf>, Sect. 6.6.

Focus. Implementation of a polynomial spectral method in MATLAB or C++ and design of numerical experiments in order to investigate the dependence of C_1, C_2 on the shape of K (angle conditions).

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References

- [1] R. HIPTMAIR, *Finite elements in computational electromagnetism*, Acta Numerica, (2002), pp. 237–339.