Seminar in Spring Semester 2023

Rational Approximation and Interpolation

Lecturer	:	Prof. Ralf Hiptmair, D-MATH
Venue	:	HG E 33.5
Time	:	Wed, 12-14
Language	:	English
Prep meeting	:	Wed, March 1, 2023, 12-14, HG E 33.5
Individual tutoring	:	Wed 14-16, HG G 58.2
First session	:	March 22, 2023, 12-14, HG E 33.5
Contact	:	R. Hiptmair, hiptmair@sam.math.ethz.ch
Prerequisites	:	Knowledge of analysis, elementary functional analysis and basic numerical methods
Audience	:	MSc & 3rd year BSc Students of Mathematics

Description:

The simplest and most widely used function system for approximation in computational mathematics are polynomials. They are ideally suited for smooth (analytic) functions. However, in many application we encounter functions with kinks and other kinds of singularities. In this case approximation by rational functions, that is, quotients of polynomials, may be vastly superior. This is why rational approximation and interpolation is receiving increased attention for the construction of surrogate models in model order reduction.

This seminar will study a number of research papers dealing with both theoretical and algorithmic aspects of rational approximation and interpolation. It covers both fundamental results in approximation theory and numerical analysis.

Presentations:

Each topic defined by one or more research articles should be covered in a roughly 60 minute presentation, based on prepared slides, and, maybe, short explanations on the blackboard or tablet. The slides should be made available as PDF beforehand.

> The seminar will comprise up to 12 student presentations of a duration of about 60 minutes. They should be held by teams of three with members presenting in turns in random order.

Quizz:

Participants of the seminar will be asked questions about the previous presentations at the beginning of each session.

Available topics:

1. Best approximation by rational functions: [PP87, Sections 2.1-2.3]: Qualitative theory of the best rational approximation, should also be connected to [PP87, Sects. 1.2-1.3]. See also [Tre13, Ch. 24].

2. Best rational approximation of $x \mapsto |x|$: [New64] and also [PP87, Sects 4.1, 4.2] and parts of [Tre13, Chapter 25].

3. Meinardus conjecture: [Bra82, Bra84] and [PP87, Sects 4.4, 4.5].

4. Approximation by composite rational functions: [NF16, Sect. 3], [GN21] and [PP87, Sect. 4.3]

5. Rational interpolation and linearized least-squares: [GPT11, PGvD12] and [Tre13, Chapter 26].

6. Padé approximationj: [Tre13, Chapter 27] and [GGT13]. See [PP87, Section 12.1] for background information.

7. Vector fitting: [DGB15, GS99], see also [DHD07]

8. The AAA algorithm for rational approximation: [NST18]

9. The RKFIT algorithm for non-linear rational approximation: [BG15, BG17]

10. Rational minimax approximation: [FNTB18]

11. Multivariate Padé approximation: [GH00, GHR98]

12. Fast least-squares Padé approximation: [BNP18, BNPP20b, BNPP20a]

Implementation and numerical experiments

Quite a few of the topics are algorithmic in nature. Many of the related papers mention open source implementations of the methods, mainly in MATLAB, often relying on the Chebfun library. It is desirable that groups presenting an algorithmic topic also conduct numerical experiments, those covered in the articles or others, and report their observations.

Speakers and dates for presentations:

- Team 1:

  - Federica Casanova

  - Arianna Guadagnini

  - Clemens Thalhammer

- Team 2:

  - Nikolaos Stamatopoulos

  - John Skelton

  - Elias Mbarek

- Team 3:

  - Emily Eberl

  - Franziska Gundersen

  - Dominik Rhyn

Date	**Speaker/Team**	Topic #	Tutoring
05.04.2023	**3**	1	22.03.
19.04.2023	**2**	2	05.04.
26.04.2023	**1**	3	19.04.
03.05.2023	**3**	4	24.04.
10.05.2023	**2**	5	03.05.
17.05.2023	**1**	6	10.05.
24.05.2023	**3**	7	17.05.
31.05.2023	**2**	12	24.05
31.05.2023	**1**	10	24.05

The extra presentation on May 31 will take place 10:15-12:00 in room HG F 26.1.

References

[BG15]   Mario Berljafa and Stefan Güttel. Generalized rational Krylov decompositions with an application to rational approximation. *SIAM J. Matrix Anal. Appl.*, 36(2):894–916, 2015.

[BG17]   Mario Berljafa and Stefan Güttel. The RKFIT algorithm for nonlinear rational approximation. *SIAM J. Sci. Comput.*, 39(5):A2049–A2071, 2017.

