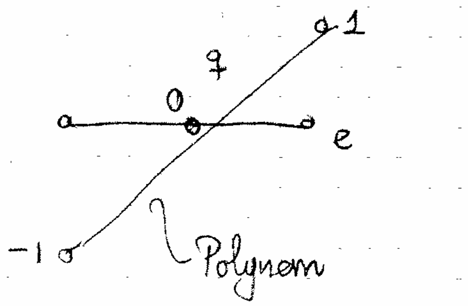
Problem Set 8

1.- $\tilde{\mathcal{I}} : H(\mathrm{curl}, \Omega) \to W^1(\mathcal{T}_h)$

$\forall e \in \mathcal{E}(\mathcal{T}_h) \quad \int_e \mathcal{I}u \, d\vec{s} = \int_e u \cdot d\vec{s}$

$\tilde{\mathcal{I}} : H(\mathrm{curl}, \Omega) \to \tilde{W}^1(\mathcal{T}_h)$



$\int_e \tilde{\mathcal{I}} u q \, d\vec{s} = \int_e \tilde{\mathcal{I}} u q \, d\vec{s}$

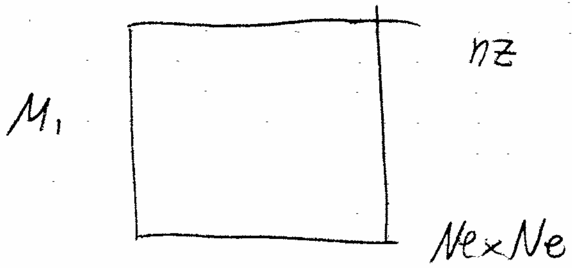
restricted on this spaces

2.- $L_h = \mathcal{I}^2 \circ L \qquad \to W^3(\mathcal{T}_h)$

$\mathrm{div} \circ L_h = \underline{\mathrm{div} \circ \mathcal{I}^2} \circ L = \mathcal{I}^3 \circ \underbrace{\mathrm{curl} \circ L} = \mathcal{I}^3 \circ Id = Id$

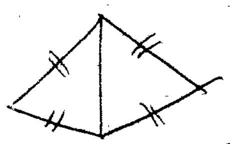
3.- M_0^{-1}: diagonal matrix

M_1 : $\underbrace{\text{edge-to-edge}}$ matrix



G^T: edge to node matrix

G: node-to-edge matrix



$M_1(e, \tilde{e}) \neq 0$ only if $\mathrm{supp}\, b_e \cap \mathrm{supp}\, b_{\tilde{e}} \neq \emptyset$

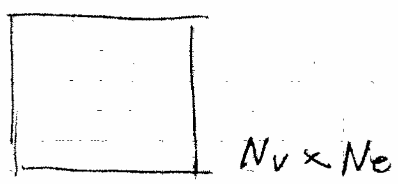
C : edge to face matrix $N_e \times N_f$

M_μ^{-1}: face to face matrix

$G(v, e) \neq 0$ only if $\mathrm{supp}\, b_v \cap \mathrm{supp}\, b_e \neq \emptyset$

$G^T M_1$ edge to node

$M_0^{-1} G^T M_1$ edge to node



$G M_0^{-1} G^T M_1$

$M_1 G M_0^{-1} G^T M_1$ edge to edge matrix

C face to face

$M_\mu^{-1} C$ edge to face

$C^T M_\mu^{-1} C$ edge to edge matrix

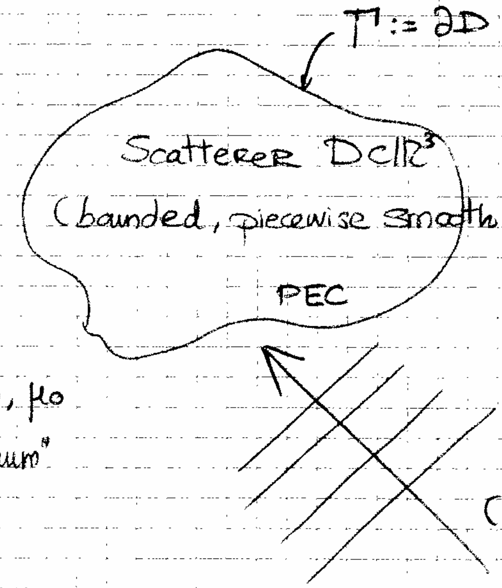
$A = C^T M_\mu^{-1} C + M_1 G M_0^{-1} G^T M_1$

