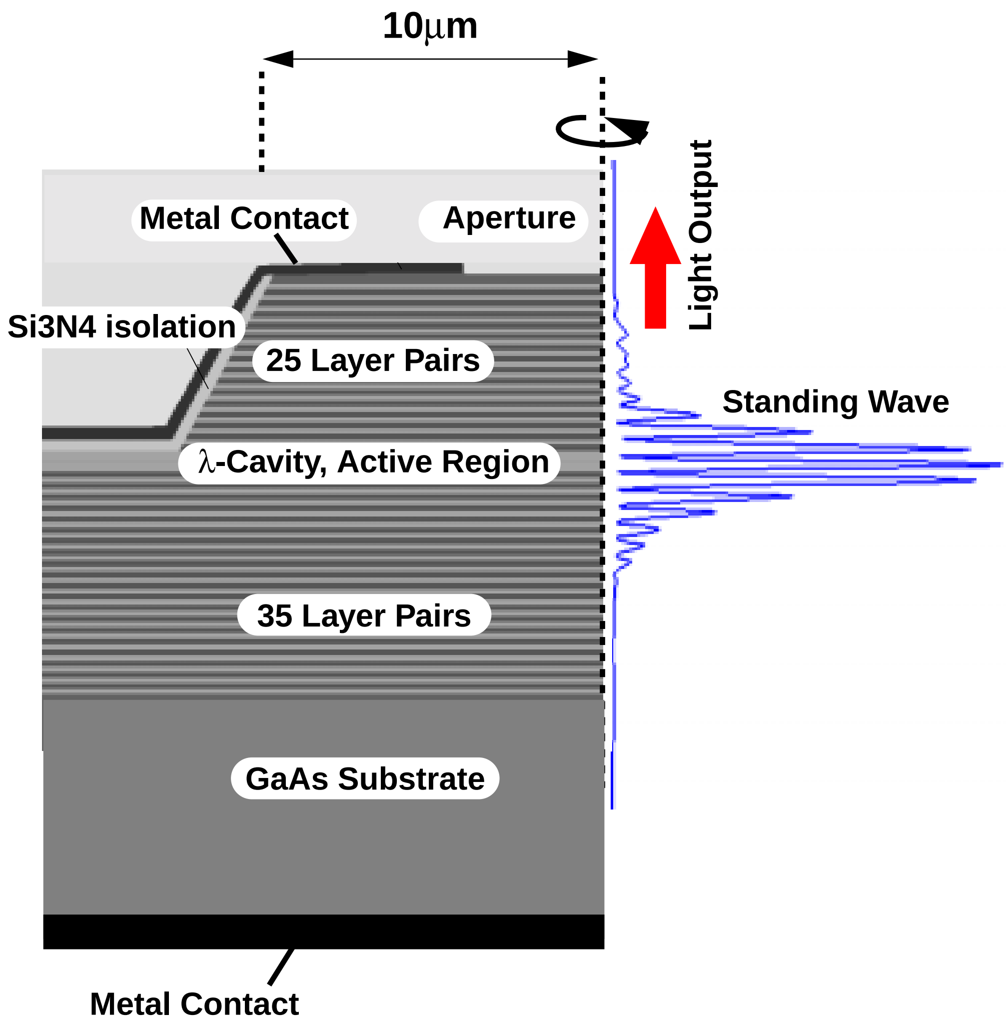
Eigenmode Analysis of Vertical-Cavity Lasers

Andreas Witzig, Matthias Streiff, Wolfgang Fichtner

Integrated Systems Laboratory
Swiss Federal Institute of Technology,
ETH Zurich Switzerland

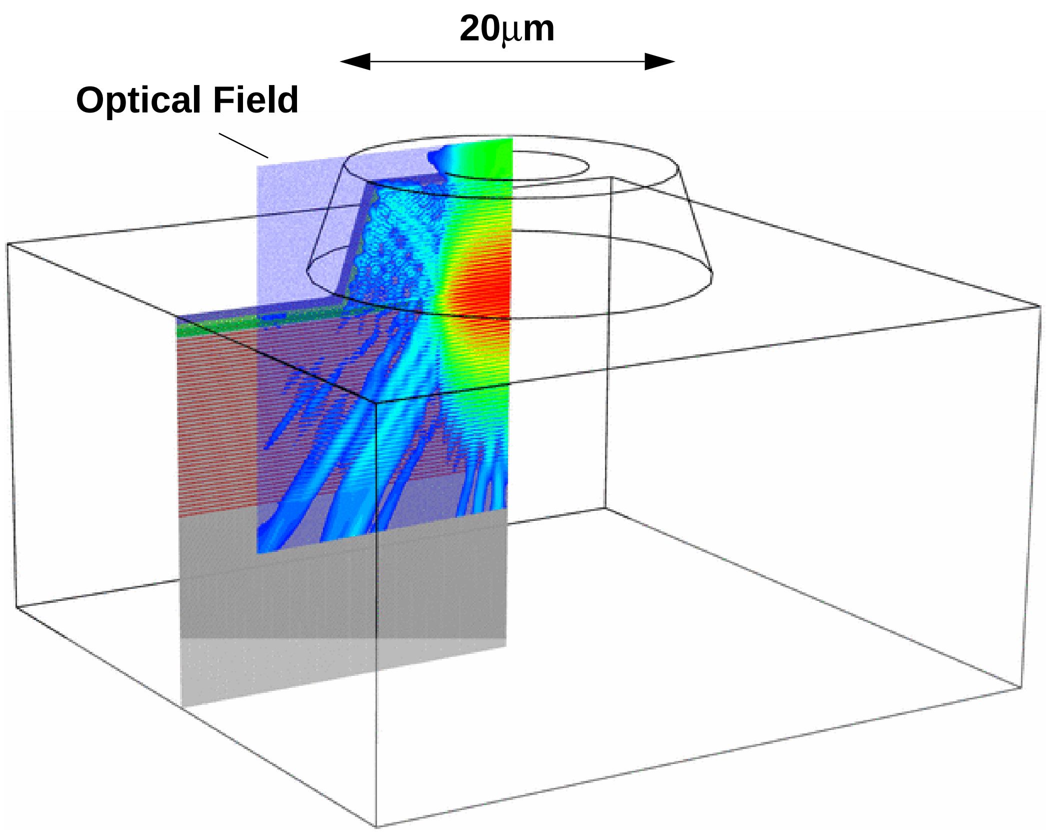
Overview

- Motivation
- Solution of the Optical Modes in Laser Cavities
  - ➜ Discretization of Vectorial Helmholtz Equation by linear and quadratic Nédélec Elements
  - ➜ Solution of the Algebraic Eigenvalue Problem by Jacobi-Davidson QZ Algorithm
- Spectral Portrait
  - ➜ Identification of Optical Eigenmodes
  - ➜ Target Values
  - ➜ Tracking of Optical Modes
- Computational Effort
  - ➜ Linear vs. Quadratic Edge-Elements
- Conclusion / Outlook



Vertical-Cavity Surface-Emitting Lasers (VCSELs)

- **Application**
  - → Promising Light Source for Optical Communication Systems
  - → Semiconductor Laser (PiN-Diode)
  - → Light Generation by Radiative Recombination of Electrons and Holes
  - → Optical Resonator (Layered Medium)
- **Challenge**
  - → Calculate Optical Eigenmodes
  - → Calculate Resonance Frequency
  - → Simulation of Opto-Electro-Thermal Device Characteristics



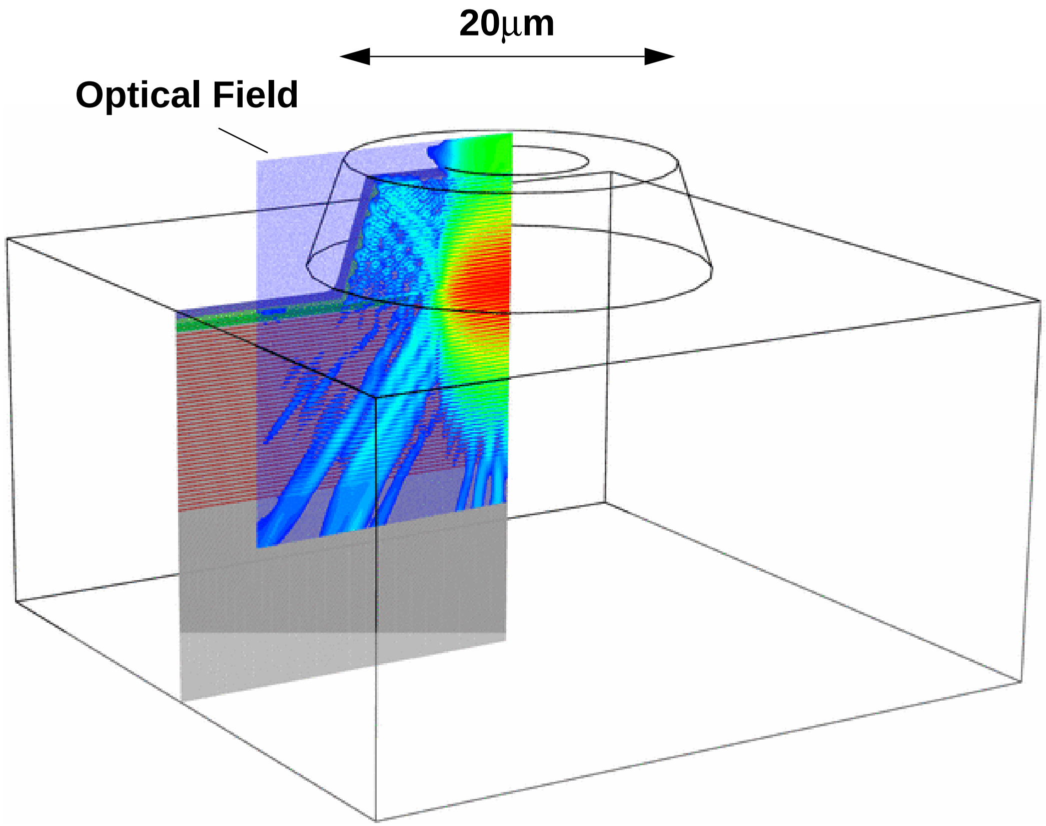
Problem Formulation:

Calculation of Optical Eigenmodes

- ➜ Frequency-Domain Solution of Homogeneous Maxwell's Equations (=Helmholtz Equations)
- ➜ Radiation Boundary Condition
- ➜ Non-Hermitian Complex Generalized Eigenproblem
- ➜ Real Part of Eigenvalue = Resonance Frequency (Free-Space Resonance Wavelength λ)
- ➜ Imaginary Part of Eigenvalue = Field Decay Constant (related to Photon Lifetime τ_{ph})

$$\nabla \times (\nabla \times \Psi_\nu(r)) - \frac{(\omega'_\nu + i\omega''_\nu)^2}{c^2} n^2(r)\Psi_\nu(r) = 0.$$

+ Radiation Boundary Condition



Application:

Coupled Opto-Electro-Thermal VCSEL Simulation

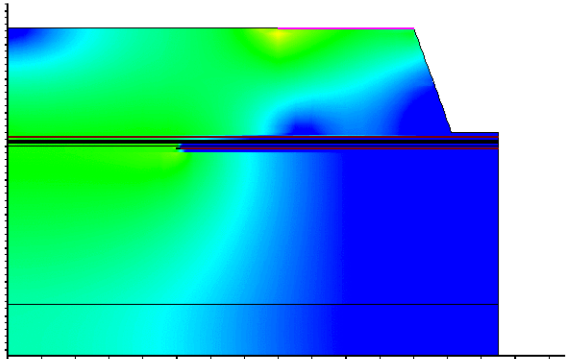
Refractive Index Depends on Temperature and Carrier Density

- → **Need Volume Discretization (Finite-Element Type, Full-Vectorial)**

Solution of Optical Cavity Problem is Iterated with Electronic Equations

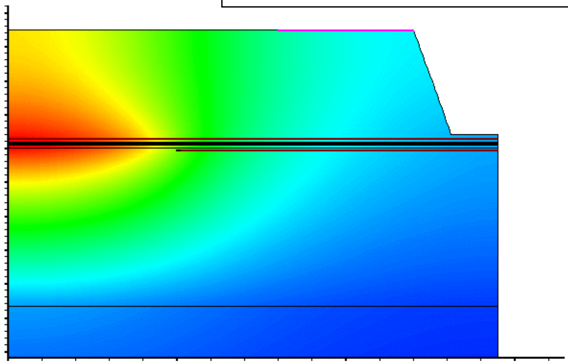
- → **Need a Fast Solver (in Compensation, allow to use a lot of memory)**

