Seminar in Spring Semester 2013

Shape Calculus

Lecturer	:	Prof. Ralf Hiptmair, D-MATH
Venue	:	HG E 33.3
Time	:	Tue 10-12
Language	:	English
Prep meeting	:	Tue, September 19, 2018, 10:15, HG E 33.3
Individual tutoring	:	Tue 8-10, HG G 58.2
First session	:	Tue, October 9, 2018
	:	
Contact	:	R. Hiptmair, hiptmair@sam.math.ethz.ch
Prerequisites	:	Knowledge of analysis, elliptic partial differential equations and functional analysis; some familiarity with numerical methods for PDEs is expected.
Audience	:	MSc/3rd year BSc Students of Mathematics

Description:

Sloppily speaking, shape calculus studies how to derive domain-dependent functionals and functions with respect to variations of the domain. The prime example are solutions of boundary value problems for partial differential equations, which are naturally associated with a domain. The seminar will study several sections of monographs and research papers dealing with analytical and numerical aspects of shape calculus.

Presentations:

The seminar will comprise up to 10 student presentations of a duration of about 60 minutes. They should be partly based on PDF slides and may also involve elaborations on the blackboard. The lecture slides in PDF format should be made available immediately after the presentation.

Quizz:

Participants of the seminar will be asked questions about the previous presentations at the beginning of each session.

Available topics:

1. *The velocity method and Eulerian shape gradients*: Main reference **[SZ92, Sect. 2.8–2.11, 2.1, 2.18]**, covers the "velocity method", the Hadamard structure theorem and formulas for shape gradients of particular functionals. Several other sections of [SZ92, Ch. 2] provide foundations and auxiliary results and should be browsed, too.

2. *Material derivatives and shape derivatives*, based on **[SZ92, Sect. 2.25–2.32]**.

3. *Shape calculus with exterior calculus*, following **[HL13]** (without Sections 5 & 6). Based on classical vector analysis the formulas are also derived in [SZ92, Sects 2,19,2.20] and [DZ10, Ch. 9, Sect. 5]. Important background and supplementary information about the shape Hessian can be found in [DZ91, BZ97] and [DZ10, Ch. 9, Sect. 6].

4. *Shape derivatives of solutions of PDEs* using exterior calculus **[HL17]**, see also [HL13, Sects. 5 & 6]. From the perspective of classical calculus the topic is partly covered in [SZ92, Sects. 3.1-3.2].

5. *Shape gradients under PDE constraints* according to **[Pag16, Sect. 2.1]** including a presentation of the adjoint method for differentiating constrained functionals **[HPUU09, Sect. 1.6]**. Related information can be found in [DZ10, Ch. 10, Sect. 2.5] and [SZ92, Sect. 3.3].

6. *Approximation of shape gradients* following **[HPS14]**. Comparison of discrete shape gradients based on volume and boundary formulas, see also [DZ10, Ch. 10, Sect. 2.5].

7. *Optimal shape design based on boundary integral equations* following **[Epp00b]**, with some additional information provided in [Epp00a].

8. *Convergence in elliptic shape optimization* as discussed in **[EHS07]**. Relies on results reported in [Epp00b] and [DP00]. Discusses Ritz-Galerkin discretization of optimality conditions for normal displacement parameterization.

9. *Shape optimization by pursuing diffeomorphisms* according to **[HP15]**, see also [Pag16, Ch. 3] for more details, and [PWF17] for extensions.

10. *Distributed shape derivative via averaged adjoint method* following **[LS16]**.

Speakers and dates for presentations:

Date	Speaker	Topic #
09.10.2018		1
16.10.2018		2
23.10.2018		3
30.10.2018		4
06.11.2018		5
13.11.2018		6
20.11.2018		7
27.11.2018		8
04.12.2018		9
11.12.2018		10

References

[BZ97] Dorin Bucur and Jean-Paul Zolsio. Anatomy of the shape hessian via lie brackets. *Annali di Matematica Pura ed Applicata*, 173:127–143, 1997. 10.1007/BF01783465.

[DP00] Marc Dambrine and Michel Pierre. About stability of equilibrium shapes. *M2AN Math. Model. Numer. Anal.*, 34(4):811–834, 2000.

[DZ91] Michel C. Delfour and Jean-Paul Zolésio. Velocity method and Lagrangian formulation for the computation of the shape Hessian. *SIAM J. Control Optim.*, 29(6):1414–1442, 1991.

[DZ10] M.C. Delfour and J.-P. Zolésio. *Shapes and Geometries*, volume 22 of *Advances in Design and Control*. SIAM, Philadelphia, 2nd edition, 2010.

[EHS07] Karsten Eppler, Helmut Harbrecht, and Reinhold Schneider. On convergence in elliptic shape optimization. *SIAM J. Control Optim.*, 46(1):61–83 (electronic), 2007.

[Epp00a] Karsten Eppler. Boundary integral representations of second derivatives in shape optimization. *Discuss. Math. Differ. Incl. Control Optim.*, 20(1):63–78, 2000. German-Polish Conference on Optimization—Methods and Applications (Żagań, 1999).

[Epp00b] Karsten Eppler. Optimal shape design for elliptic equations via BIE-methods. *Int. J. Appl. Math. Comput. Sci.*, 10(3):487–516, 2000.

[HL13] Ralf Hiptmair and Jingzhi Li. Shape derivatives in differential forms I: an intrinsic perspective. *Ann. Mat. Pura Appl. (4)*, 192(6):1077–1098, 2013.

[HL17] R. Hiptmair and J.-Z. Li. Shape derivatives in differential forms II: Application to scattering problems. Report 2017-24, SAM, ETH Zürich, 2017. To appear in Inverse Problems.

[HP15] Ralf Hiptmair and Alberto Paganini. Shape optimization by pursuing diffeomorphisms. *Comput. Methods Appl. Math.*, 15(3):291–305, 2015.

[HPS14] R. Hiptmair, A. Paganini, and S. Sargheini. Comparison of approximate shape gradients. *BIT Numerical Mathematics*, 55:459–485, 2014.

[HPUU09] M. Hinze, R. Pinnau, M. Ulbrich, and S. Ulbrich. *Optimization with PDE constraints*, volume 23 of *Mathematical Modelling: Theory and Applications*. Springer, New York, 2009.

[LS16] Antoine Laurain and Kevin Sturm. Distributed shape derivative *via* averaged adjoint method and applications. *ESAIM Math. Model. Numer. Anal.*, 50(4):1241–1267, 2016.

[Pag16] A. Paganini. *Numerical shape optimization with finite elements*. Eth dissertation 23212, ETH Zurich, 2016.

[PWF17] A. Paganini, F. Wechsung, and P.E. Farell. Higher-order moving mesh methods for pde-constrained shape optimization. Preprint arXiv:1706.03117 [math.NA], arXiv, 2017.

[SZ92] J. Sokolowski and J.-P. Zolesio. *Introduction to shape optimization*, volume 16 of *Springer Series in Computational Mathematics*. Springer, Berlin, 1992.

Link for accessing literature: `https://polybox.ethz.ch/index.php/s/43SCcqjJqty6je0`, password: `ShapeCalc`