$A(e, \tilde{e}) \neq 0$ if $\mathrm{supp}\, b_e \cap \mathrm{supp}\, b_{\tilde{e}} \neq 0$

12-12-08

V.- Boundary Element Methods for Time-Harmonic Electromagnetic Scattering

5.1.- Electromagnetic scattering



$\Omega := \mathbb{R}^3 \setminus \overline{D}$ "complement domain"

ε_0, μ_0 "vacuum"

PEC ≙ Perfect electric conducting

complex amplitudes!

$(\underline{e}_{inc}, \underline{h}_{inc})$ Incident wave

Excitation by time-harmonic incident wave

→ Treatment in frequency domain.

Assume: $(\underline{e}_{inc}, \underline{h}_{inc})$ solve Maxwell's equations* (in frequency domain with angular frequency $\omega > 0$) in Ω

* $\mathrm{curl}\, \underline{e} = -i\omega\mu_0 \underline{h}$

$\mathrm{curl}\, \underline{h} = i\omega\varepsilon_0 \underline{e}$

$\leftrightarrow \quad \mathrm{curl}\, \mu_0^{-1} \mathrm{curl}\, \underline{e} - \omega^2 \varepsilon_0 \underline{e} = 0$

$\leftrightarrow \quad \boxed{\mathrm{curl}\, \mathrm{curl}\, \underline{e} - K^2 \underline{e} = 0}$

$K := \omega \sqrt{\varepsilon_0 \mu_0}$ wave number

$\lambda = \frac{2\pi}{K}$ wavelength

Boundary conditions : $\underline{e}_t = 0$ on Γ

Need for radiation or decay conditions at infinity

Radiation conditions:

Common choice: $\underbrace{e_{inc}(x) = \underline{p} \exp(i k\, \underline{d} \cdot \underline{x})}$

plane wave, direction $\underline{d}$, $|\underline{d}| = 1$; polarization $\underline{p} \in \mathbb{R}^3$, $p \cdot \underline{d} = 0$

→ $\underline{e_{inc}}$ does not satisfy radiation conditions (energy goes out) as $\underline{e_{inc}}$ is « inflow of energy ».

→ total field $\underline{e}$ does not satisfy radiation condition

→ Silver - Müller radiation conditions only apply to scattered field (part of the field that radiates energy into infinity)

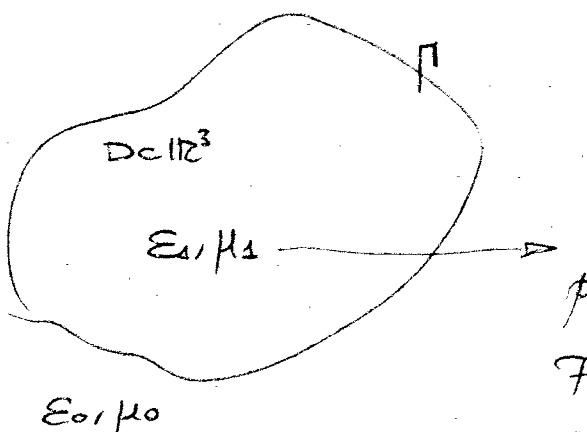
$$\underline{e_s} = \underline{e} - \underline{e_{inc}}$$

Boundary Value Problem BVP for $\underline{e_s}$:

$$\left.\begin{array}{l} \operatorname{curl}\operatorname{curl} \underline{e_s} - \kappa^2 \underline{e_s} = 0 \quad \text{in } \Omega \\ (e_s)_t = -(\underline{e_{inc}})_t \quad \text{on } \Gamma \\ \operatorname{curl} \underline{e_s} \times \underline{x} - i\kappa |\underline{x}|\, \underline{e_s} \to 0 \quad \text{for } |x| \to \infty \end{array}\right\} \quad (5.1.a)$$

Thm 5.1.A: (5.1.a) has a unique solution for all $\kappa > 0$ (Based on spherical harmonic expansion and analytical expansion).

Extension: dielectric EM scattering



→ Homogeneous material properties

PEC b.c. are replaced by transmission conditions.