Current Density



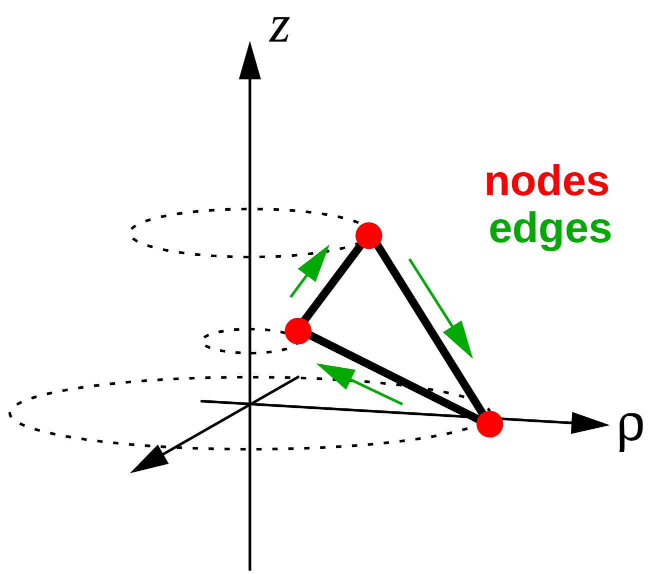
+

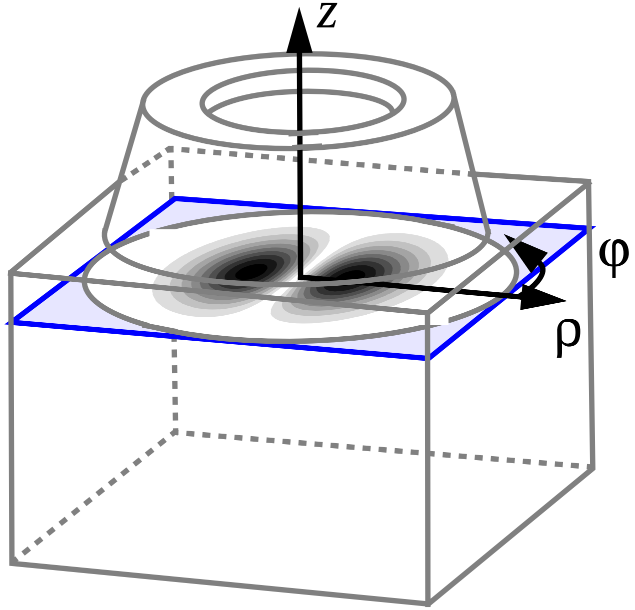
Temperature



→

Peak Change in Refractive Index by ≈ 2%





Discretization

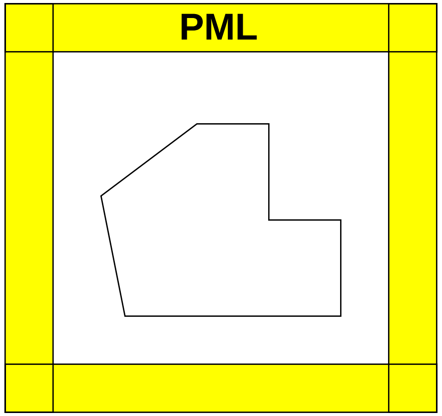
- **Discretization by Nédélec Elements**
  - ➜ **Unstructured Grid (typ. order 500'000)**
  - ➜ **Full-Vectorial Complex Optical Field**
  - ➜ ***"curl"*-Conforming Discretization**
  - ➜ **Linear and Quadratic Elements**

- **Solution in 2D or *"2.5D"***
  - ➜ **Body-of-Revolution**
    Fourier Series Expansion in ϕ-Direction

Solution of
The Open Cavity Problem

- **Radiation to Infinity**
  - ➜ **Essential Result of the Simulation (Directly proportional to Photon Life Time, Laser Threshold, Filter Linewidth, etc)**
- **Perfectly Matched Layer (PML) Concept**
  - ➜ **Artificial Material for Absorption**
  - ➜ **Outside of PML, use Dirichlet Boundary Conditions.**

  Very good Results, Problem with Layered Media



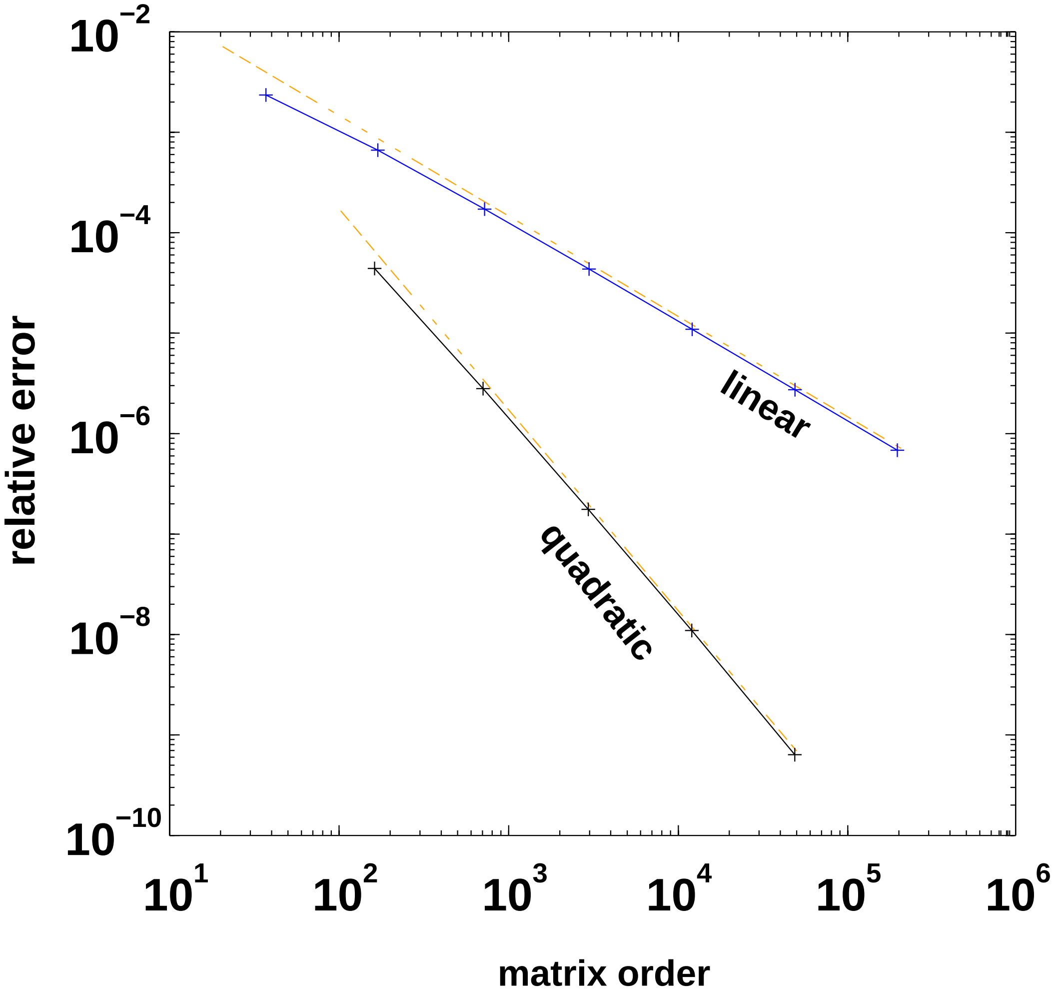
$$\nabla \times \left([\mu_r]^{-1} \cdot \nabla \times \Psi_\nu\right) - k_0^2[\varepsilon_r]\Psi_\nu = 0.$$

$$[\mu_r] = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha^{-1} \end{bmatrix} \qquad [\varepsilon_r] = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha^{-1} \end{bmatrix}$$

Solution of
Algebraic Eigenproblem

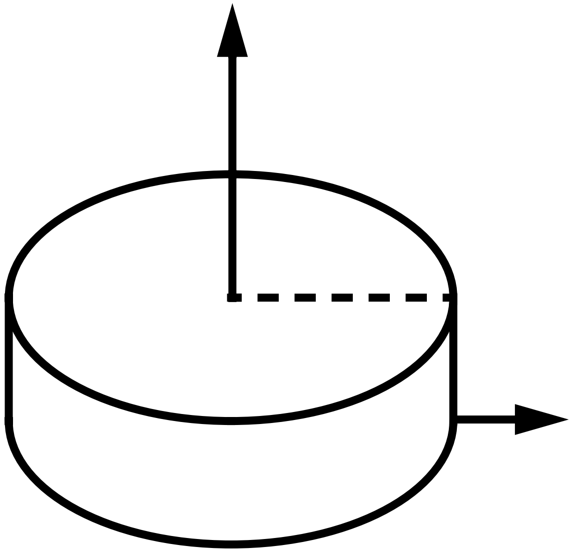
- **Jacobi-Davidson QZ Algorithm**
  - ➜ **Low-Dim. Search Subspace (e.g. order 10)**
  - ➜ **High-Order Correction Equation (BiCGstab)**
  - ➜ **Preconditioner: Matrix Inversion, using Direct Solver *PARDISO***

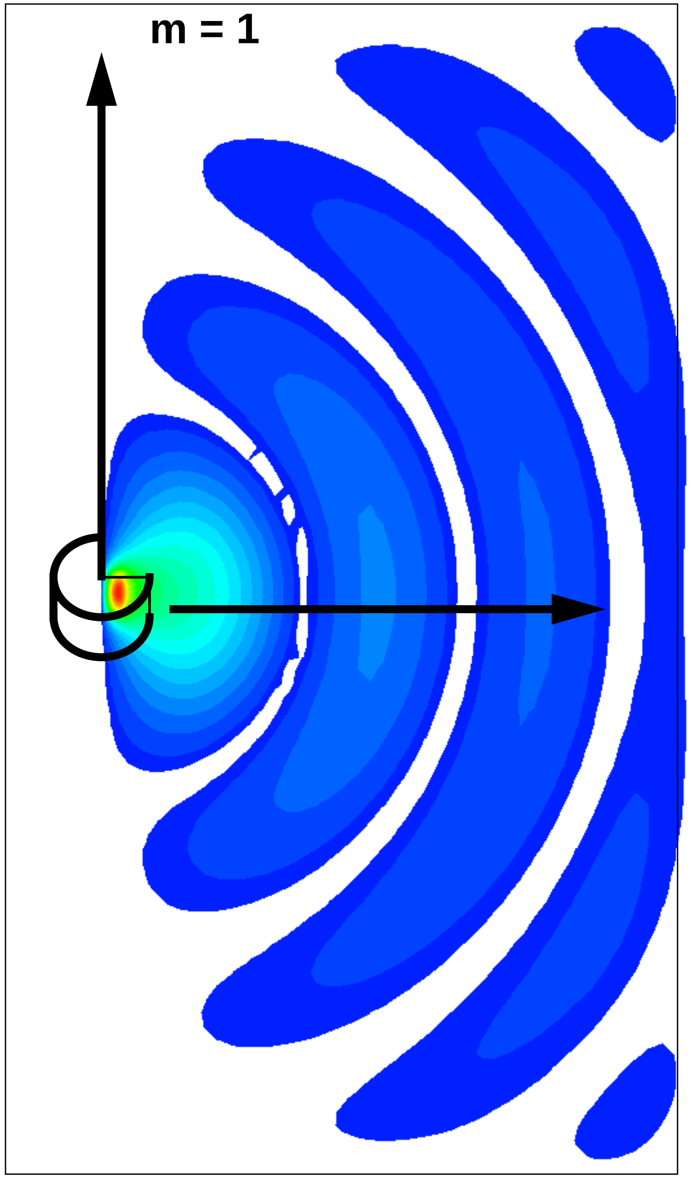
- **In Practice...**
  - ➜ **Find a few *inner* Eigenmodes (1..5)**
  - ➜ **Need Good Target Value (know λ within 0.1%)**
  - ➜ **Need a Lot of Memory for Computation (6GB)**
  - ➜ **Quick Method to Find Eigenvalue (20min)**

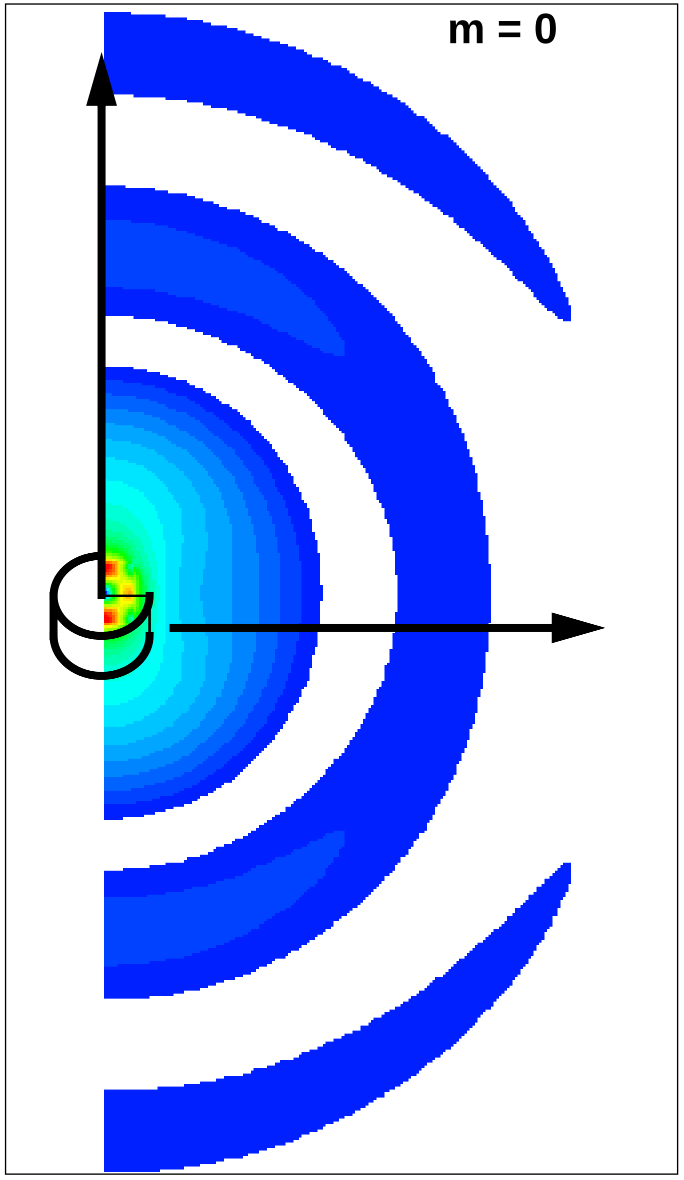


Reference I: Metal Box Resonator

- **Analytical Solution Known**
- **Comparison of Linear and Quadratic Elements**
- **Symmetry similar to VCSEL(Body-of-Revolution, Fourier Expansion of 3D Fields)**



