Problem 8.1 Heun Method

Let the Heun method:
\[
\tilde{y}_{n+1} = y_n + hf(t_n, y_n), \\
y_{n+1} = y_n + \frac{h}{2}[f(t_n, y_n) + f(t_{n+1}, \tilde{y}_{n+1})]
\]

(8.1a) Show that the method is consistent.

(8.1b) Implement in Matlab a program for the method using template Heun.m, and verify with scriptHeun.m that the order of accuracy with respect to \( h \) for the problem is 2:
\[
y' = \sin(t) + y, \quad t \in (0, 1] \\
y(0) = 0
\]

Problem 8.2 One Example of Runge-Kutta Method

Let us consider the following Runge-Kutta method:
\[
y_{n+1} = y^n + h\left(\frac{1}{6}k_1 + \frac{2}{3}k_2 + \frac{1}{6}k_3\right) \\
k_1 = f(t_n, y_n) \\
k_2 = f(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_1) \\
k_3 = f(t_n + h, y_n - hk_1 + 2hk_2)
\]

(8.2a) Show that the method is consistent.

(8.2b) Implement in Matlab a program for the method using template rk3.m and verify with RKmethodsCscript.m the order of accuracy is 3 with respect to \( h \) for the problem
\[
y' = \sin(t) + y, \quad t \in (0, 1] \\
y(0) = 0
\]
Problem 8.3 Vibrant system

The displacement $y(t)$ of a vibrant system is given by a body with a certain weight and a spring and is subject to a resistive force proportional with the velocity. This can be described by the differential equation $y'' + 5y' + 6y = 0$.

Using the Heun method implemented in the Problem 1, solve and plot the solution of the differential equation for $y(0) = 1$, $y'(0) = 0$, $t \in [0, 5]$, $h_n = 0.3$.

**Hint:** Write the equation in the form of 1st order ODE and implement in MATLAB using template ex3.m. Please copy Heun.m in Problem 1 to the same directory as ex3.m, so that the function can be called.

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