On Γ transmission conditions:

$$[e_t]_\Gamma = 0$$
$$[h_t]_\Gamma = 0$$

$[u]_\Gamma = u^{ext} - u^{int} \;\hat{=}\; \text{jump}$

→ BVP (for scattered field in Ω + total field in D)

$$\left\{\begin{array}{l} \operatorname{curl}\operatorname{curl} \underline{e_s} - \kappa^2 \underline{e_s} = 0 \quad \text{in } \Omega \\ \operatorname{curl}\operatorname{curl} \underline{e} - \kappa_0^2 \underline{e} = 0 \quad \text{in } D, \; \kappa_0 = \omega\sqrt{\varepsilon_1 \mu_1} \\ \underbrace{(e_s - e)_t}_{\text{by def } \underline{e_s}} = -(\underline{e}_{inc})_t, \; (\mu_0^{-1} \operatorname{curl} e_s - \mu_1^{-1} \operatorname{curl} \underline{e})_t = -(i\omega\, h_{inc})_t \\ + \text{ Silver Müller radiation conditions for } \underline{e_s} \end{array}\right.$$

Existance and uniqueness of solutions is granted for by PDE theory.

"We will use boundary conditions on Γ, which will allow discretization of a finite domain."

5.2.- Traces and trace spaces

Recall: trace of l-forms on $\Omega \subset \mathbb{R}^d$

$$\gamma_{\partial\Omega} : \mathcal{F}^l(\Omega) \to \mathcal{F}^l(\partial\Omega)$$

$l = 0 \Rightarrow$ 0-form $w \in \mathcal{F}^0(\Omega)$ continuous

$\Rightarrow \gamma_{\partial\Omega} w \in \mathcal{F}^0(\partial\Omega)$ continuous

Goal: Characterize trace spaces of Sobolev spaces $H(d_l, \Omega)$ of l-forms

Technique: Completion with respect to suitable trace norm.
(it should be related to energy norm)

→ "The energy can be addressed through the potential of a field with the desired trace, which gives a minimal energy"

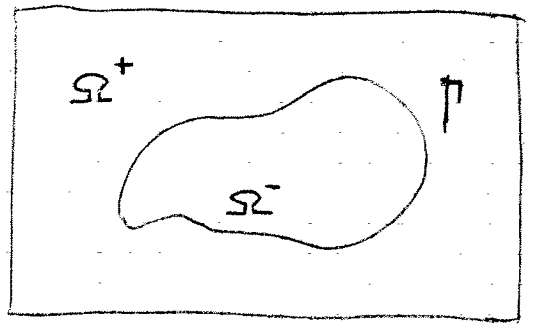
Trace norm: $\|\eta\|_{H^{1/2}(de,\partial\Omega)} := \inf_{\gamma_{\partial\Omega} w = \eta} \|w\|_{H(de;\Omega)}$

→ = energy of minimum energy extension (→ equilibrium extension)

(→ Trace space ~~by completion~~ $H^{1/2}(de,\partial\Omega) :=$) * No

→ Trace space $H^{1/2}(de,\partial\Omega)$ by completion of $\gamma_{\partial\Omega}\mathcal{DF}^{l,\infty}(\Omega)$ with respect to $\|\cdot\|_{H^{1/2}(de,\partial\Omega)}$

→ Hilbert space (can be shown, inner product spaces)



$\boxed{\text{Recall:}}$

continuous extension operator

$E_e : H(de,\Omega^-) \to H(de,\Omega^+)$

$\gamma_{\partial\Omega^-} w = \gamma_{\partial\Omega^+} E w$

"The union of Ω^- and Ω^+ has to be a global l-form"

$$\inf_{\gamma_{\partial\Omega^+} w^+ = \eta} \|w^+\|_{H(de,\Omega^+)} \leq \inf_{\gamma_{\partial\Omega^+} E_e w^- = \eta} \|E w^-\|_{H(de,\Omega^+)}$$

"Trace form from outside" "Smaller space"

$$\leq \|E_e\| \underbrace{\inf_{\gamma_{\partial\Omega^-} w^- = \eta}}_{\text{by def}} \|w^-\|_{H(de,\Omega^-)}$$

"Trace form from inside"

↓ Continuity

→ trace norms from inside and outside are equivalent (to a cte).