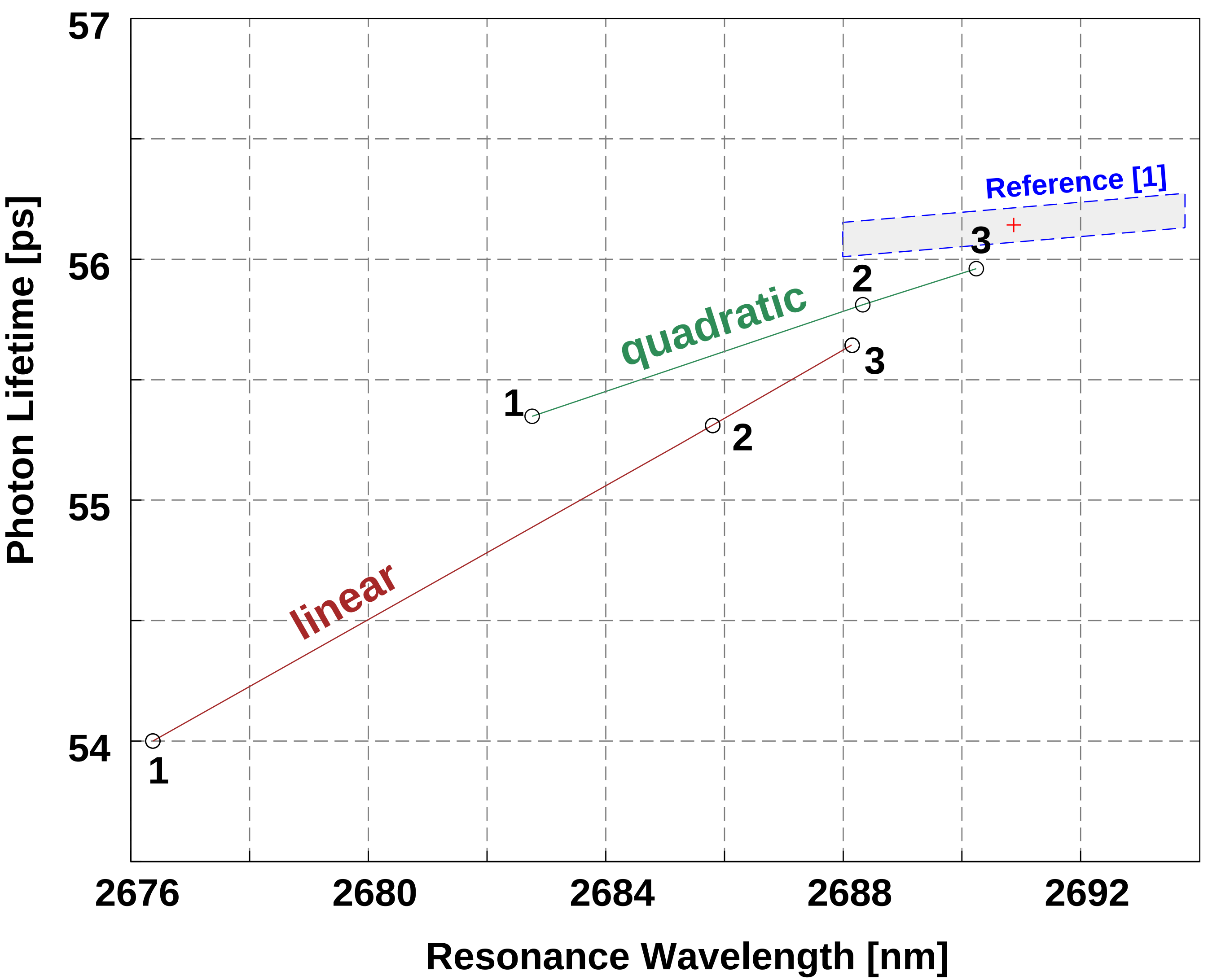




Real Part of the Electric Field

Reference II: Dielectric Pill Resonator

- Cylinder with *n* = 5.9 in Vacuum
- Includes Radiation
  - ➜ Good Test for Absorbing Boundaries
  - ➜ Non-Hermitian Matrices, Complex Eigenvalue
- Symmetry similar to VCSEL
  - ➜ Body-of-Revolution, Fourier Expansion

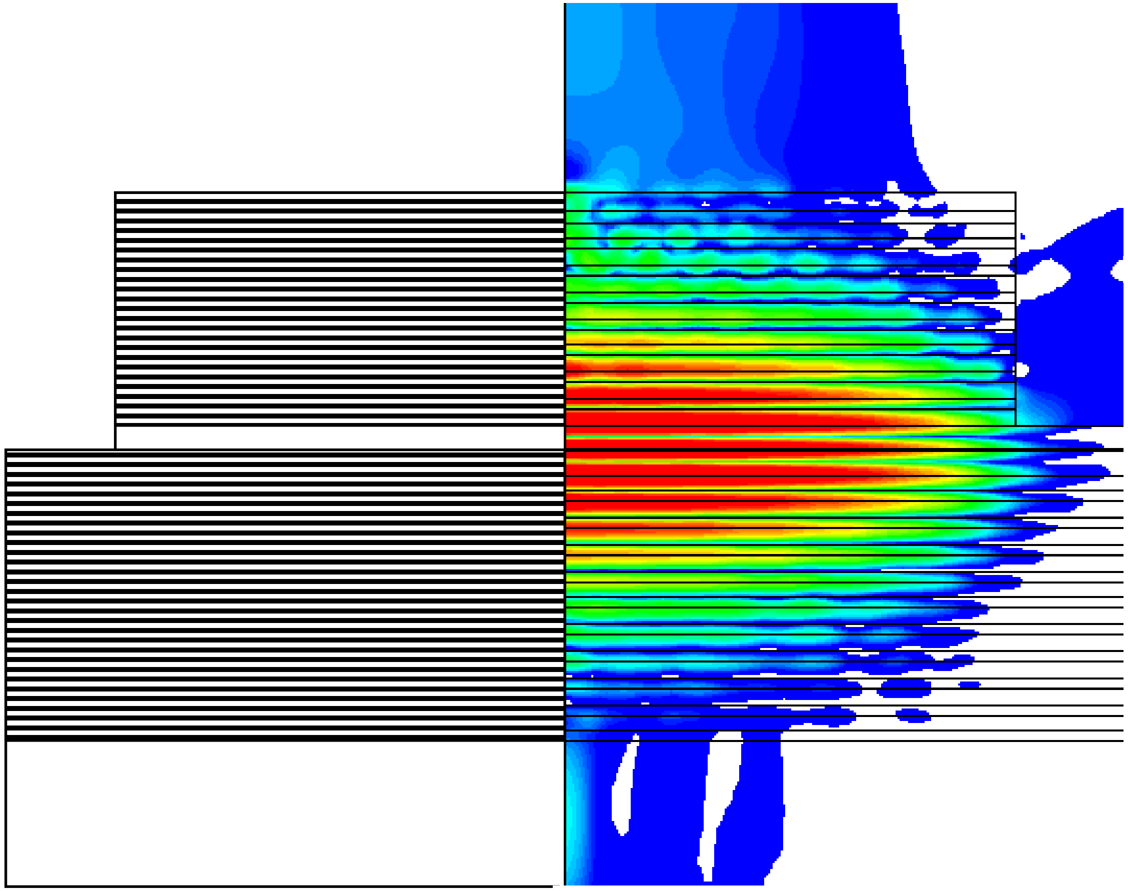


Reference II: Dielectric Pill Resonator

- Accuracy Dependent on Mesh Refinement:
- 1: 10 Cells/λ (8x8 in Pill)
  linear: order 9’000
  quad.: order 36’000
- 2: 20 Cells/λ (16x16 in Pill)
  linear: order 35’000
  quad.: order 140’000
- 2: 40 Cells/λ (32x32 in Pill)
  linear: order 139’000
  quad.: order 556’000

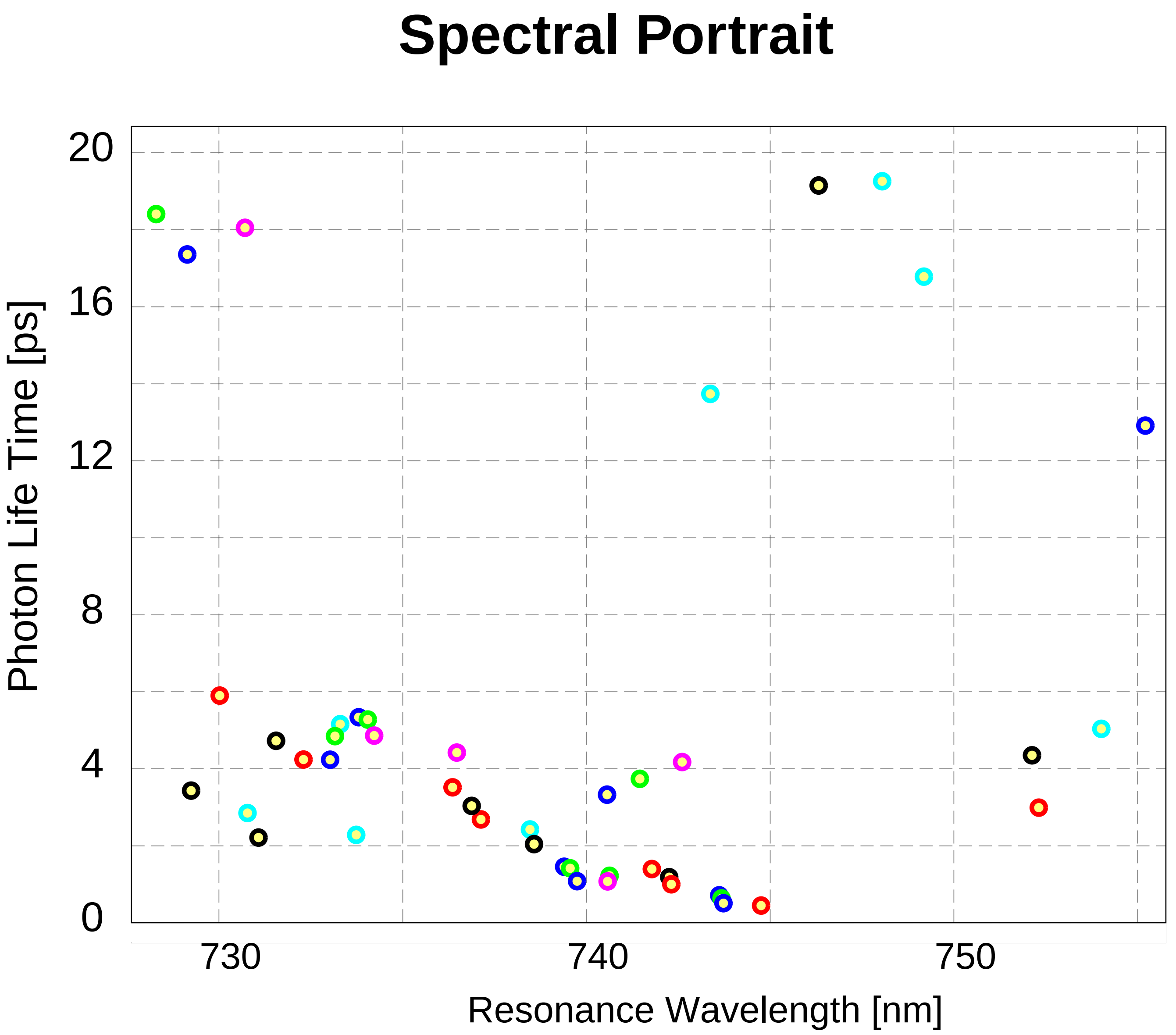
[1] Tsuji et. al. "On the Complex Resonant Frequency of Open Dielectric Resonators", IEEE Trans. on Microwave Theory and Techniques, Vol. 31, No. 5 May 1983

