1.3 Topological electrodynamic Laws | 26-09-2008 Lecture II | P_{11}

Faraday's Law:

$$\int_{\partial\Sigma} \underline{e} = -\frac{d}{dt}\int_{\Sigma} \underline{b}$$

Ampère's Law

$$\int_{\partial\Sigma} \underline{h} = \frac{d}{dt}\int_{\Sigma} \underline{d} + \int_{\Sigma} \underline{j}$$

holds for any oriented 2-surfaces Σ

1.4 Exterior derivative

Def 1.4.A $\int_{\partial\Sigma} \omega = \int_{\Sigma} d\omega \qquad \forall\, l+1$-oriented surface Σ. $\forall\, l$-form ω.

RK: view integration as duality pairing:

$$\langle \Sigma \mid d\omega \rangle = \langle \partial\Sigma \mid \omega \rangle$$ d, ∂ are adjoint to each other

But for pullback operator

$$\langle \phi(\Sigma) \mid \omega \rangle = \langle \Sigma \mid \phi^* \omega \rangle \qquad \boxed{\partial\circ\phi = \phi\circ\partial}$$

Then

$$\langle \Sigma \mid \phi^* d\omega \rangle = \langle \phi(\Sigma) \mid d\omega \rangle = \langle \partial\phi(\Sigma) \mid \omega \rangle = \langle \phi(\partial\Sigma) \mid \omega \rangle$$
$$= \langle \partial\Sigma \mid \phi^*\omega \rangle = \langle \Sigma \mid d\phi^*\omega \rangle$$

$$\boxed{\phi^*\circ d = d\circ\phi^*}$$

$$\boxed{d\circ d = 0}$$

For (local) differential forms

$$d\omega(x)(v_1, \ldots v_{l+1}) = \sum_{i=1}^{l+1} (-1)^{i-1} \big(D\omega(x) v_i\big)(v_1, \ldots, \check{v}_i, \ldots v_{l+1})$$

Ex: derivative in terms of Euclidean vector proxies in 3D

$l = 0$. $\quad VP(d\omega)(x)\cdot v = d\omega(x)(v) = D\omega(x)v = \big(\operatorname{grad} VP(\omega)(x)\big)\cdot v$

Δ $l = 1$ $\quad VP(d\omega)(x)\cdot(v_1 \times v_2) \overset{?}{=} \big(\operatorname{curl} VP(\omega)(x)\big)\cdot(v_1\times v_2)$

Ex1. $l = 2$ $\quad VP(d\omega)(x)\cdot \det(v_1, v_2, v_3) \overset{?}{=} \big(\operatorname{div} VP(\omega)(x)\big)\cdot \det(v_1, v_2, v_3)$

(Grand Unification)

Def 1.4.A $\hat{=}$ $\begin{cases} \text{Fundamental} \\ \text{Stokes} \\ \text{Gauss} \end{cases}$ — Theorem for vector proxies

Thm 1.4.E. (Product rule)

$\omega \in DF^{l,1}(\Lambda_n) \quad \eta \in DF^{m,1}(\Lambda_n)$

$$\boxed{d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^l \omega \wedge d\eta}$$

$\xrightarrow{+\text{ Def } 1.4.A}$ Integration by Parts formula (Generalized Green's Formula)

$$\boxed{\int_{\partial\Omega} \omega \wedge \eta = \int_{\Omega} d\omega \wedge \eta + (-1)^l \int_{\Omega} \omega \wedge d\eta} \qquad \underline{(1.4.a)}$$

For Euclidean vector proxies $n=3 \quad l=m=1$.

$\omega \leftrightarrow \underline{u}. \quad \nu \leftrightarrow \underline{v}$

$$\int_{\partial\Omega} \underline{u} \times \underline{v}\, ds = \int_{\Omega} \operatorname{curl}\underline{u} \cdot \underline{v} - \underline{u} \cdot \operatorname{curl}\underline{v}\, dx$$

Topo. EM Laws $\Rightarrow$ in form's calculus.

