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Numerical Analysis II

ETH Zürich
D-MATH

Problem Sheet 8

Problem 8.1 Heun Method

The Heun method is the following:

$$\begin{aligned}\tilde{y}_{n+1} &= y_n + hf(t_n, y_n),\\ y_{n+1} &= y_n + \frac{h}{2}[f(t_n, y_n) + f(t_{n+1}, \tilde{y}_{n+1})]\end{aligned}$$

(8.1a) Show that the method is consistent.

(8.1b) Implement in Python a program for the method by filling in the template `heun.py`, and verify with `scriptheun.py` that the order of accuracy with respect to h is 2, with the following IVP:

$$\begin{aligned}y' &= \sin(t) + y, t \in (0, 1]\\ y(0) &= 0\end{aligned}.$$

Hint: Show that the IVP has the exact solution $y(t) = (\exp(t) - \sin(t) - \cos(t))/2$ and use that exact solution to compute the error.

Problem 8.2 One Example of Runge-Kutta Method

Let us consider the following Runge-Kutta method:

$$\begin{aligned}y^{n+1} &= y^n + h(\frac{1}{6}k_1 + \frac{2}{3}k_2 + \frac{1}{6}k_3)\\ k_1 &= f(t_n, y_n)\\ k_2 &= f(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_1)\\ k_3 &= f(t_n + h, y_n - hk_1 + 2hk_2)\end{aligned}$$

(8.2a) Show that the method is consistent.

(8.2b) Implement in Python a program for the method by filling in the template `rk3.py` and verify with `RKmethodscript.m` that the order of accuracy is 3 with respect to h for the problem

$$\begin{aligned}y' &= \sin(t) + y, t \in (0, 1]\\ y(0) &= 0\end{aligned}$$

Problem 8.3 Vibrant system

The displacement $y(t)$ of a vibrant system is given by a body with a certain weight and a spring and is subject to a resistive force proportional with the velocity. This can be described by the differential equation $y'' + 5y' + 6y = 0$.

Modify the Heun method implemented in Problem 1, so that it takes vectors and plot the solution of the differential equation for $y(0) = 1, y'(0) = 0, t \in [0, 5], h = 0.3$.

Hint: Write the equation in the form of 1st order ODE and implement it Python by filling in the template `ex3.py`. Remember to copy the script `Heun.py` from Problem 1 to the same directory as `ex3.py`, so that the right function is called.

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