Fundamental Mode



Example I: Airpost VCSEL

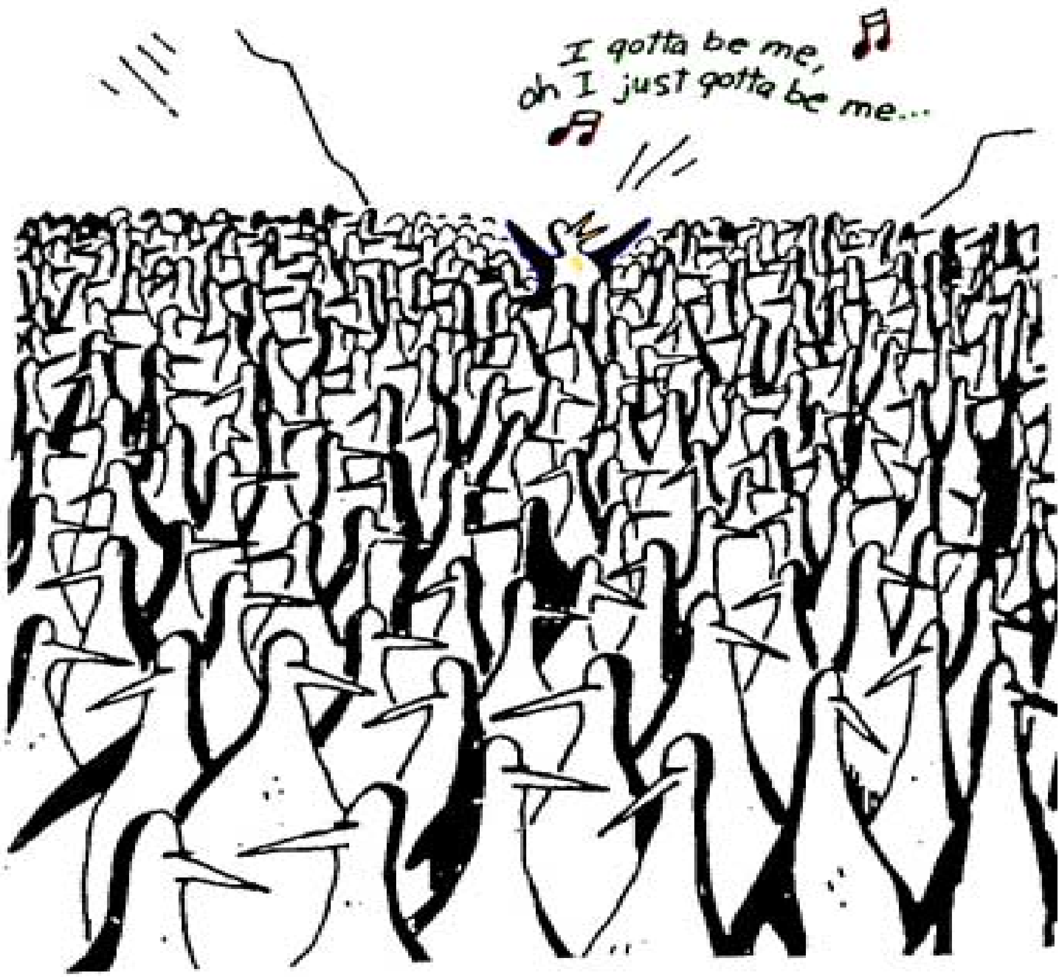
- **Semiconductor Stack**
  - ➜ **Low Refractive Index Contrast**
  - ➜ **Many Layers (30..40)**
- **How to Find Fundamental and Higher Order Modes?**
  - ➜ **Spectral Portrait**



Example I: Airpost VCSEL

- **Spectral Portrait**
  - ➜ **Mode Selection??**
  - ➜ **Mode with Smallest Imaginary Part and Large Overlap with Active Region...**
- **Note: Different Expansion Number m**

Spectral Portrait

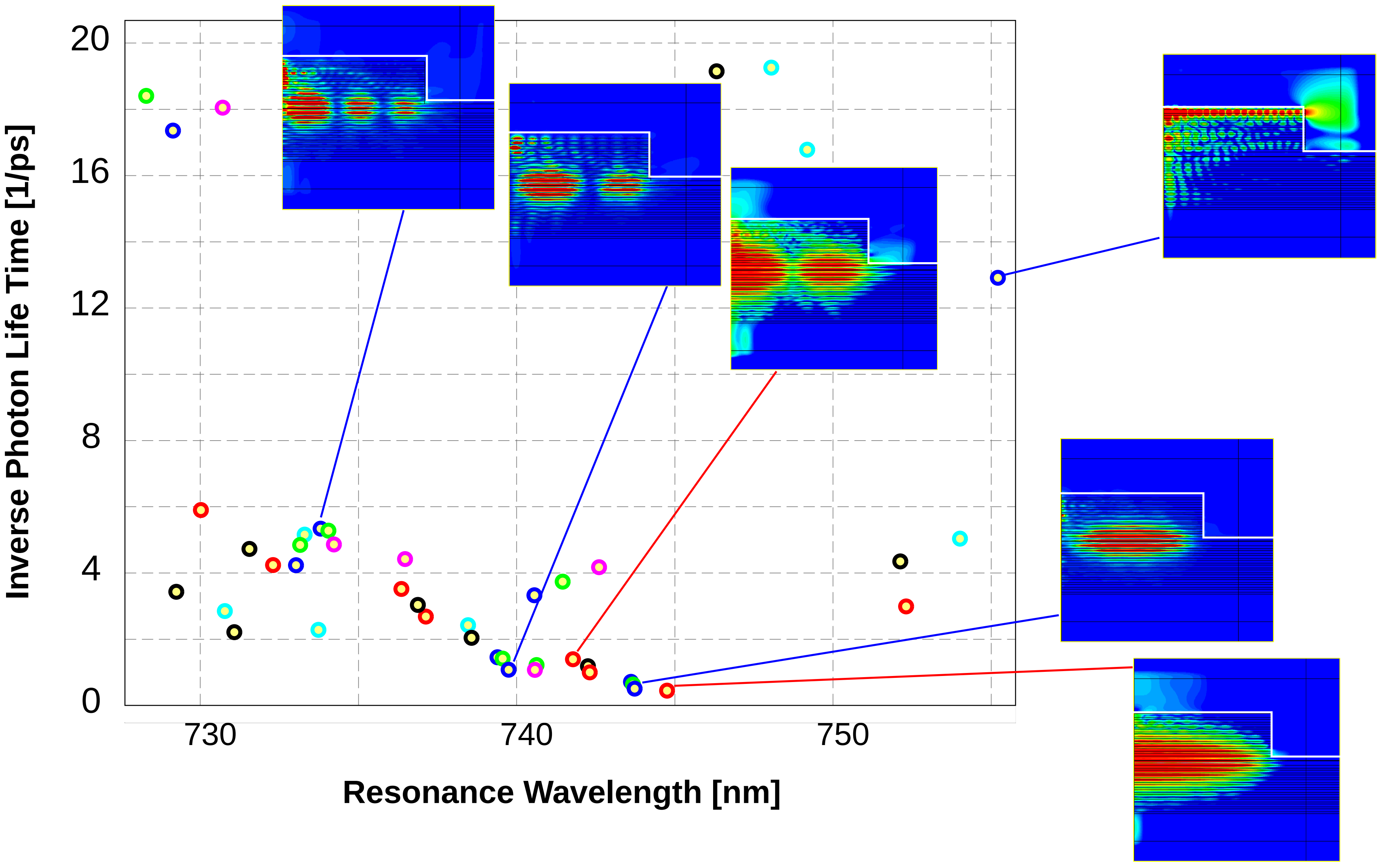


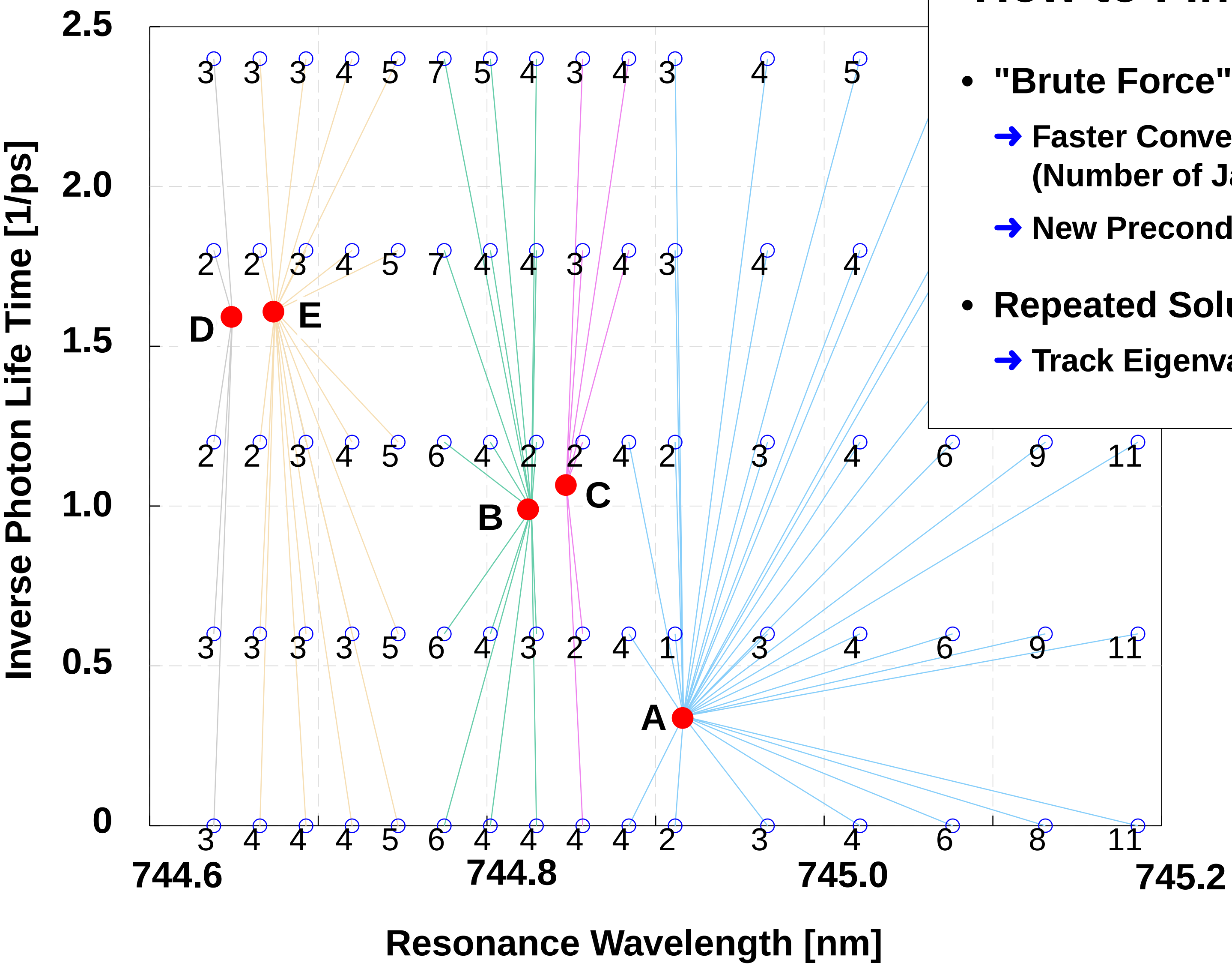
Example I: Airpost VCSEL

- **Spectral Portrait**
  ➜ **Mode Selection??**

- **Mode with Smallest Imaginary Part and Large Overlap with Active Region...**

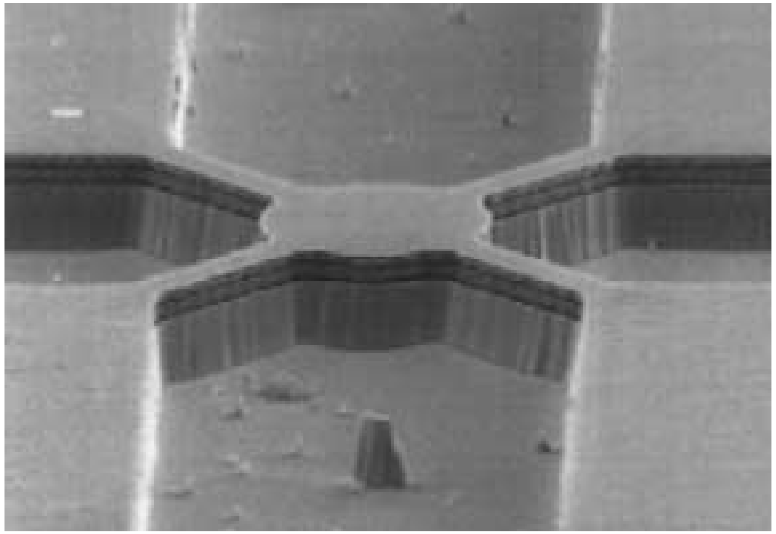
Spectral Portrait

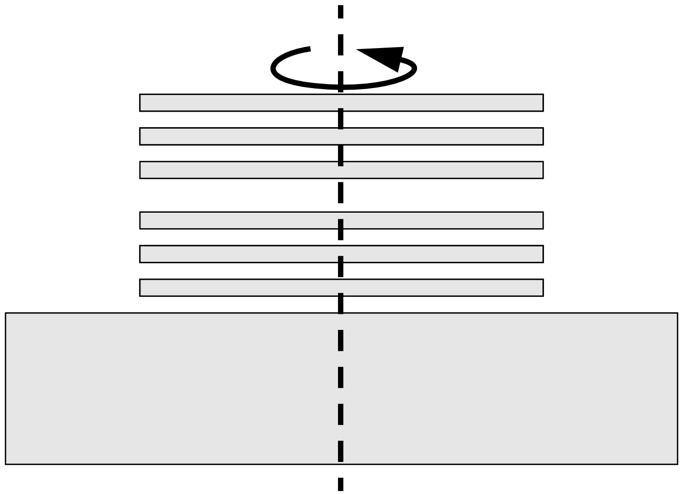




How to Find Complete Spectrum?