Faraday : (F.L.) $\quad de = -\partial_t b \qquad \underline{(1.4.b)}$

Ampere : (A.L.) $\quad dh = \partial_t d + j \qquad \underline{(1.4.c)}$

} invariant under any transformation of space

Since $d \circ \Phi^* = \Phi^* \circ d$. $\triangleright$ if e.g. $\underline{e}$. $\underline{b}$ satifies F.L.

$\Rightarrow \Phi^* \underline{e} \quad \Phi^* \underline{b}$ also satifies F.L.

RK: (1) vector proxies relation:

F.L. : $\quad \operatorname{curl}\underline{e} = -\partial_t \underline{b}$

A.L. : $\quad \operatorname{curl}\underline{h} = \partial_t \underline{d} + \underline{j}$

(2) Consequence of (1.4.b.) (1.4.c)

apply d on (1.4.b) $\Rightarrow -\partial_t d\underline{b} = 0$ if $\underline{b}(t=0)=0 \Rightarrow \boxed{d\underline{b}=0}$ (no magnetic monopole) $\underline{(1.4.d)}$

apply d on (1.4.c) $\Rightarrow \partial_t \underbrace{d\underline{d}}_{\text{3-forms } \rho} + \underline{j} = 0 \Rightarrow \boxed{d\underline{j} = -\partial_t \rho}$ (conservation of charge) (1.4.e)

(3). If in (1.4.b) $\quad \underline{e} \leftarrow$ 0-form p, $\quad \underline{b} \leftarrow$ 1-form $\underline{s}$ $\quad \triangleright dp = -\partial_t \underline{s} \Leftrightarrow \boxed{\operatorname{grad} p = -\partial_t \underline{s}}$ (*1)

balance of momentum; pressure; momentum

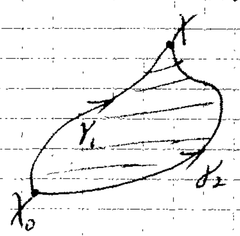
If in (1.4.c) $\quad \underline{h} \leftarrow$ 2-form $\underline{v}$, $\quad \underline{d} \leftarrow$ 3-form ρ $\quad \triangleright d\underline{v} = \partial_t \rho \Leftrightarrow \boxed{\operatorname{div}\underline{v} = \partial_t \rho}$ (*2)

mass conservation; velocity; density

☆ $\boxed{(*1)\ (*2) \Leftrightarrow \text{"Acoustics"}}$ $\leftarrow$ "Finite Elements in Computational Acoustics"

1.5. Potentials.

Motivation: consider $\omega \in \mathcal{F}^1(A_3)$. with $\boxed{d\omega = 0}$ (← closed form)



$$\int_\Sigma d\omega = 0. \quad \forall \text{ oriented } 2\text{-surfaces } \Sigma$$

$$\Downarrow$$

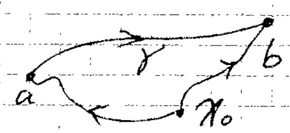
$$\int_{\partial\Sigma} \omega = 0.$$

$\gamma_1 - \gamma_2$ is a boundary of shading surface $\triangleright$ $\int_{\gamma_1 - \gamma_2} \omega = 0 \Rightarrow \int_{\gamma_1} \omega = \int_{\gamma_2} \omega$

Well-define: $\boxed{\eta(x) = \int_{\gamma(x_0,\, x)} \omega}$ $\boxed{\eta \text{ is } 0\text{-form}}$

arbitrary oriented curve starting at x_0, ending at x.

For any oriented γ.



$$\int_\gamma d\eta = \eta(b) - \eta(a) = \int_{\gamma(x_0, a)} \omega - \int_{\gamma(x_0, b)} \omega = \int_\gamma \omega$$

$$\Rightarrow \boxed{d\eta = \omega.}$$

(exact form)

Generalization: (Now from perspective of differential forms)

$\boxed{\text{Poincaré Lifting}}$: k : $\left\{ \begin{array}{l} D\mathcal{F}^l(A_n) \longrightarrow D\mathcal{F}^{l-1}(A_n) \\ \omega \to k(\omega), \\ k(\omega)(x)(v_1, \cdots, v_{l-1}) = \int_0^1 \tau^{l-1} \omega(\tau x)(x, v_1, \cdots, v_{l-1})\, d\tau \end{array} \right.$ (1.5.a)

lower the order of $D\mathcal{F}$

Theorem 1.5.A: $d \circ k + k \circ d = Id$.