→ "By completion they give the same space" → $H^{1/2}(de,\Gamma)$ is intrinsic

("does not depend that Γ is boundary, looks the same from inside and outside")

By definition:

$\gamma_{\partial\Omega} : H(de,\Omega) \longrightarrow H^{1/2}(de,\partial\Omega)$
is continuous
is surjective

From $d\circ\gamma_{\partial\Omega} = \gamma_{\partial\Omega}\circ d$

$d : H(de,\Omega) \longrightarrow H(de+1,\Omega)$ continuous

exterior derivative continuous in trace spaces

$\underbrace{d : H^{1/2}(de,\partial\Omega) \to H^{1/2}(de+1,\partial\Omega)}_{\text{continuous}}$

Trace spaces in 3D (vector proxies):

	trace for vector proxies	Sobolev space	trace space (standard notation)
$l=0$:	$V.P.(\gamma_{\partial\Omega} w) = u_{\vert\partial\Omega}$, $u := V.P(w)$ Pointwise trace	$H^1(\Omega)$	$H^{1/2}(\Gamma)$
$l=1$:	$V.P.(\gamma_{\partial\Omega} w) = \begin{cases} \underline{u}_t \\ \underline{u}\times\underline{n} \end{cases}$, $\underline{u} := V.P.(w)$ (two conventions) "Tangential component" $\underline{n}$: Exterior unit normal Notation: $n \hat{=}$ unit normal vector field	$H(curl,\Omega)$	$\begin{cases} H^{-1/2}(\underline{curl}_\Gamma,\Gamma) = \chi_{el} \\ H^{-1/2}(div_\Gamma,\Gamma) = \chi_{mag} \end{cases}$
$l=2$:	$V.P.(\gamma_{\partial\Omega} w) = \underline{u}\cdot\underline{n}$, $\underline{u} := V.P.(w)$ "Normal component"	$H(div,\Omega)$	$H^{-1/2}(\Gamma)$

Exterior derivative on trace spaces (for vector proxies)

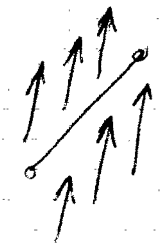
$l = 0 : d \longleftrightarrow \begin{cases} \mathrm{grad}_\Gamma \\ \underline{\mathrm{curl}}_\Gamma \end{cases}$ "Again two conventions"

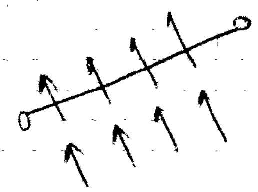
$l = 1 : d \longleftrightarrow \begin{cases} \mathrm{curl}_\Gamma \\ \mathrm{div}_\Gamma \end{cases}$

Remark: Euclidean vector proxies for 1-forms in 2D

$$\omega(\underline{x})(\underline{v}) = \cancel{V.P(\omega)(x)v} \quad V.P(\omega)(\underline{x})\cdot\underline{v} \quad \forall \underline{v} \in \mathbb{R}^2 \quad \text{First convention}$$

$$\omega(\underline{x})(\underline{v}) = V.P.(\omega)(x)^{*} \cdot \underline{v}^{\perp}, \quad \perp \mathrel{\hat{=}} \text{rotation by } ^{*}\frac{\pi}{2}\cdot(-1) \quad 2^{nd} \text{ convention}$$


2D


2D

First convention (a)
("Voltage as tangential component of field integrated along path)

Second convention (b)
Flux, normal component
(from fluid mechanics)

$R \mathrel{\hat{=}}$ pointwise rotation by $\frac{\pi}{2}$

$R : \mathcal{X}_{el} \longrightarrow \mathcal{X}_{mag}$ bijective isometric

$\triangleright \quad \underline{\mathrm{curl}}_\Gamma := R \circ \mathrm{grad}_\Gamma$

$\qquad \mathrm{curl}_\Gamma := \mathrm{div}_\Gamma \circ R$

(One contains traces of electric fields, the other traces of magnetic field)

L^2-duality:

"trace spaces X, Y on Γ are L^2-dual", if $\forall \varphi \in X' : \exists y \in Y :$ " Continuous linear functional on one space

$: \varphi(x) = (y, x)_{L^2(\Gamma)}$

"Just a notion, the spaces we are working with do not need always be contained in L^2"