- **"Brute Force": Scan Target Value**
  - ➜ **Faster Convergence for Better Target Value (Number of Jacobi Correction Equations Solved)**
  - ➜ **New Preconditioner for Each Eigenvalue?**
- **Repeated Solution of Perturbed Problem**
  - ➜ **Track Eigenvalue**

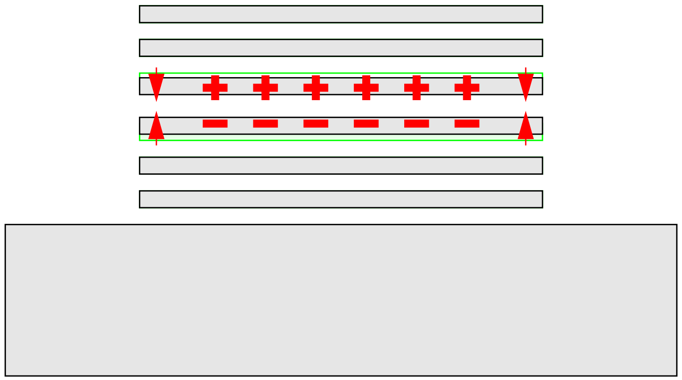




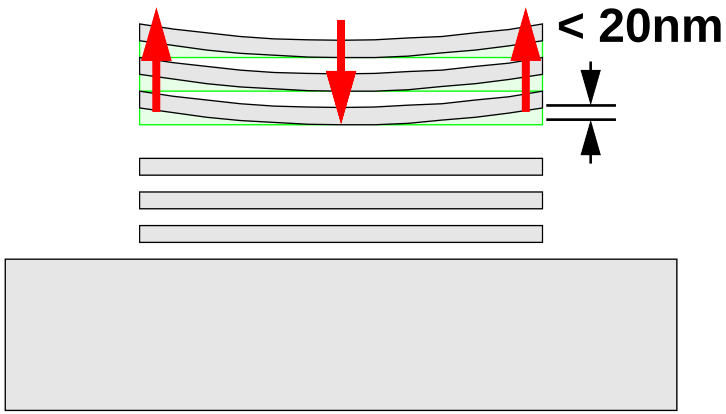
Example II:
Tunable Filter

- **Air-Gap Mirrors**
  - ➜ **High Refractive Index Contrast**
  - ➜ **Need Only 3..4 Layers**
- **Fabrication:**
  - ➜ **Selectively Ethching Sacrifice Layers**
  - ➜ **Air Bridges Hold the Layers**
- **Concept Allows Tuning**
- **Filter: Incident Light from Top**
  - ➜ **Resonances in the Cavity Cause Sharp Spectral Dip in the Reflected Beam**

Tuning

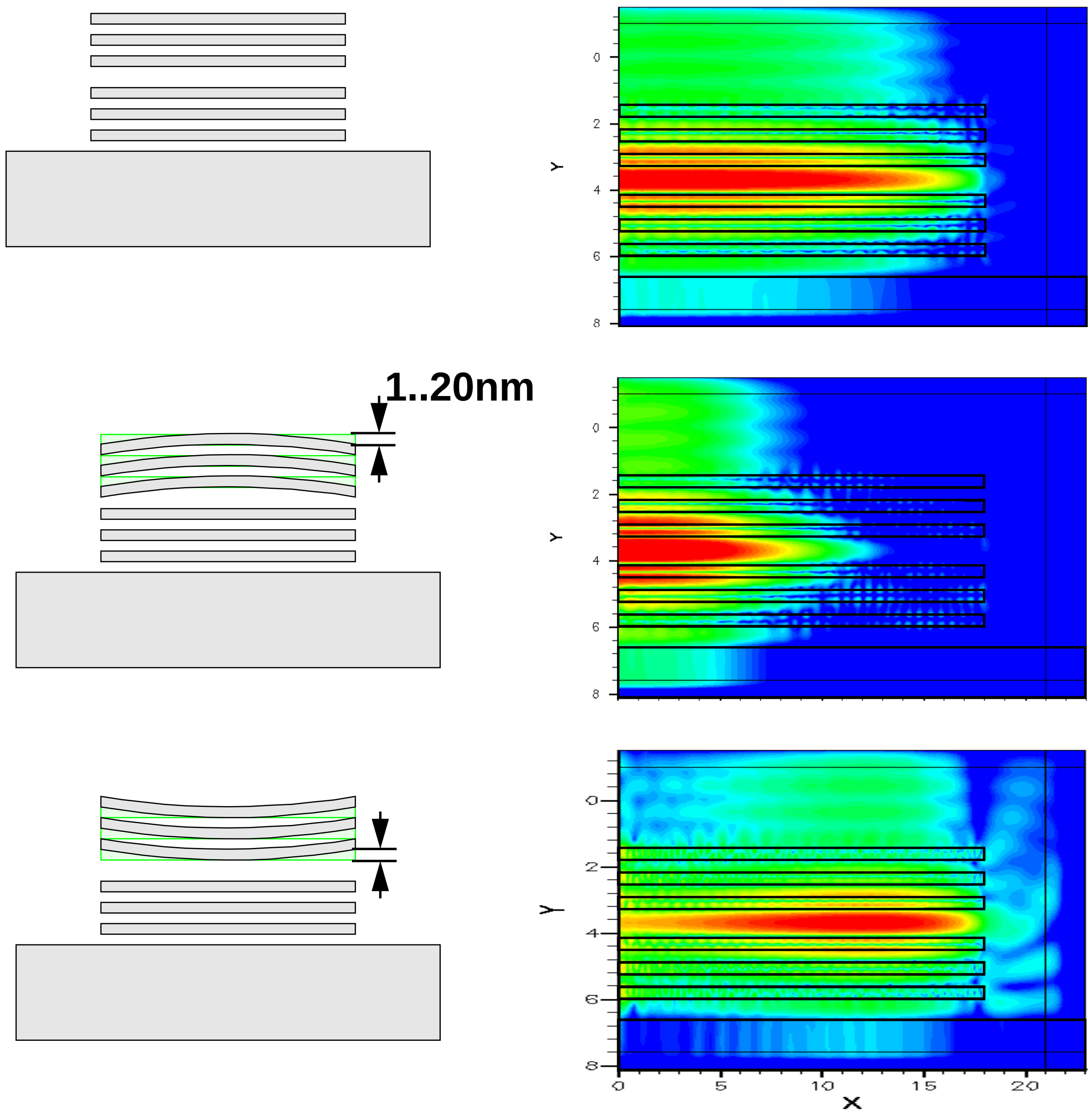


Bend



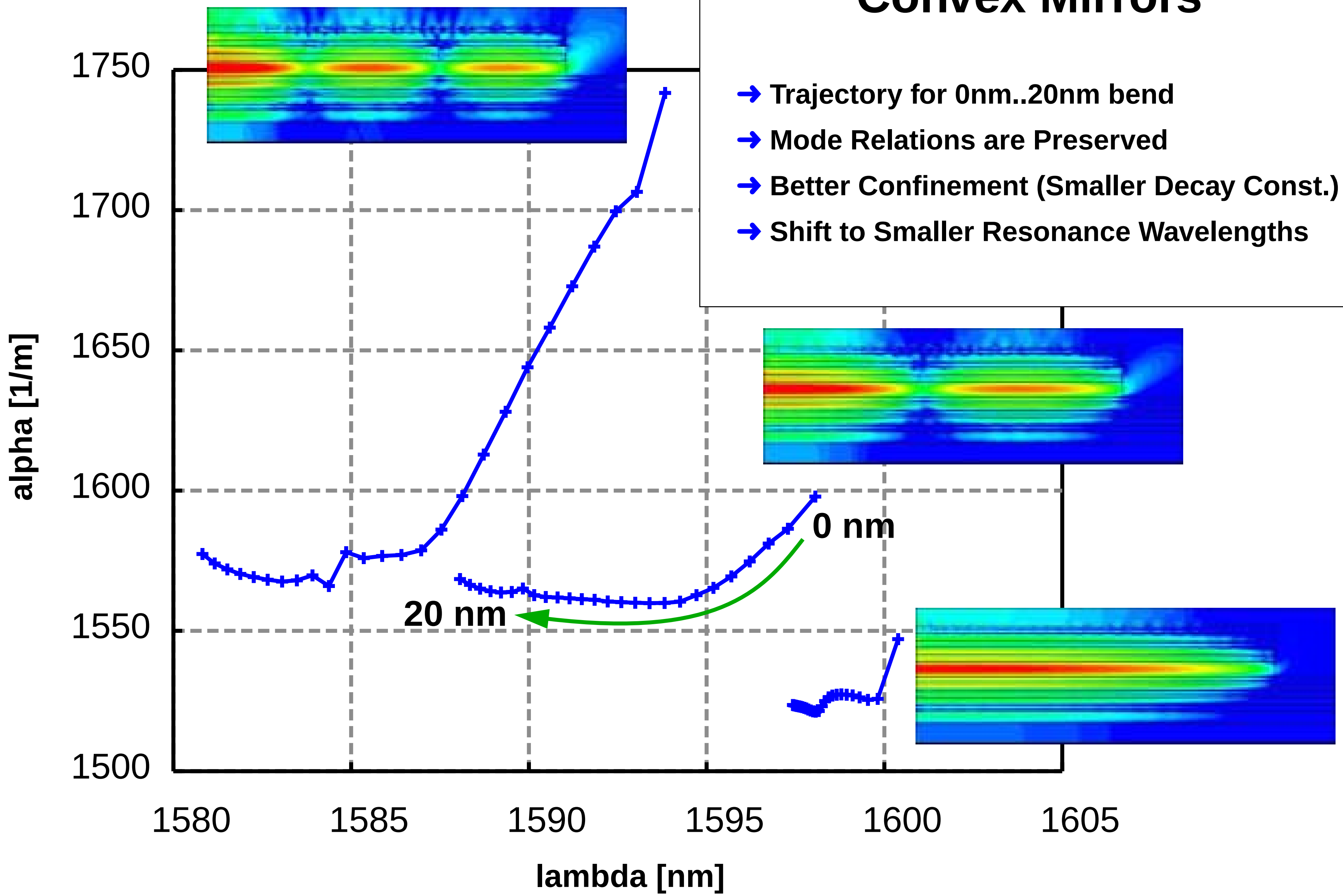
Example II: Tunable Filter

- **Charge Center Layers to Control Air-Gap Distance**
- **Critical Design Issue: Bending of the Layers**
- **Sensitivity Analysis by Simulation**