(Ex. 4/Sheet 1) $\Rightarrow$ $d\omega = 0$. $\Rightarrow$ $dk(\omega) = \omega$. (k

Ex 4. sheet 1

(i.e. for closed forms. k is a right inverse of d.)

(if $\Omega \subset A_n$. $d\omega = 0$ depends on topo. structures)

<u>RK</u>: The sequence is exact.

$$\mathcal{F}^0(A_n) \xrightarrow{d} \mathcal{F}^1(A_n) \xrightarrow{d} \cdots \xrightarrow{d} \mathcal{F}^n(A_n) \xrightarrow{d} \{0\}$$

Interpretation: $\mathrm{Kern}\left(d|_{\mathcal{F}^l(A_n)}\right) = \mathrm{Imag}\left(d|_{\mathcal{F}^{l-1}(A_n)}\right)$ (*)

(*) holds for every two adjacent pairs in this sequence.

<u>Ex</u>: $\xrightarrow{\text{grad}}$ $\xrightarrow{\text{curl}}$ $\xrightarrow{\text{div}}$

de Rahm exact sequence.

<u>Electromagnetic potential</u>:

$d\underline{b} = 0 \Rightarrow \underline{b} = d\underline{a}$, $\underline{a} \in \mathcal{F}^1(A_n) \;\hat{=}\;$ <u>magnetic vector potential</u>.

$\downarrow$

$d\underline{e} = -\partial_t \underline{b} \Rightarrow d(\underline{e} + \partial_t \underline{a}) = 0$.

$\Rightarrow \underline{e} + \partial_t \underline{a} = -dv$. $v \in \mathcal{F}^0(A_n) \;\hat{=}\;$ <u>electric scalar potential</u>.

F.L. $\Rightarrow \underline{e} = -dv - \partial_t \underline{a}$

<u>Note</u>: $\boxed{\begin{array}{l} \underline{a}' \leftarrow \underline{a} + dw \\ v' \leftarrow v - \partial_t w \end{array}}$ $\triangleright$ $\underline{a}'$, v' generates same fields as $\underline{a}$, v.

$\Uparrow$

<u>gauge freedom</u>

1.6 Energy and Material Law

Assume that energy functionals are given:

$E_{el} : \mathcal{DF}^2(A_3) \longrightarrow \mathbb{R} \implies$ Energy $E_{el}(\underline{d})$ of displacement field.

Assume: E_{el} is smooth, $\underbrace{\text{uniformly strictly convex}}$.

Hessian $D^2 E_{el}$ is uniformly positive definite.

i.e. $D^2 E_{el}(\underline{d})(\underline{v}_1, \underline{v}_2) \geq r > 0 \quad \forall \underline{v}_1, \underline{v}_2 \in \mathcal{DF}^2(A_3)$

$\triangleright$: $\boxed{\underset{\text{mapping}}{\underline{DE_{el}}} : \mathcal{DF}^2(A_3) \longrightarrow \underbrace{(\mathcal{DF}^2(A_3))'}_{\text{space of bdd linear form of } \mathcal{DF}^2(A_3)} \text{ is bijective}}$

[Abstract idea: $J : V \to \mathbb{R}$ uniformly strictly convex ;

$f \in V'$. $\quad x = \underset{y \in V}{\arg\min}\, J(y) - f(y)$ is unique $\implies$ mapping $f \to x$.

J smooth $\implies DJ(x) - f = 0$]

Lemma 1.6.A: The pairing: $\begin{cases} \mathcal{DF}^1(A_3) \times \mathcal{DF}^2(A_3) \to \mathbb{R} \\ (\omega, \eta) \longrightarrow \int_{A_3} \omega \wedge \eta \end{cases}$

induces an isomorphism $\mathcal{DF}^1(A_3) \cong (\mathcal{DF}^2(A_3))'$

Meaning ($\Rightarrow$) For each bdd linear functional for $\mathcal{DF}^2(A_3)$, we find some $\omega \in \mathcal{DF}^1(A_3)$, s.t.

