

Seminar in Spring Semester 2010

Structure Preserving Discretizations of PDEs

Lecturers : Prof. R. Hiptmair
: Prof. S. Mishra
Venue : HG E 33.3
Time : Tue 15-17
Language : English
First session : March 22, 2011
Prep meeting : Feb 22, 2011, 15:15, HG E 33.3
Contact : R. Hiptmair, hiptmair@sam.math.ethz.ch
: S. Mishra, mishra@sam.math.ethz.ch

Prerequisites : Knowledge about numerical methods for PDEs as conveyed in the courses *Numerical methods for elliptic partial differential equations* or *Numerical methods for partial differential equations*

Audience : BSc/MSc Students of Mathematics, RW/CSE from the 3rd year

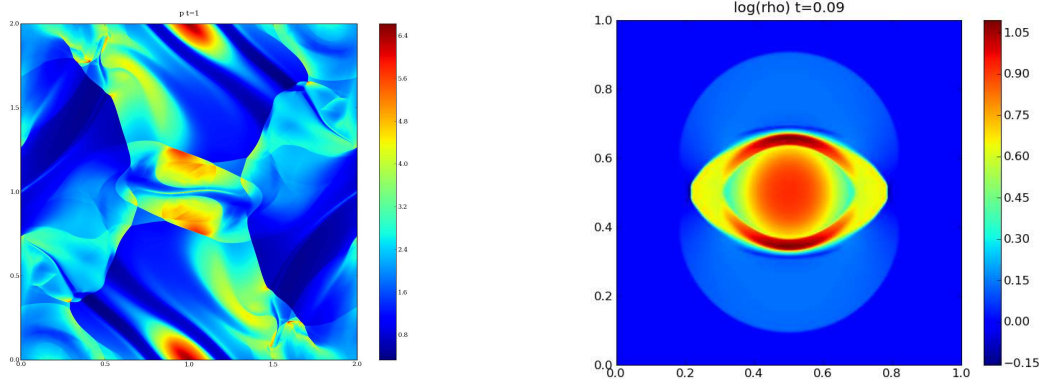
Description:

Partial differential equations that arise from continuum models of physical systems feature rich structural properties that are usually connected to fundamental physical properties. Hence it is essential that these traits are preserved in the process of discretizing the boundary value problems for the PDE. This seminar studies a few prominent examples of PDE models that are rich in structure and examines discretization approaches that respect it.

In detail the following topics may be addressed:

- Maxwell equations and co-chain models

- Low-order discrete differential forms
- Edge element discretization of Maxwell's equations
- Maxwell eigenvalue problem and the discrete compactness property
- Discrete Lie derivatives
- Vortex methods for incompressible Euler equations
- Vorticity preserving finite differences for Euler flow
- Projection methods for magnetohydrodynamics (MHD)
- MHD: Structure preserving finite difference schemes
- MHD: Generalized Lagrangian multipliers



MHD flow patterns simulated with structure preserving finite volume method (S. Mishra)

Presentations:

The seminar will comprise up to 10 student presentations of a duration of about 60 minutes. They should be partly based on PDF slides prepared using the BEAMER \LaTeX package (or \LaTeX based tools under MacOS). The presentations should be done using a laptop computer (which can be provided). Speakers are advised to elaborate technical manipulations and proofs on the blackboard. MATLAB demonstration of simple numerical experiments is expected whenever appropriate. The lecture slides in PDF format should be made available immediately after the presentation.

Available topics:

1. "The Maxwell house" [Bos05, Chapter 2]
2. "Discrete Maxwell's equations" [Bos05, Chapter 3], [Ton01]
3. "Discrete differential forms" [Hip02, Sect. 3], [Bos05, Chapter 4], [AFW06, Sects. 3-5]
4. "Discrete Hodge operators" [Hip01]
5. "Maxwell eigenvalue problem" [Bof10, Part 5]

6. Projection methods (SM & UF)
7. Vorticity preserving methods (Lerat 2008)
8. Divergence preserving methods (SM & ET, 2011)
9. Divergence cleaning methods (SM, 2011)
10. Structure preserving transport [?]

Speakers and dates for presentations:

Date	Speaker	Topic #
22.3.2011	Okan Koc	1
29.3.2011	Simon Lanthaler	2
5.4.2011	Franziska Weber	3
12.4.2011	Jan Mikelson	4
19.4.2011	Marc Lickes	5
3.5.2011	Reto Haeberli	7
10.5.2011	Sophie Haug	6
17.5.2011	Elke Spindler	8
31.5.2011	Luc Grosheintz	9
1.6.2011	Steve Hou	10

References

- [AFW06] D.N. Arnold, R.S. Falk, and R. Winther. Finite element exterior calculus, homological techniques, and applications. *Acta Numerica*, 15:1–155, 2006.
- [Bof10] D. Boffi. Finite element approximation of eigenvalue problems. *Acta Numerica*, 19:1–120, 2010.
- [Bos05] A. Bossavit. Discretization of electromagnetic problems: The “generalized finite differences”. In W.H.A. Schilders and W.J.W. ter Maten, editors, *Numerical Methods in Electromagnetics*, volume XIII of *Handbook of numerical analysis*, pages 443–522. Elsevier, Amsterdam, 2005.
- [Hip01] R. Hiptmair. Discrete Hodge operators. *Numer. Math.*, 90:265–289, 2001.
- [Hip02] R. Hiptmair. Finite elements in computational electromagnetism. *Acta Numerica*, 11:237–339, 2002.
- [Ton01] E. Tonti. Finite formulation of the electromagnetic field. In F.L. Teixeira, editor, *Geometric Methods for Computational Electromagnetics*, volume 32 of *PIER*, pages 1–44. EMW Publishing, Cambridge, MA, 2001.

The articles can be obtained from

<http://www.sam.math.ethz.ch/~hiptmair/Seminars/STRUCPRES/Articles/>.