1..20nm
Y
X
0
2
4
6
8
0
5
10
15
20

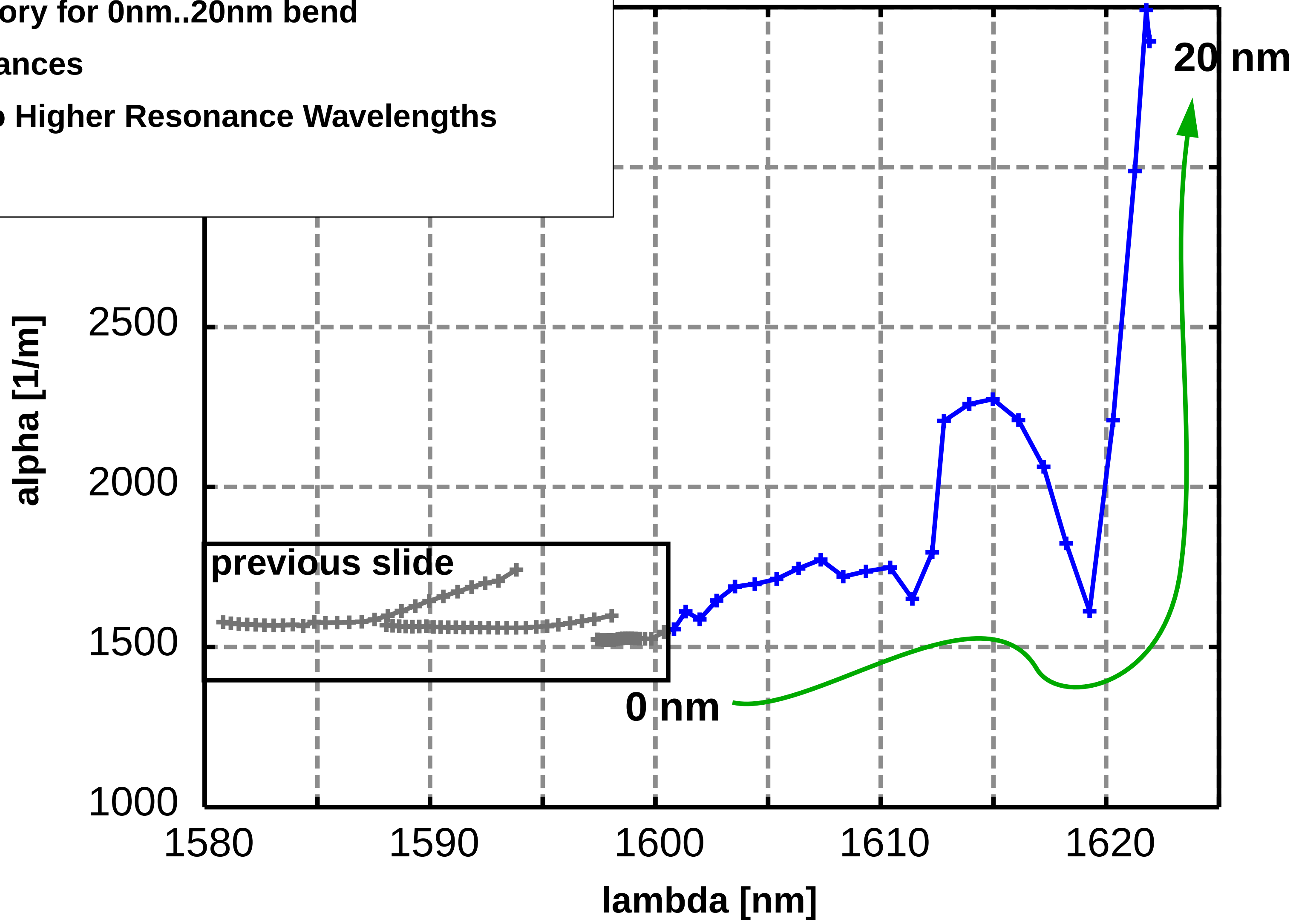
Convex Mirrors

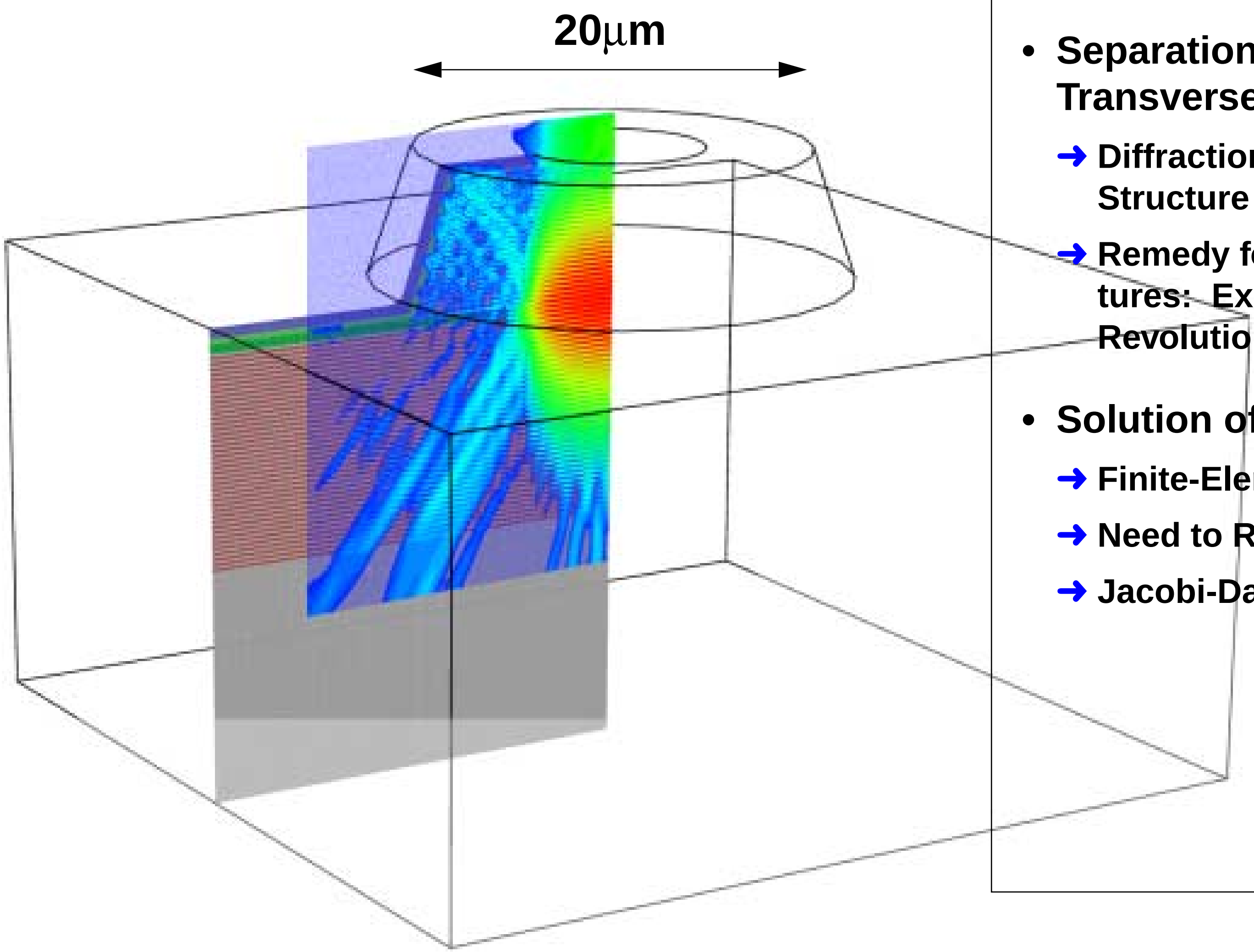
- ➜ Trajectory for 0nm..20nm bend
- ➜ Mode Relations are Preserved
- ➜ Better Confinement (Smaller Decay Const.)
- ➜ Shift to Smaller Resonance Wavelengths



Concave Mirrors

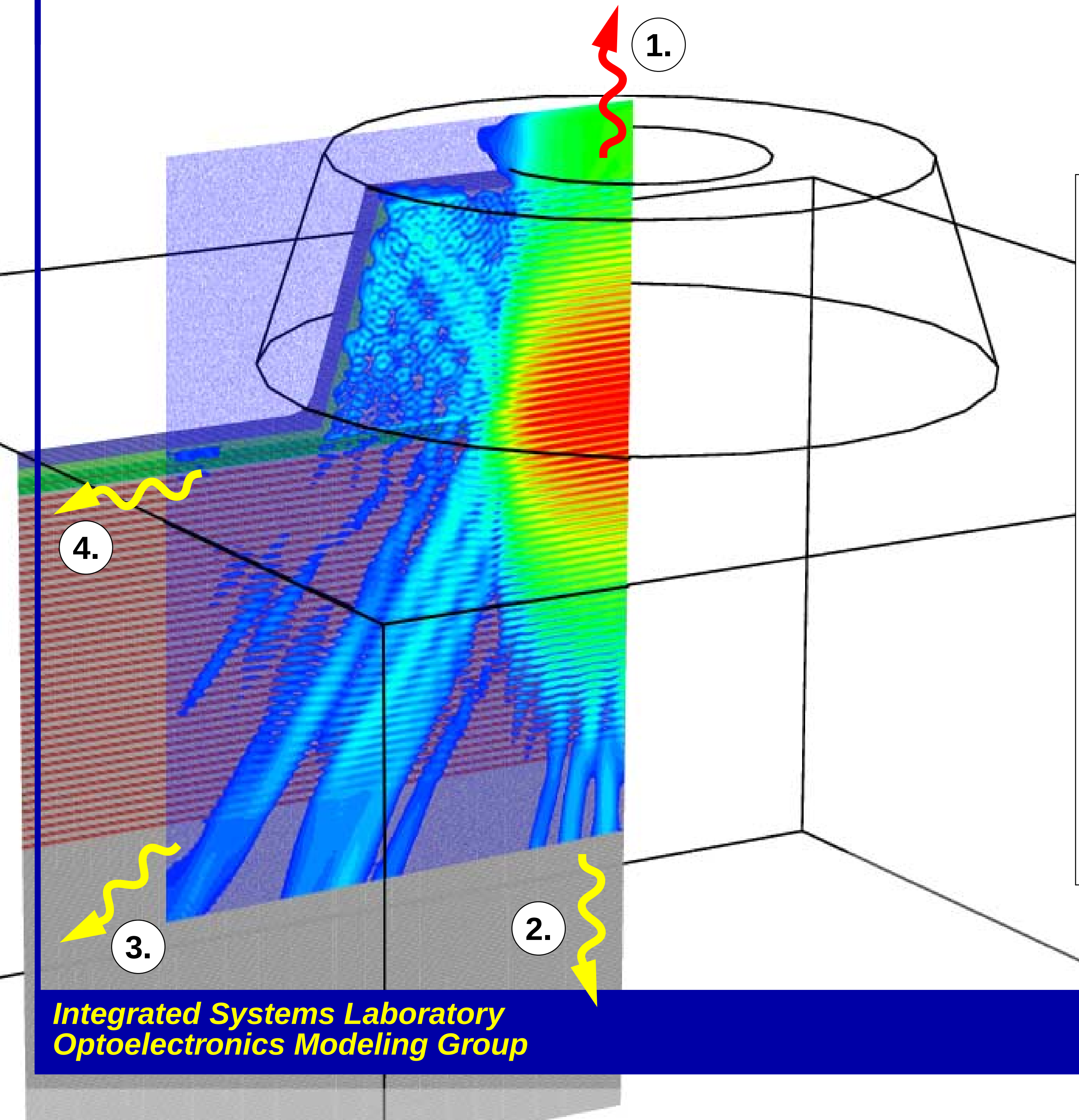
- → Trajectory for 0nm..20nm bend
- → Resonances
- → Shift to Higher Resonance Wavelengths





Example III: VCSEL Simulation

- Separation between Longitudinal and Transverse Direction Impossible
  - ➜ Diffraction at Oxide Confinement or Mesa Structure
  - ➜ Remedy for Rotationally Symmetric Structures: Expand by Fourier Series (Body-of-Revolution)
- Solution of Large Eigenvalue Problem
  - ➜ Finite-Element Discretization
  - ➜ Need to Resolve Bragg Mirrors
  - ➜ Jacobi-Davidson Method

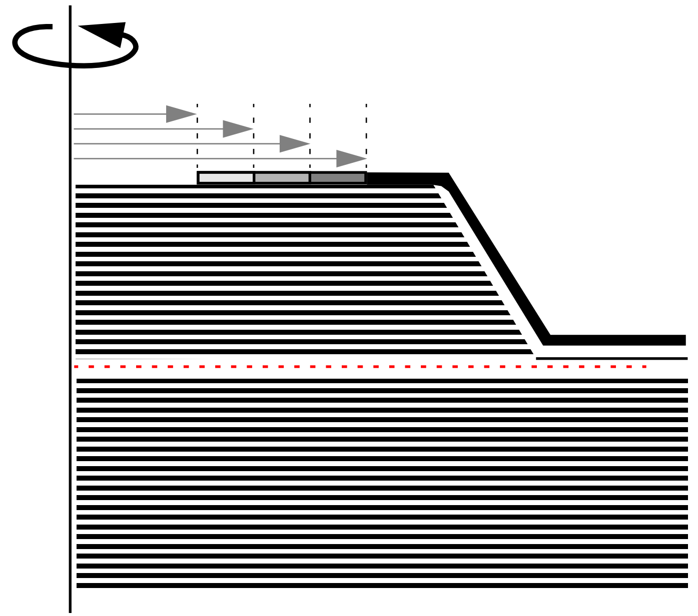


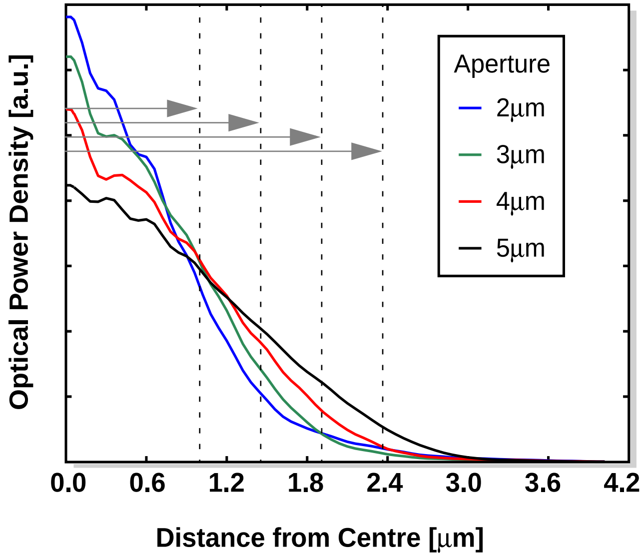
Zoom-In:

- **Standing Waves**
  ➜ **High Field Intensities in the Centre**
- **Traveling Waves**
  1.Laser Output
  2.Bragg Losses
  3.Radiating Waves
  4.Guided Surface Waves
- **Diffraction at the Oxide Confinement**
  ➜ **Important to Calculate Photon Lifetime and Threshold Current**

