Semesterarbeit/Term Project

(Rechnergestützte Wissenschaften / Angewandte Mathematik)

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Wavelet FEM for problems with stochastic data

Issues.

Strongly elliptic operator equations with stochastic data in a domain $D \subset \mathbb{R}^d$ are considered. We discretize the mean field problem using a wavelet FEM with N degrees of freedom.

To compute the k-th moment $\mathcal{M}^k u$ of the random solution u both, stochastic and deterministic methods are analyzed. Key tool in both types of algorithms are k-fold sparse tensor products of the FEM in D.

Monte-Carlo FEM with M samples (i.e., deterministic solves) is proved to yield approximations to $\mathcal{M}^k u$ converging with optimal rate in work $O(MN(\log N)^{k-1})$.

Direct deterministic FEM approximation for $\mathcal{M}^k u$ is based on equations for $\mathcal{M}^k u$ in $D^k \subset \mathbb{R}^{kd}$. These equations are derived and their strong ellipticity and regularity in scales of anisotropic Sobolev spaces is established. Their Galerkin approximation using sparse tensor products of the FE spaces in D allows approximation of $\mathcal{M}^k u$ with $N(\log N)^{k-1}$ degrees of freedom converging at an optimal rate.

Multilevel preconditioning and, for nonlocal operators, wavelet compression of the stiffness matrices is shown to yield deterministic approximations to $\mathcal{M}^k u$ in complexity in $N(\log N)^c$ for some $c > 0$ proportional to k.

Prerequisites.

Numerik der Differentialgleichungen (RW) or Numerik Partieller Differentialgleichungen, MATLAB or C.

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