Implicit-Explicit Runge-Kutta schemes for hyperbolic systems with stiff relaxation in the hyperbolic and diffusive limit

Giovanni Russo
Department of Mathematics and Computer Science (DMI),
University of Catania, Catania, Italy

Several mathematical models are described by hyperbolic systems with stiff relaxation, defined by a small relaxation time ϵ. In the so-called hyperbolic relaxation, as the ϵ vanishes, the system relaxes to another hyperbolic system with smaller number of equations. Classical method of line approach based on finite difference schemes present the double difficulty: the stiffness of the source term, and the requirement that in the limit of infinite stiffness the scheme becomes a consistent discretization of the relaxed system.

Implicit-Explicit (IMEX) schemes constitute a powerful tool to attach this problem (see for example [4] and references therein).

In such schemes, the (non stiff) hyperbolic terms are treated explicitly, while the (stiff) source terms are treated implicitly. It will be shown that under certain conditions, IMEX schemes provide the correct asymptotic behavior as the stiffness parameter ϵ vanishes. Furthermore, singular perturbation method can be employed to derive order conditions which will provide uniform accuracy in ϵ [1].

In the case of diffusive relaxation the situation is much more complicated. In this case the characteristic speeds of the hyperbolic part depend on ϵ and diverge as $\epsilon \to 0$. There are methods that allow to overcome such stiffness, and that allow the construction of asymptotic preserving shemes that, in the limit of infinite stiffness, reduce to a consistent explicit scheme for the underlying diffusion (or convection-diffusion) equation [3].

Here we present two techniques for the construction of IMEX schemes which capture the diffusive limit, without the classical stability restriction on the time step $\Delta t = O(\Delta x^2)$. The first one, developed in collaboration with L.Pareschi, is based on an implicit treatment of some hyperbolic terms, which can however be explicitly computed, while the second one, which treats the hyperbolic terms explicitly, is obtained by applying additional condition on the RK coefficients, obtained by a singular perturbation expansion of both the solution and the numerical solution [2]. Several numerical tests will be presented that illustrate the robustness and generality of the methods.

Most of the results presented have been developed in collaboration with Sebastiano Boscarino, DMI, University of Catania.

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[4] Pareschi L., Russo G.: Implicit-Explicit Runge-Kutta schemes and applications to hyperbolic systems with relaxations. Journal of Scientific Computing **25**, 129-155 (2005).