Typical Simulation Tasks

- **Device Optimization**
  - ➜ Mode Discrimination
  - ➜ High Output Power
  - ➜ Good Fiber Coupling
- **Design of Top Contact**
  - ➜ Aperture Diameter
  - ➜ Material
- **However...**
  - ➜ Results Questionable for Optics-Only Simulation
  - ➜ Need To Simulate Fully-Coupled Opto-Electro-Thermal Physics

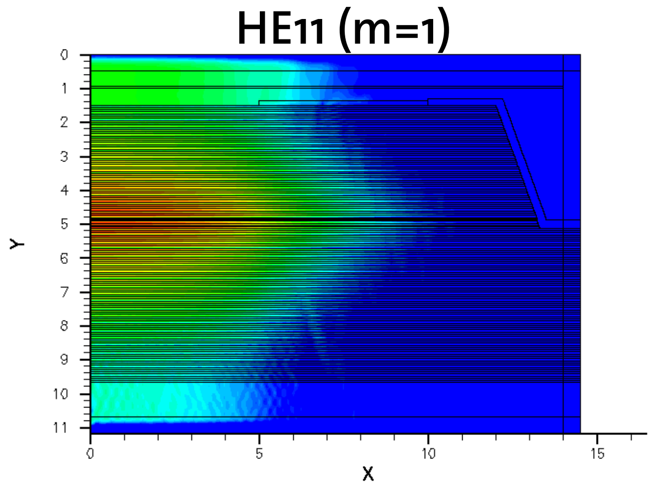


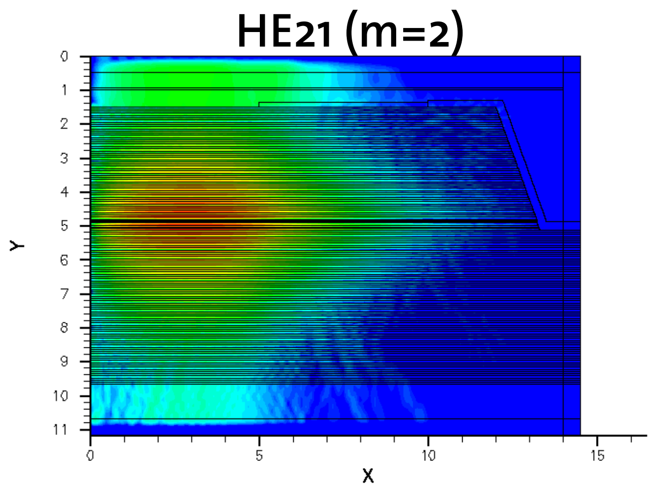


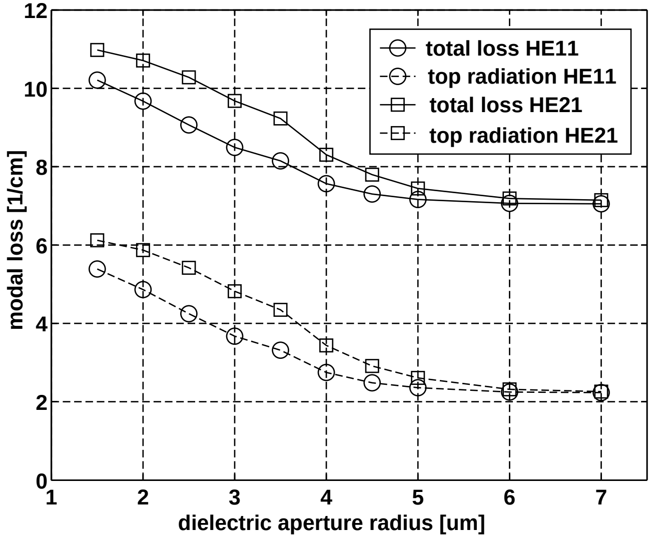
Simulation Results (Optical): Mode Discrimination

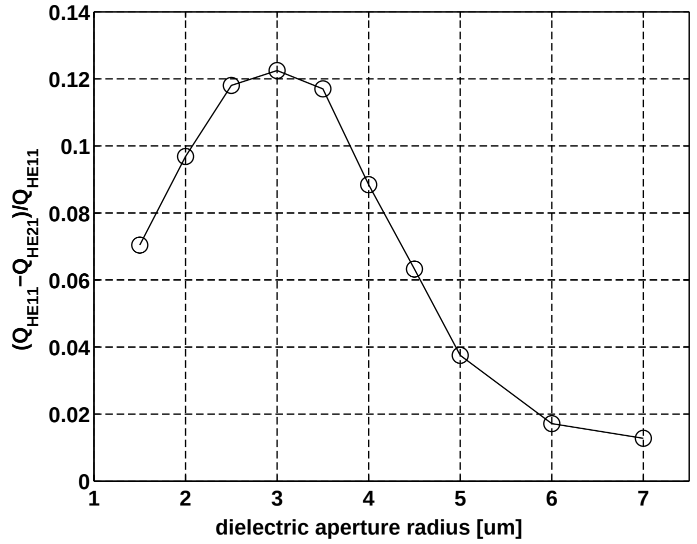
Furukawa Electric 850 nm VCSEL with oxide aperture:
Optimal single-mode performance (cold cavity)

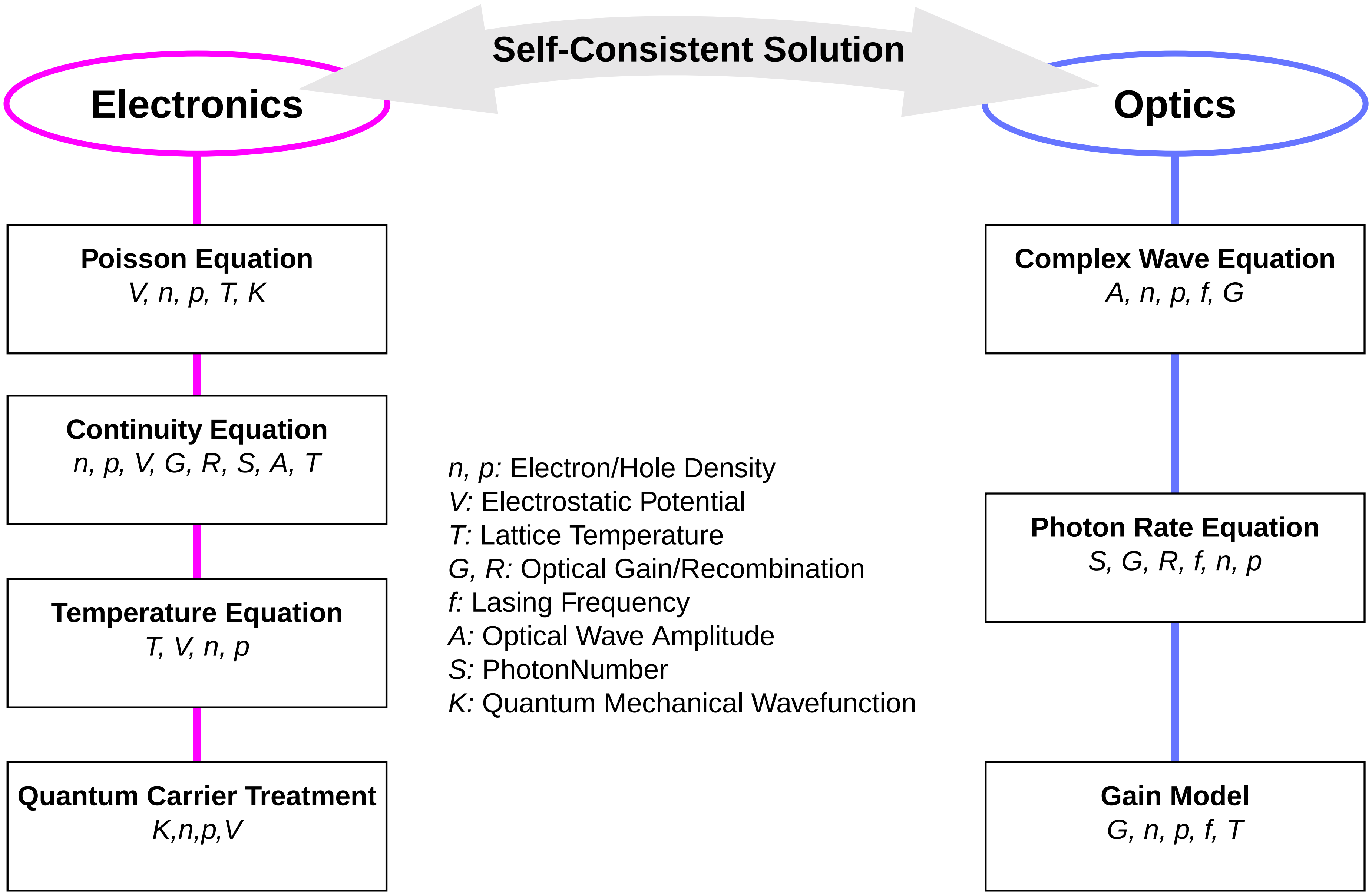
➜ **Adjust dielectric aperture to obtain maximum separation between fundamental HE11 and next higher order HE21 optical mode.**










Self-Consistent Solution
Electronics
Optics
Poisson Equation
V, n, p, T, K
Continuity Equation
n, p, V, G, R, S, A, T
Temperature Equation
T, V, n, p
Quantum Carrier Treatment
K,n,p,V
Complex Wave Equation
A, n, p, f, G
Photon Rate Equation
S, G, R, f, n, p
Gain Model
G, n, p, f, T
n, p: Electron/Hole Density
V: Electrostatic Potential
T: Lattice Temperature
G, R: Optical Gain/Recombination
f: Lasing Frequency
A: Optical Wave Amplitude
S: PhotonNumber
K: Quantum Mechanical Wavefunction

$$\nabla \cdot (\epsilon \nabla \phi) = -q \left(p - n + N_D^+ - N_A^-\right)$$
$$-\nabla \cdot \boldsymbol{S} = H + c_{tot} \partial_t T$$
$$\nabla \cdot \boldsymbol{j}_n = q \left(R + \partial_t n\right)$$
$$-\nabla \cdot \boldsymbol{j}_p = q \left(R + \partial_t p\right)$$

$$\boldsymbol{S} = -\kappa_{th} \nabla T$$
$$\boldsymbol{j}_n = -q \left(\mu_n n \nabla \phi - D_n \nabla n + \mu_n n P_n \nabla T\right)$$
$$\boldsymbol{j}_p = -q \left(\mu_p p \nabla \phi + D_p \nabla p + \mu_p p P_p \nabla T\right)$$

$$\frac{d}{dt} S_\nu(t) = \left(G_\nu - L_\nu\right) S_\nu(t) + R_{sp}$$

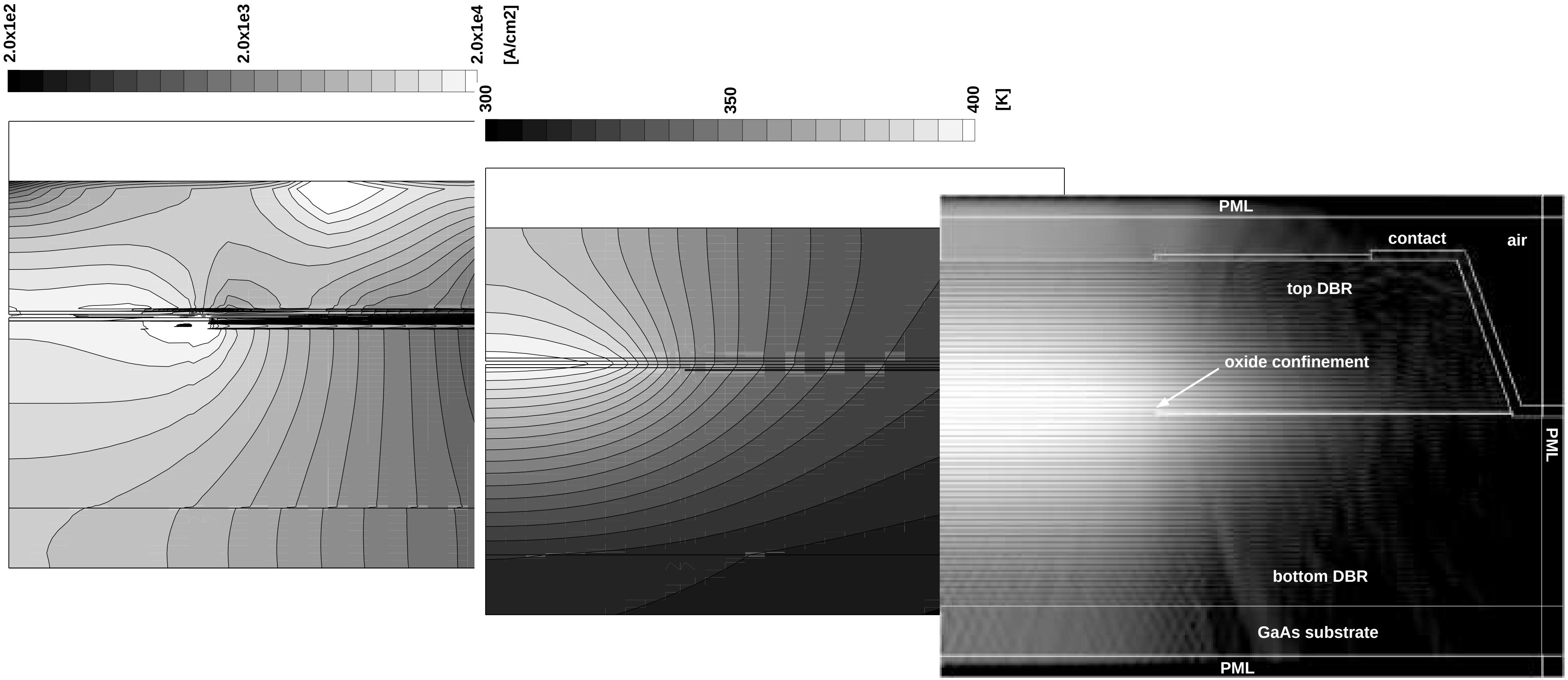
Example III: Opto-Electro-Thermal VCSEL Simulation