[BNP18]   Francesca Bonizzoni, Fabio Nobile, and Ilaria Perugia. Convergence analysis of Padé approximations for Helmholtz frequency response problems. *ESAIM Math. Model. Numer. Anal.*, 52(4):1261–1284, 2018.

[BNPP20a] Francesca Bonizzoni, Fabio Nobile, Ilaria Perugia, and Davide Pradovera. Fast least-squares Padé approximation of problems with normal operators and meromorphic structure. *Math. Comp.*, 89(323):1229–1257, 2020.

[BNPP20b] Francesca Bonizzoni, Fabio Nobile, Ilaria Perugia, and Davide Pradovera. Least-squares Padé approximation of parametric and stochastic Helmholtz maps. *Adv. Comput. Math.*, 46(3):Paper No. 46, 28, 2020.

[Bra82]   Dietrich Braess. On the conjecture of Meinardus on rational approximation of e^x. *J. Approx. Theory*, 36(4):317–320, 1982.

[Bra84]   Dietrich Braess. On the conjecture of Meinardus on rational approximation of e^x. II. *J. Approx. Theory*, 40(4):375–379, 1984.

[DGB15]   Z. Drmač, S. Gugercin, and C. Beattie. Quadrature-based vector fitting for discretized $\mathcal{H}_\in$ approximation. *SIAM J. Sci. Comput.*, 37(2):A625–A652, 2015.

[DHD07]   Dirk Deschrijver, Bart Haegeman, and Tom Dhaene. Orthonormal vector fitting: A robust macromodeling tool for rational approximation of frequency domain responses. *IEEE Transactions on Advanced Packaging*, 30(2):216–225, 2007.

[FNTB18] Silviu-Ioan Filip, Yuji Nakatsukasa, Lloyd N. Trefethen, and Bernhard Beckermann. Rational minimax approximation via adaptive barycentric representations. *SIAM J. Sci. Comput.*, 40(4):A2427–A2455, 2018.

[GGT13]   Pedro Gonnet, Stefan Güttel, and Lloyd N. Trefethen. Robust Padé approximation via SVD. *SIAM Rev.*, 55(1):101–117, 2013.

[GH00]   Philippe Guillaume and Alain Huard. Multivariate Padé approximation. *J. Comput. Appl. Math.*, 121(1-2):197–219, 2000. Numerical analysis in the 20th century, Vol. I, Approximation theory.

[GHR98]   Philippe Guillaume, Alain Huard, and Vincent Robin. Generalized multivariate Padé approximants. *J. Approx. Theory*, 95(2):203–214, 1998.

[GN21]   Evan S. Gawlik and Yuji Nakatsukasa. Approximating the pth root by composite rational functions. *J. Approx. Theory*, 266:Paper No. 105577, 16, 2021.

[GPT11]   Pedro Gonnet, Ricardo Pachón, and Lloyd N. Trefethen. Robust rational interpolation and least-squares. *Electron. Trans. Numer. Anal.*, 38:146–167, 2011.

[GS99]   B. Gustavsen and A. Semlyen. Rational approximation of frequency domain responses by vector fitting. *IEEE Transactions on Power Delivery*, 14(3):1052–1061, 1999.

[New64]   D. J. Newman. Rational approximation to $|x|$. *Michigan Math. J.*, 11:11–14, 1964.

[NF16]   Yuji Nakatsukasa and Roland W. Freund. Computing fundamental matrix decompositions accurately via the matrix sign function in two iterations: the power of Zolotarev's functions. *SIAM Rev.*, 58(3):461–493, 2016.

[NST18]   Yuji Nakatsukasa, Olivier Sète, and Lloyd N. Trefethen. The AAA algorithm for rational approximation. *SIAM J. Sci. Comput.*, 40(3):A1494–A1522, 2018.

[PGvD12]   Ricardo Pachón, Pedro Gonnet, and Joris van Deun. Fast and stable rational interpolation in roots of unity and Chebyshev points. *SIAM J. Numer. Anal.*, 50(3):1713–1734, 2012.

[PP87]   P. P. Petrushev and V. A. Popov. *Rational approximation of real functions*, volume 28 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, Cambridge, 1987.

[Tre13]   Lloyd N. Trefethen. *Approximation theory and approximation practice*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2013.

Link for accessing literature and slides: https://polybox.ethz.ch/index.php/s/Dj4hMVnATNZoIfE, password: RAP_FS23