$$f(\eta) = \int_{A_3} \omega \wedge \eta.$$

($\Leftarrow$) 2-dual $\implies$ 1-form

Material Law: (local view)

$$\boxed{\underline{e} = \underset{\substack{\Updownarrow \\ \text{isomorphism}\left((\mathcal{DF}^2(A_3))' \to \mathcal{DF}^1(A_3)\right)}}{\mathcal{I}} \overset{\text{Frechet}}{D} E_{el}(\underline{d})}$$

Alternatively: (global view)

$$\boxed{D E_{el}(\underline{d})(\underline{d}') = \int_{A_3} \underline{e} \wedge \underline{d}'} \quad \underset{\text{(global Law)}}{(1.6.a)}$$

[$f(x) = f(0) + \int_0^t Df(\tau x)(x)\,d\tau$ $\quad E_{el}(\underline{d}) = \int_0^1 \int_{A_3} \underline{e}(\tau \underline{d}) \wedge \underline{d}\, d\tau$ (1.6.b) special case: Local field energy:

$$\mathcal{E}_{el}(\underline{d}) = \int_{A_3} W_{el}(x)(\underline{d}(x))\, dx$$

local energy density

$\Rightarrow$ Local material Law

$$\big(D W_{el}(x)(\underline{d}(x))\big)(\varphi) = \underline{e} \wedge \varphi \,. \quad \varphi \in \Lambda^2(\mathbb{R}^3)$$

In Euclidean vector proxies:

$$\mathrm{grad}\, W_{el}(x)(\underline{d}) = \underline{e}(\underline{d}) \qquad (1.6.d)$$

Special case: quadratic local energy

$$W_{el}(x; \underline{d}(x)) = \frac{1}{2} a_\varepsilon(x)(\underline{d}(x), \underline{d}(x))$$

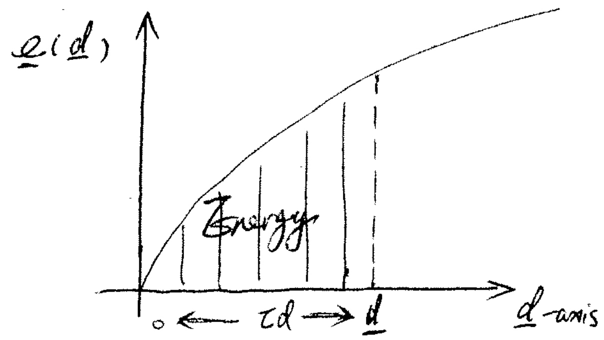
with a s.p.d. bilinear form $a_\varepsilon(\cdot,\cdot) : \Lambda^2(A_3) \times \Lambda^2(A_3) \to R$

In vector proxies:

$$W_{el}(x, \underline{d}(x)) = \frac{1}{2} \underline{d}^T \underline{\underline{\varepsilon}}^{-1}(x)\, \underline{d} \quad \forall \underline{d} \in R^3$$

$$\Rightarrow \quad \boxed{\underline{\underline{\varepsilon}}^{-1}(x)\, \underline{d}(x) = \underline{e}(x)}$$

RK: i) Local meaning of (1.6.b)



$$\int_0^1 \int_{A_3} e(\tau \underline{d}) \wedge \underline{d}\, d\tau$$

$$\Updownarrow$$

$$\int_0^1 f(\tau x)\, x\, d\tau = \int_0^x f(y)\, dy$$

ii) For quadratic energy. $\Rightarrow$ "linear material Laws"

vector proxies
$$\begin{cases} \underline{d}(x) = \underline{\underline{\varepsilon}}(x)\, \underline{e}(x) \\ \underline{b}(x) = \underline{\underline{\mu}}(x)\, \underline{h}(x) \end{cases}$$

$\underline{\underline{\varepsilon}}(x)$: dielectric tensor

$\underline{\underline{\mu}}(x)$: permeability tensor

iii) Material Law $\hat{=}$ bijective relationship between 1-form $\leftrightarrow$ 2-form.