A consequence of L^2-duality:

$$x \in X \cap L^2(\Gamma) : (x, y)_{L^2(\Gamma)} = 0 \quad \forall y \in Y \cap L^2(\Gamma) \Rightarrow x = 0$$

A criterion for L^2-duality: ("Search γ!")

$$\sup_{x \in X \cap L^2} \frac{(x, y)_{L^2}}{\|x\|_X} \geq \gamma \|y\|_Y \quad \forall y \in L^2 \cap Y$$

Thm 5.2.A:

$$\left.\begin{matrix} H^{1/2}(\Gamma) - H^{-1/2}(\Gamma) \\ \mathcal{X}_{el} - \mathcal{X}_{mag} \end{matrix}\right\} \text{ are } L^2(\Gamma)\text{-dual}$$

Proof (sketch):

$\varphi \in H^{-1/2}(\Gamma) \longrightarrow v \in H^1(\Omega)$

$\qquad -\Delta v + v = 0$ in Ω

$\qquad \frac{\partial v}{\partial n} = \varphi$ on $\partial\Omega$

One proofs v exists

$\downarrow$

$\exists \underline{u} \in H(\mathrm{div}, \Omega)$

$\underline{u} \cdot n = \varphi$

$$\longrightarrow (\varphi, v)_{L^2(\Gamma)} = \int_{\partial\Omega} \frac{\partial v}{\partial n} v \, dS = \int_\Omega \Delta v \, v + |\mathrm{grad}\, v|^2 \, dx =$$

Green's Formula

$$= \|v\|^2_{H^1(\Omega)}$$

$$\sup_{w \in H^{1/2}(\Gamma)} \frac{(\varphi, w)_{L^2}}{\|w\|_{H^{1/2}}} \geq \frac{(\varphi, v|_{\partial\Omega})_{L^2}}{\|v|_{\partial\Omega}\|_{H^{1/2}}} \geq \frac{\|v\|^2_{H^1(\Omega)}}{\|v\|_{H^1(\Omega)}} \geq \|v\|_{H^1(\Omega)} \geq =$$

$$= \|\mathrm{grad}\, v\|_{H(\mathrm{div}, \Omega)} \geq \|\varphi\|_{H^{-1/2}(\Gamma)}$$

By def of v $\qquad$ $\mathrm{grad}\, v \cdot n = \varphi$

(For the other proof one would use div instead of grad and Laplace theorem)

5.3.- Representation formula

[Rigorous treatment $\to$ Mc Lean "Strongly elliptic BVP"]

$\underline{e}$ $\hat{=}$ (Radiating) Maxwell solution in $D \cup \Omega$

$\hookrightarrow$ satisfies Radiation conditions at ∞

$\to$ $\operatorname{curl}\operatorname{curl}\underline{e} - \kappa^2 \underline{e} = 0$ in $D \cup \Omega$

$\to$ $\operatorname{div}\underline{e} = 0$ in $D \cup \Omega$

Recall : distributions

e.g. "$w = \operatorname{curl}\underline{u}$ in the sense of distributions" on $\mathbb{R}^3$"

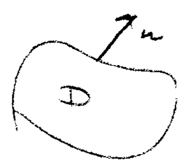
$$\int_{\mathbb{R}^3} w \cdot \phi \cdot dx = \int \underline{u} \cdot \operatorname{curl}\phi \, dx \quad \forall \phi \in (C_0^\infty(\Omega))^3$$

$\underline{u} \in H(\operatorname{curl}, D \cup \Omega)$

$$\int_{\mathbb{R}^3 = D\cup\Omega} \underline{u} \cdot \operatorname{curl}\phi \, dx = \int_{D\cup\Omega} \operatorname{curl}u \cdot \phi \, dx + \underbrace{\int_\Gamma [u \times n]_\Gamma \phi \, dS}_{= \int_{\mathbb{R}^3} [u \times]_\Gamma \delta_\Gamma \phi \, dx}$$

$$\left[\int_{\partial\Omega} (\underline{u} \times n) \underline{v} \, dS = \int_\Omega \operatorname{curl}\underline{u} \cdot \underline{v} - \underline{u} \operatorname{curl}\underline{v} \, dx \right]$$

exterior unit normal w.r.t Ω



$$\to \operatorname{curl}u = \operatorname{curl}u|_{D\cup\Omega} - [u \times n]_\Gamma \delta_\Gamma \quad (5.3.b)$$

