Seminar in Autumn Semester 2022

Rational Approximation and Interpolation

Lecturer	: Prof. Ralf Hiptmair, D-MATH
Venue	: HG E 22
Time	: Mon, 16-18
Language	: English
Prep meeting	: Mon, Sep 19, 2022, 18:00 on ZOOM
	: Meeting ID: 698 4220 0325, Password: RAP_HS22
Individual tutoring	:
First session	: Oct 10, 2022
Contact	: R. Hiptmair, hiptmair@sam.math.ethz.ch
Prerequisites	: Knowledge of analysis, elementary functional analysis and basic numerical methods
Audience	: MSc & 3rd year BSc Students of Mathematics

Description:

The simplest and most widely used function system for approximation in computational mathematics are polynomials. They are ideally suited for smooth (analytic) functions. However, in many application we encounter functions with kinks and other kinds of singularities. In this case approximation by rational functions, that is, quotients of polynomials, may be vastly superior. This is why rational approximation and interpolation is receiving increased attention for the construction of surrogate models in model order reduction.

This seminar will study a number of research papers dealing with both theoretical and algorithmic aspects of rational approximation and interpolation. It covers both fundamental results in approximation theory and numerical analysis.

Presentations:

Each topic defined by one or more research articles should be covered in a roughly 60 minute presentation, based on prepared slides, and, maybe, short explanations on the blackboard or tablet. The slides should be made available as PDF beforehand.

The seminar will comprise up to 12 student presentations of a duration of about 60 minutes. They should be held by teams of three with members presenting in turns in random order.

Quizz:

Participants of the seminar will be asked questions about the previous presentations at the beginning of each session.

Available topics:

1. [PP87, Sections 2.1-2.3]: Qualitative theory of the best rational approximation, should also be connected to [PP87, Chapter 1]. See also [Tre13, Ch. 24].

2. [New64, Bra84b, Bra84a]: Rational approximation of particular functions, see also [PP87, Chapter 4] and [Tre13, Chapter 25].

3. [GPT11]: Robust rational interpolation and least-squares, see also [Tre13, Chapter 26].

4. [Tre13, Chapter 27] and [GGT13]. See [PP87, Section 12.1] for background information.

5. [SG00, GS99, DHD07]: Rational approximation by vector fitting

6. [DGB15]: Vector fitting for matrix-valued rational approximation

7. [BG17]: RKFIT algorithm for non-linear rational approximation

8. [NST18]: The AAA algorithm for rational approximation

9. [Bad21]: Trigonometric AAA

10. [FNTB18]: Rational minimax approximation

11. [GH00, GHR98]: Multivariate Padé approximation

12. [BNPP20]: Fast least-squares Padé approximation

Speakers and dates for presentations:

Date	Speaker/Team	Topic #	Dry run
10.10.2018		1	03.10.
17.10.2018		2	10.10.
24.10.2018		3	17.10.
31.10.2018		4	24.10.
07.11.2018		5	07.11.
14.11.2018		6	07.11.
21.11.2018		7	14.11.
28.11.2018		8	21.11.
05.12.2018		9	28.11.
12.12.2018		10	05.12.
19.12.2018		11 & 12	12.12.

If there are 12 students who want to give presentations, one additional talk will be scheduled for 19.12.2022, 18:00-19:30.

References

[Bad21]   Peter J. Baddoo. The AAAtrig algorithm for rational approximation of periodic functions. *SIAM J. Sci. Comput.*, 43(5):A3372–A3392, 2021.

[BG17]   Mario Berljafa and Stefan Güttel. The RKFIT algorithm for nonlinear rational approximation. *SIAM J. Sci. Comput.*, 39(5):A2049–A2071, 2017.

[BNPP20]   Francesca Bonizzoni, Fabio Nobile, Ilaria Perugia, and Davide Pradovera. Fast least-squares Padé approximation of problems with normal operators and meromorphic structure. *Math. Comp.*, 89(323):1229–1257, 2020.

[Bra84a]   Dietrich Braess. On rational approximation of the exponential and the square root function. In *Rational approximation and interpolation (Tampa, Fla., 1983)*, volume 1105 of *Lecture Notes in Math.*, pages 89–99. Springer, Berlin, 1984.

[Bra84b]   Dietrich Braess. On the conjecture of Meinardus on rational approximation of e^x. II. *J. Approx. Theory*, 40(4):375–379, 1984.

[DGB15]   Z. Drmač, S. Gugercin, and C. Beattie. Vector fitting for matrix-valued rational approximation. *SIAM J. Sci. Comput.*, 37(5):A2346–A2379, 2015.

[DHD07]   Dirk Deschrijver, Bart Haegeman, and Tom Dhaene. Orthonormal vector fitting: A robust macromodeling tool for rational approximation of frequency domain responses. *IEEE Transactions on Advanced Packaging*, 30(2):216–225, 2007.

[FNTB18]   Silviu-Ioan Filip, Yuji Nakatsukasa, Lloyd N. Trefethen, and Bernhard Beckermann. Rational minimax approximation via adaptive barycentric representations. *SIAM J. Sci. Comput.*, 40(4):A2427–A2455, 2018.

[GGT13]   Pedro Gonnet, Stefan Güttel, and Lloyd N. Trefethen. Robust Padé approximation via SVD. *SIAM Rev.*, 55(1):101–117, 2013.

[GH00]   Philippe Guillaume and Alain Huard. Multivariate Padé approximation. *J. Comput. Appl. Math.*, 121(1-2):197–219, 2000. Numerical analysis in the 20th century, Vol. I, Approximation theory.

[GHR98]   Philippe Guillaume, Alain Huard, and Vincent Robin. Generalized multivariate Padé approximants. *J. Approx. Theory*, 95(2):203–214, 1998.

[GPT11]   Pedro Gonnet, Ricardo Pachón, and Lloyd N. Trefethen. Robust rational interpolation and least-squares. *Electron. Trans. Numer. Anal.*, 38:146–167, 2011.

[GS99]   B. Gustavsen and A. Semlyen. Rational approximation of frequency domain responses by vector fitting. *IEEE Transactions on Power Delivery*, 14(3):1052–1061, 1999.

[New64] D. J. Newman. Rational approximation to $|x|$. *Michigan Math. J.*, 11:11–14, 1964.

[NST18] Yuji Nakatsukasa, Olivier Sète, and Lloyd N. Trefethen. The AAA algorithm for rational approximation. *SIAM J. Sci. Comput.*, 40(3):A1494–A1522, 2018.

[PP87]   P. P. Petrushev and V. A. Popov. *Rational approximation of real functions*, volume 28 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, Cambridge, 1987.

[SG00]   Adam Semlyen and Bjø rn Gustavsen. Vector fitting by pole relocation for the state equation approximation of nonrational transfer matrices. *Circuits Systems Signal Process.*, 19(6):549–566, 2000.

[Tre13] Lloyd N. Trefethen. *Approximation theory and approximation practice*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2013.

Link for accessing literature and slides: `https://polybox.ethz.ch/index.php/s/X8VY1qCUOKK8srL`, password: `RAP_HS22`