→ ~~bijective~~ linear mappings for local quadratic material Laws (P17)

Hodge − operator

D: Hodge operator: (linear bijective mapping) $\ast : \Lambda^l(\mathbb{R}^n) \longrightarrow \Lambda^{n-l}(\mathbb{R}^n)$

p.w. in A_n $\ast : DF^l(A_n) \longrightarrow DF^{n-l}(A_n)$

Rk: easy to derive V.F. even for nonlinear material laws

Supplement: Ohm's Law

$$\underline{j} = \ell \circ D\underset{\uparrow}{P}(\underline{e})$$

dissipative power

→ local, linear relation for vector proxies

$\underline{j}(x) = \underline{\sigma}(x) \cdot \underline{e}(x)$ $\quad \underline{\sigma}$: conductivity tensor,

RK. Poynting's Theorem: Given $\Omega \subset A_3$, bdd.

$$\underbrace{\frac{d}{dt}\left(E^{\Omega}_{el}(\underline{d}) + E^{\Omega}_{mg}(\underline{b})\right)}_{\text{change of energy in } \Omega \quad \text{time-invariant energy}} = \langle DE^{\Omega}_{el}(\underline{d}), \partial_t \underline{d} \rangle + \langle DE^{\Omega}_{mg}(\underline{b}), \partial_t \underline{b} \rangle$$

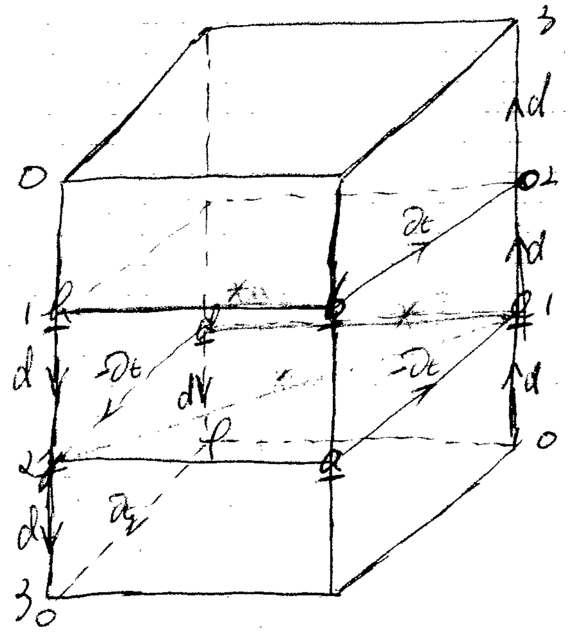
$$\stackrel{M.L.}{=} \int_{\Omega} \underline{e} \wedge \partial_t \underline{d} + \underline{h} \wedge \partial_t \underline{b}$$

$$\stackrel{F.L.\ A.L.}{=} \int_{\Omega} \underline{e} \wedge (d\underline{h} - \underline{j}) + \underline{h} \wedge (-d\underline{e})$$

$$\stackrel{IBP}{=} \int_{\partial\Omega} \underbrace{\underline{e} \wedge \underline{h}}_{\text{energy flux}} - \underbrace{\int_{\Omega} \underline{e} \wedge \underline{j}}_{\text{energy dissipation}}$$

Poynting vector 2-form

Maxwell's House by Allan Bossavit



1.7. Variational Form: (Formal practice)

Recall material Laws:

$$\langle DE_{el}^{\Omega}(\underline{d}), \underline{d}'\rangle = \int_\Omega \underline{e}\wedge\underline{d}' \qquad \forall d'\in\mathcal{F}^2(\Omega)$$

$$\langle D\bar{E}_{el}^{\Omega}(\underline{e}), \underline{e}'\rangle = \int_\Omega \underline{d}\wedge\underline{e}' \qquad \forall e'\in\mathcal{F}^1(\Omega)$$

$$\langle DE_{mg}^{\Omega}(\underline{b}), \underline{b}'\rangle = \int_\Omega \underline{h}\wedge b' \qquad \forall b'\in\mathcal{F}^2(\Omega)$$

$$\langle DE_{mg}^{\Omega}(\underline{h}), \underline{h}'\rangle = \int_\Omega \underline{b}\wedge h' \qquad \forall h'\in\mathcal{F}^1(\Omega)$$

Combine these material laws with those topological laws.