Also: "$w = \operatorname{div}\underline{u}$ in the sense of distributions"

$$\int_{\mathbb{R}^3} w \cdot \phi \, dx = - \int_{\mathbb{R}^3} \underline{u} \operatorname{grad}\phi \, dx \quad \forall \phi \in C_0^\infty(\Omega)$$

$\to$ For $\underline{u} \in H(\operatorname{div}, \Omega \cup D)$

$$\operatorname{div}\underline{u} = \operatorname{div}\underline{u}|_{\Omega\cup D} + [\underline{u} \cdot n]_\Gamma \partial_\Gamma$$

Only if there is a jump in normal component

In the sense of distributions:

$$\to -\Delta \underline{e} - \kappa^2 \underline{e} = \operatorname{curl}\operatorname{curl}\underline{e} - \operatorname{grad}\operatorname{div}\underline{e} - \kappa^2 \underline{e} =$$

("Here there is no problem in using grad div")

$$= \operatorname{curl}(\operatorname{curl}\underline{e}|_{D\cup\Omega} - [\underline{e} \times \underline{n}]_\Gamma \delta_\Gamma) - \operatorname{grad}(\underbrace{\cancel{\operatorname{div}\underline{e}|_{D\cup\Omega}}}_{=0} + [e \cdot n]_\Gamma) - \kappa^2 \underline{e}$$

$$= \operatorname{curl}\operatorname{curl}\underline{e}|_{D\cup\Omega} - \operatorname{curl}[\underline{e} \times \underline{n}]_\Gamma \delta_\Gamma - \operatorname{curl}([e \times n]\delta_\Gamma) - \operatorname{grad}([e \, n]_\Gamma \delta_\Gamma) - \kappa^2 \underline{e}$$

$$= -[\operatorname{curl} e \times n]_\Gamma \delta_\Gamma - \operatorname{curl}([e \times n]_\Gamma \delta_\Gamma) - \operatorname{grad}([e \cdot n]_\Gamma \delta_\Gamma)$$

Fundamental solution of Helmholtz equation or Helmholtz operator:

$$G(\underline{x}) = \frac{e^{i\kappa|x|}}{4\pi|x|} \qquad (-\Delta - \kappa^2) G = \delta$$

A formal computation: Assume $x \notin \Gamma$

$$e_i(x) = \int_{\mathbb{R}^3} \underline{e}(y) \cdot (\delta(x-y)\varepsilon_i) \, dy = \int_{\mathbb{R}^3} e(y)(-\Delta_y - \kappa^2) G(x-y) \varepsilon_i \, dy =$$

$$\varepsilon_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \varepsilon_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \ldots$$

$$= \int_{\mathbb{R}^3} ((-\Delta_y - \kappa^2)\underline{e})(y) \underbrace{[G(x-y)\varepsilon_i]}_{\text{will play role of } \phi} \, dy$$

"Laplace self-adjoint operator"

$$= -\int_\Gamma [\operatorname{curl} e \times n]_\Gamma^{(y)} (G(x-y)\varepsilon_i) \, dS(y) - \int_\Gamma [e \times n]_\Gamma (y) \operatorname{curl}_y (G(x-y)\varepsilon_i) \, dS(y) +$$

$$+ \int_\Gamma [e \cdot n]_\Gamma (y) \underbrace{\operatorname{div}_y [G(x-y)\varepsilon_i]}_{\frac{\partial}{\partial y_1} G(x-y)} \, dS(y)$$

Note: $\frac{\partial}{\partial y_i} G(x-y) = -\frac{\partial}{\partial x_i} G(x-y)$ "Even function, derivative odd function (chain rule)"

$$\Rightarrow e_i(x) = -\int_\Gamma [\operatorname{curl}\underline{e} \times n]_\Gamma G(x-y) \, dS(y) \cdot \varepsilon_i + \operatorname{curl}_x \int_\Gamma [e \times n]_\Gamma (y) G(x-y) \, dS(y)$$

$$- \frac{\partial}{\partial x_i} \int_\Gamma [e \cdot n]_\Gamma (y) G(x-y) \, dS(y)$$

