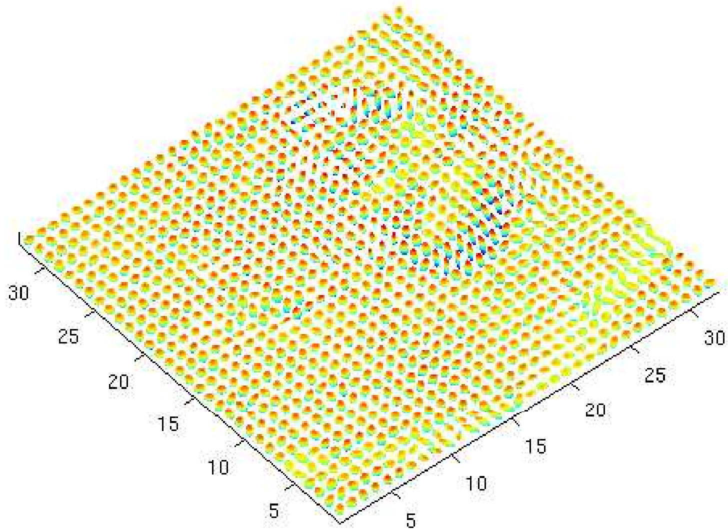

Figure 1: A slice of a DTI-MRI brain image. The diffusion tensors are visualized as covariance ellipsoids.

Type Bachelor or Master Thesis

Title

Multiscale Edge Detection for Diffusion Tensor MRI

Advisor PROF. PHILIPP GROHS

Description. Diffusion Tensor Magnetic Resonance Imaging (DT-MRI) is a relatively recent method to visualize the directional diffusion of water molecules across brain cells. This non-invasive imaging method provides important information on the complex arrangement of fiber tracts in the brain which cannot be obtained by conventional MRI imaging. For each voxel, the direction-dependent diffusion rate is described by a symmetric positive definite matrix, the so-called diffusion tensor, see Figure 1 where these matrices are vizualized as covariance ellipsoids.

A diffusion tensor image can be modeled as a discrete function

$$u : h\mathbb{Z}^d \to \mathrm{SPD}(3),$$

where $\mathrm{SPD}(3)$ denotes the space of 3×3 symmetric positive definite matrices.

Given such a u it is of interest to detect edges and other features in this image. For real-valued images u, a popular strategy is to estimate the gradient of a smoothed version of u and to study the maxima of these gradient magnitudes. These points are candidates for edge points (Mallat 2003).

For DTI-MRI the problem is that u takes its values in a nonlinear space, which implies that concepts such as smoothing or gradients do not make sense in general.

On the other hand, the space $\mathrm{SPD}(3)$ possesses some geometric structure: it constitutes a Riemannian manifold, or more precisely a Riemannian symmetric space (Do Carmo 1992). Exploiting this structure, it becomes possible to define geometric analogs to all the operations needed for defining a multiscale edge detection scheme, see for instance (Rahman, Drori, Stodden, Donoho and Schröder 2006, Pennec, Fillard and Ayache 2006).

The goal of this project is to develop and implement (in MATLAB or C++) a multiscale edge detection scheme capable of handling geometric images such as DT-MRI data. A successful completion may result in a journal publication.

Prerequisites. Numerical analysis, (basic) differential geometry, programming skills (MATLAB and/or C++).

Contact. Prof. Philipp Grohs, Seminar for Applied Mathematics, Email: pgrohs@sam.math.ethz.ch, Web: `http://www.math.ethz.ch/~pgrohs`, Office: HG G 59.2.

References

M. Do Carmo (1992), *Riemannian geometry*, Birkhäuser.

S. Mallat (2003), *A wavelet tour of signal processing*, Academic Press.

X. Pennec, P. Fillard and N. Ayache (2006), 'A Riemannian framework for tensor computing', *International Journal of Computer Vision* **66**(1), 41–66.

I. U. Rahman, I. Drori, V. C. Stodden, D. Donoho and P. Schröder (2006), 'Multiscale representations for manifold-valued data', *Multiscale modeling and Simulation* **4**(4), 1201–1232.