$$d\underline{e} = -\partial_t\underline{b}$$
$$d\underline{h} = \partial_t\underline{d} + \underline{j}$$

Option 1: test A.L. with arbitrary $\underline{e}'\in\mathcal{F}^1(\Omega)$

$$\int_\Omega d\underline{h}\wedge\underline{e}' = \int_\Omega \partial_t\underline{d}\wedge\underline{e}' + \underline{j}\wedge\underline{e}' \qquad \forall\underline{e}'\in\mathcal{F}^1(\Omega)$$

By Integration By Parts,

$$\int_\Omega h\wedge d\underline{e}' + \int_{\partial\Omega} h\wedge e' = \partial_t\int_\Omega \underline{d}\wedge\underline{e}' + \int_\Omega \underline{j}\wedge\underline{e}'$$

plugging into energy

$$\langle DE_{mg}^{\Omega}(\underline{b}), d\underline{e}'\rangle + \int_{\partial\Omega} h\wedge e' = \partial_t\langle D\bar{E}_{el}^{\Omega}(\underline{e}), \underline{e}'\rangle + \int_\Omega \underline{j}\wedge\underline{e}'$$

Differentiate w.r.t time & use F.L.

$$<DE_{mg}(-d\underline{e}), d\underline{e}'> + \int_{\partial\Omega} \partial_t \underline{h} \wedge e'$$

$$= \partial_t^2 < DE_{el}(\underline{e}), \underline{e}'> + \int_\Omega j \wedge e' \qquad \underline{(1.7.a)}$$

$\hat{=}$ <u>e-based variational formulation.</u>

$\boxed{RK}$: in $\underline{e}$-base formulation
i) $\longrightarrow$ A.L. taken into account weakly
ii) $\longrightarrow$ F.L. taken — — — strongly.

Ex: $\underline{h}$-based V.F.

<u>Special case</u>. local quadratic energy $\Longleftrightarrow$ linear material laws.

In vector proxies (1.7.a) translates into

$$-\int_\Omega \underline{\underline{\mu}}^{-1} \operatorname{curl} \underline{e} \cdot \operatorname{curl} \underline{e}\, dx + \int_{\partial\Omega} (\partial_t \underline{h} \times \underline{e}') \cdot n\, dS$$

$$= \frac{d}{dt^2} \int_\Omega \underline{\underline{\varepsilon}}\, \underline{e} \cdot \underline{e}'\, dx + \int_\Omega j \wedge e'\, dx$$

Option II: test F.L. with arbitrary $\underline{h}' \in \mathcal{F}'(\Omega)$

$$\int_\Omega d\underline{e} \wedge \underline{h}' = \int_\Omega -\partial_t \underline{b} \wedge \underline{h}'$$

By I.B.P.

$$\int_\Omega \underline{e} \wedge d\underline{h}' + \int_{\partial\Omega} \underline{e} \wedge \underline{h}' = \int_\Omega -\partial_t \underline{b} \wedge \underline{h}'$$

Plugging into energy:

$$< DE_{el}^\Omega(\underline{d}), d\underline{h}'> + \int_{\partial\Omega} \underline{e} \wedge \underline{h}' = -\partial_t < DE_{mg}^\Omega(\underline{h}), \underline{h}'>$$

Differentiate w.r.t time & use A.L.

$$< DE_{el}^\Omega(d\underline{h} - j), d\underline{h}'> + \int_\Omega \partial_t \underline{e} \wedge \underline{h}' = -\partial_t^2 < DE_{mg}^\Omega(\underline{h}), \underline{h}'>.$$

$\hat{=}$ <u>$\underline{h}$-based V.F.</u>

$\boxed{RK}$: $\longrightarrow$ F.L. weakly
$\longrightarrow$ A.L. strongly

specialcase: linear material laws in vector proxies.

$$\int_\Omega \underline{\underline{\varepsilon}}^{-1}(\mathrm{curl}\,\underline{h}-\underline{j})\cdot\mathrm{curl}\,\underline{h}' + \int_{\partial\Omega}(\partial_t \underline{e}\times\underline{h}')\cdot n\,dS$$
$$= -\frac{d^2}{dt^2}\int_\Omega \mu\,\underline{h}\cdot\underline{h}'\,dx$$

