Master/Diploma Thesis Project
(Mathematics)

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Spectral lumping for discrete polar coordinates

Prerequisites. Knowledge about the numerical treatment of partial differential equations, as acquired in the respective courses, see [1, 2].

Problem description. We consider boundary value problems on the circular domain $\Omega := \{\mathbf{x} \in \mathbb{R}^2, |\mathbf{x}| < 1\}$:

- the potential equation

$$-\Delta u = f \quad \text{in } \Omega \quad , \quad u = 0 \quad \text{on } \partial\Omega \; . \tag{1}$$

- a non-linear 2nd-order elliptic boundary value problem

$$-\Delta u + u^3 = f \quad \text{in } \Omega \quad , \quad u = 0 \quad \text{on } \partial\Omega \; . \tag{2}$$

- a pure advection problem

$$\boldsymbol{b} \cdot \mathbf{grad}\, u = 0 \quad \text{in } \Omega \quad , \quad u = g \quad \text{on inflow boundary.} \tag{3}$$

- Burger's equation as a specimen of an initial-boundary value problem for a non-linear conservation law

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(\tfrac{1}{2}u^2) + \frac{\partial}{\partial y}(\tfrac{1}{2}u^2) = 0 \; , \tag{4}$$

$$u(\mathbf{x}, 0) = u_0(\mathbf{x}) \quad \text{and} \quad \text{periodic bounday conditions.} \tag{5}$$

Discretization of each of these problems will be based on a tensor product grid in polar coordinates, see Fig. 1. In particular, we use

- piecewise linear Lagrangian finite elements in the case of (1) and (2)
- a simple upwind finite volume scheme in the case of (3).
- Godunov's method in the case of (4).

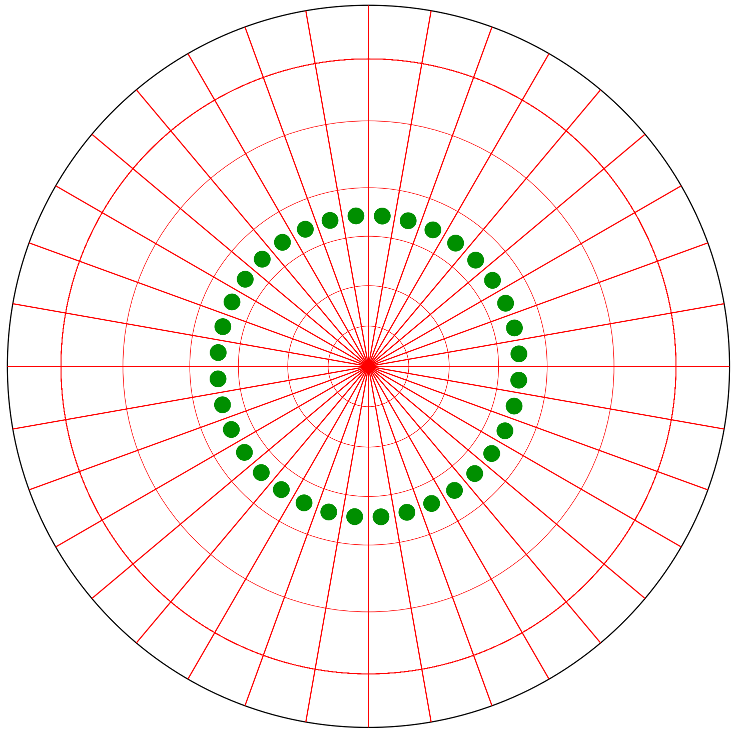

Figure 1: Tensor product mesh in polar coordinates. The green dots mark d.o.f. located at the same radius.

In angular direction we use a Fourier basis representation for degrees of freedom having the same radial coordinate. This amounts to a mere basis transformtion applied to the discrete system of equations.

In a second step some high frequency angular Fourier components are discarded (spectral lumping). This can be done depending on the radial position of the lumped degrees of freedom.

Task. Implementation of the discrete problems before and after spectral lumping in MATLAB based on a simple finite element toolbox. Empirical and mathematical analysis of the impact of spectral lumping on the various problems.

References

[1] D. Braess, *Finite Elements*, Cambridge University Press, 2nd ed., 2001.

[2] R. LeVeque, *Finite Volume Methods for Hyperbolic Problems*, Cambridge Texts in Applied Mathematics, Cambridge University Press, Cambridge, UK, 2002.