Thm 5.3.B: [Stratton - Chu representation formula]

$$\underline{e} = \overset{1\ *}{\Psi_V([\operatorname{curl} \underline{e} \times n]_\Gamma)} + \overset{2\ *}{\operatorname{curl} \Psi_V([e \times n]_\Gamma)} - \overset{3\ *}{\operatorname{grad} \Psi_V([en]_\Gamma)}$$ in $\Omega \cup D$

Where $\Psi_V(u)(x) := \int_\Gamma u(y)\, G(x-y)\, dS(y)$

holds for any radiating Maxwell solution in $D \cup \Omega$

The terms 1,2,3 all call boundary potentials (map "boundary jumps" into a solution for the full domain field)
(don't confuse with potentials as seen so far)

$*$ → boundary potentials

$\Psi_V \hat{=}$ single layer potential

$$\Psi_V(u) = \int_\Gamma \underbrace{\frac{e^{ik(x-y)}}{4\pi|x-y|}} u(y)\, dS(y)$$

kernel $\in L^1(\Gamma)$ ("even with the weak singularity, it cancels in polar coordinates with metric term")

→ u bounded → Ψ_V continuous (Calculus of parametric integrals)

Thm 5.3.C: $u \in H^{-1/2}(\Gamma) \Rightarrow \Psi_V \in H^1_{BL}(\mathbb{R}^3)$
in each bounded domain

$$\underline{e} \cdot n = -\frac{1}{\kappa^2} \operatorname{curl} \operatorname{curl} \underline{e} \cdot n = -\frac{1}{\kappa^2} \operatorname{div}_\Gamma(\operatorname{curl} e \times n)$$

1form

trace

→ Modified Stratton-Chu representation formula:

$$\underline{e} = \Psi_{DL}(\overset{\text{jump}}{[\underline{e}]}) - \Psi_{SL}([\operatorname{curl} e \times n]_\Gamma)$$

$$\Psi_{SL}(\underline{u}) = \Psi_V(\underline{u}) + \frac{1}{\kappa^2} \operatorname{grad} \Psi_V(\operatorname{div}_\Gamma \underline{u})$$

$$\Psi_{DL}(\underline{u}) = \operatorname{curl} \Psi_V(R\underline{u})$$

rotation

Note: $\cdot$ $\operatorname{curl} \underline{e} \times n \in \mathcal{H}_{mag}$

→ $\operatorname{div}_\Gamma(\operatorname{curl} \underline{e} \times n) \in H^{-1/2}(\Gamma)$

$\cdot$ $(H^1(\Omega))^3 \subset H(\operatorname{curl}, \Omega)$

↓

component-wise trace

$(H^{1/2}(\Gamma))^3_t \subset \mathcal{H}_{el}$

↓ duality

$(H^{-1/2}(\Gamma))^3 \supset \mathcal{H}_{imag}$

→ $\Psi_V([\operatorname{curl} e \times n]_\Gamma)$ is meaningful

Theorem 5.3.D: $\Psi_{SL}: \mathcal{H}_{imag} \to H_{loc}(\operatorname{curl}, \Omega)$ is continuous
$\Psi_{DL}: \mathcal{H}_{el} \to H_{loc}(\operatorname{curl}, \Omega)$ is continuous
Local

Consider solutions of: $\operatorname{curl} \operatorname{curl} \underline{u} - \kappa^2 \underline{u} = 0$ in $\Omega \cup D$
$[u_t]_\Gamma = 0$
$[\operatorname{curl} u \times n]_\Gamma = \underline{v} \in \mathcal{H}_{imag}$
$\underline{u}$ radiation conditions at ∞

▷ $\underline{u} = \Psi_{SL}(\underline{v})$ by representation formula
→ piecewise Maxwell solution

If one plugs it for the magnetic field one gets Ψ_{DL}

Thm 5.3.E: $\Psi_{SL}(\underline{v})$, $\underline{v} \in \mathcal{H}_{imag}$ are radiating Maxwell solutions
$\Psi_{DL}(\underline{w})$, $\underline{w} \in \mathcal{H}_{el}$ in $\Omega \cup D$

Remark: Ψ_{SL} → Maxwell single layer potential
Ψ_{DL} → Maxwell double layer potential