1.8. Boundary Conditions

$\Omega \in A_3$. boundary b.d.d. p.w. smooth. $\omega \in \mathcal{F}^l_\Delta(\Omega)$

trace $\gamma_{\partial\Omega}\,\omega \in \mathcal{F}^l_\Delta(\partial\Omega)$

is the simple restriction of ω

$$(\gamma_{\partial\Omega}\,\omega)(\Sigma) = \omega(\Sigma) \qquad \forall\, \Sigma \in \mathcal{S}_l(\partial\Omega)$$

l-dim oriented p.w. smooth submanifolds of $\partial\Omega$

traces for vector proxies in 3D (assuming "standard" vector proxies in 2D)

A_3- space

$l = 0$: $VP(\gamma_{\partial\Omega}\omega)(x) = VP(\omega)(x) \qquad x \in \partial\Omega$

$l = 1$: $VP(\gamma_{\partial\Omega}\omega)(x) = [VP(\omega)(x)]_t \qquad x \in \partial\Omega$

$[\cdot]_t$: tangential components

$l = 2$: $VP(\gamma_{\partial\Omega}\omega)(x) = [VP(\omega)(x)]_n \qquad x \in \partial\Omega$

$[\cdot]_n$: normal components.

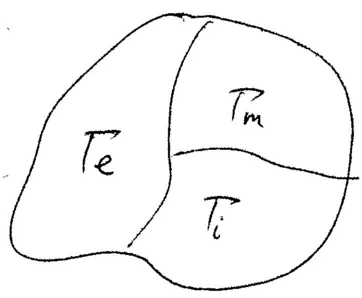
? $l = 3$

Electromagnetic boundary conditions

$$\partial\Omega = \overbrace{\Gamma_e \cup \Gamma_m \cup \Gamma_i}^{\text{(mutually disjoint)}}$$

- Electric B.C. $\quad \underline{e}_t = g_{el} \quad$ on $\Gamma_e \subset \partial\Omega$
- Magnetic B.C. $\quad \underline{b}_t = g_{mag} \quad$ on $\Gamma_m \subset \partial\Omega$
- Impedance B.C. $\quad \underline{b}_t = \phi(\underline{e}_t) \quad$ on $\Gamma_i \subset \partial\Omega$

one on any part of $\partial\Omega$.



Incorporate B.C.s in $\underline{e}$-based V.F.

→ Electric B.C. → strongly imposed on ~~that~~ trial field $\underline{e}$.

Seek $\underline{e} \in \tilde{g}_{el} + \{\hat{e} \in \mathcal{F}^1(\Omega)\ \gamma_{\partial\Omega}\hat{e} = 0\}$

↑ extension of $g_{el}|_{\partial\Omega}$ ~~into~~ Ω

→ require test field $\underline{e}'$ $\gamma_{\partial\Omega}\underline{e}' = 0$ on Γ_e.

No need to test ~~what~~ we have known

→ Magnetic B.C., Impedance B.C. ⇒ taken into account weekly in V.F.

(1.7.9) ⇒

$$< DE_{mg}(-d\underline{e}), d\underline{e}' > + \underbrace{\int_{\Gamma_m} g_{mg} \wedge \underline{e}'}_{\text{linear form of } \underline{e}'} + \int_{\Gamma_i} \Phi(\gamma_{\partial\Omega}\underline{e}) \wedge \underline{e}' = R.H.S$$

↑ move source term to R.H.S.

Ex. acoustic case.

$E_k(\underline{s})$

$E_p($

Acoustics :

kinetic energy $E_{kin}(\underline{s}) \leftrightarrow E_{kin}(\underline{v})$

$$< DE_{kin}(\underline{s}), \underline{s}' > = \int_\Omega \underline{v} \wedge \underline{s}' \quad \forall s' \in \mathcal{F}^1$$

potential energy $E_{pot}(\rho) \leftrightarrow E_{pot}(p)$

$$< DE_{pot}(\rho), \rho' > = \int \rho' \wedge p \quad \forall \rho' \in \mathcal{F}^3$$

- Variational formulation (vector proxies)
- Boundary cond. in terms of vector proxies