- **Separate meshes and interpolate electro-thermal: ~ 10’000 FE optical: ~ 200’000 FE**
- **Newton scheme to solve coupled non-linear set of electro-thermal equations. Parallelised direct solver (PARDISO) to solve linear equations in Newton scheme**
- **Photon Rate Equation Accounts for Optics**
- **Photon Life Time (Imaginary Part of Eigenvalue) as a Variable in the Equation**

Simulation Results (Coupled)

Electro-thermo-optical simulation:

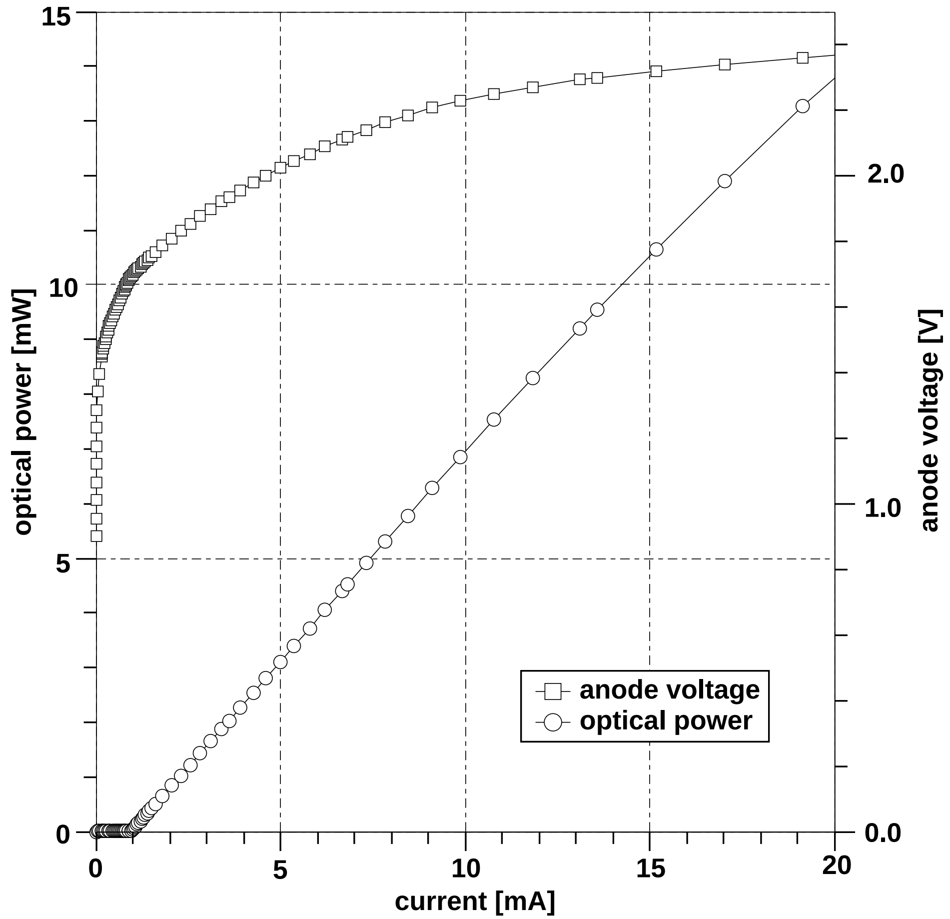
- ➜ **Distribution of current density**
- ➜ **temperature and optical intensity at 16.3 mA.**

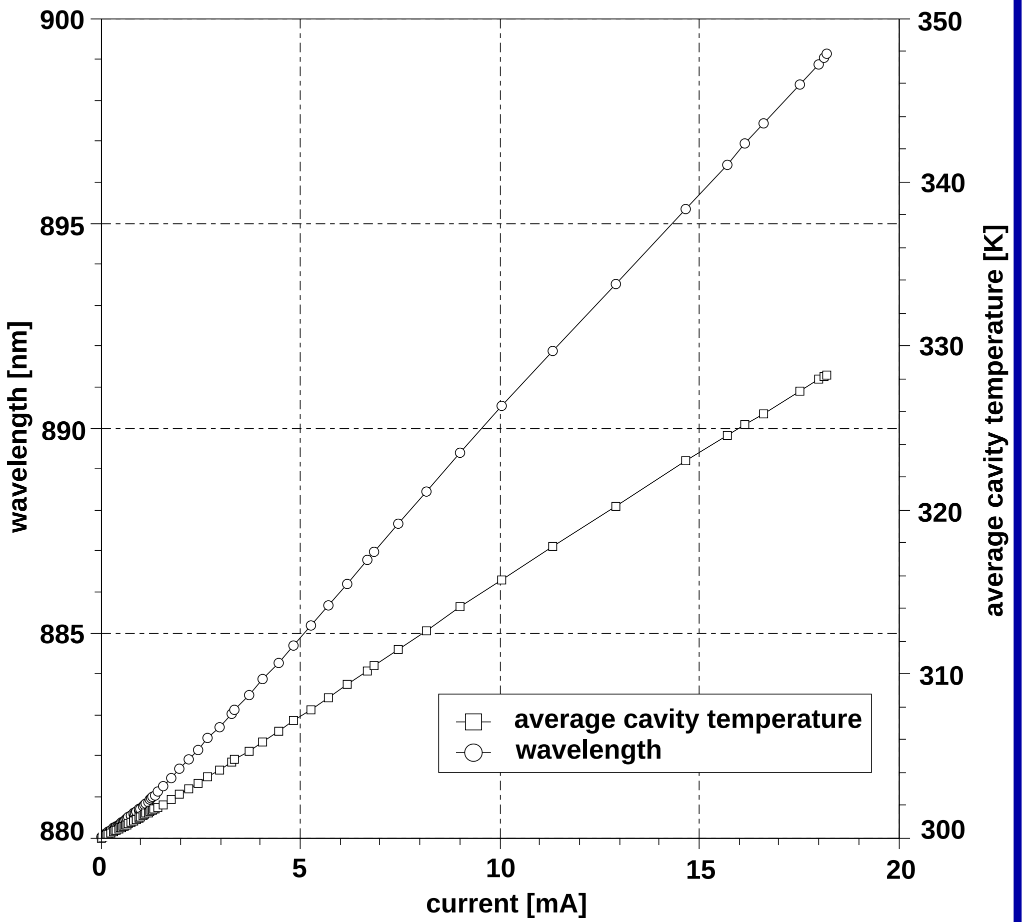


Simulation Results (Coupled)

Electro-thermo-optical simulation:

- ➜ **I/P_opt and I/V curve**
- ➜ **Wavelength tuning of HE11 mode**
- ➜ **average cavity temperature versus laser current.**

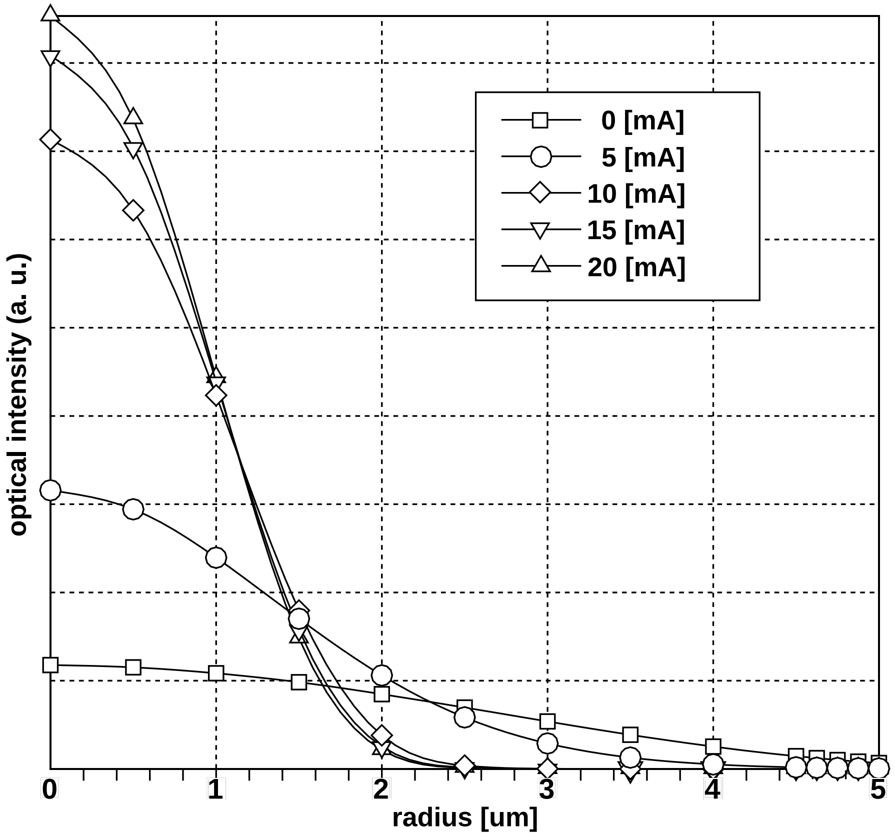


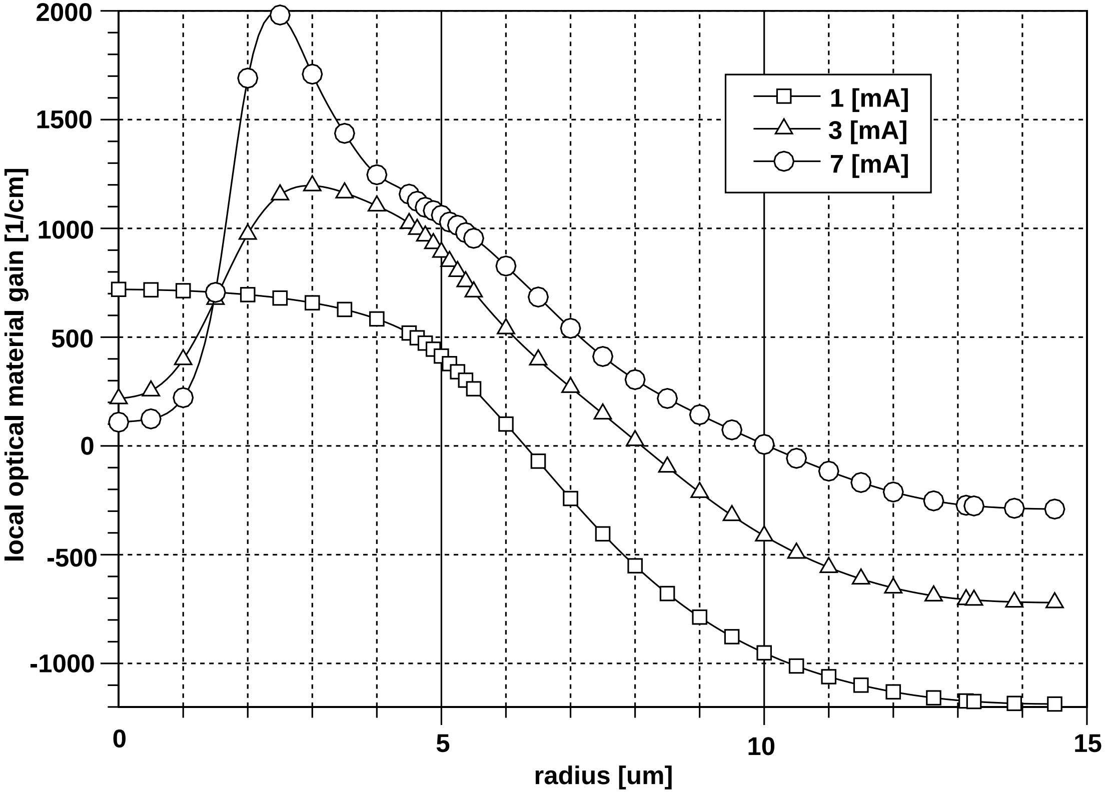


Simulation Results (Coupled)

Electro-thermo-optical simulation:

- ➜ **Normalised optical intensity of HE11 mode in active region versus current (thermal lensing)**
- ➜ **Optical material gain in active region versus current (spatial hole burning).**





Numerical Performance

Simulation Results (Optical):

- # optical vertices: 90’000
- job size: ~ 2 GBytes
- Total CPU Time Usage: ~3 m

Simulation Results (Coupled):

- # optical vertices: 90’000
  # electro-thermal vertices: 7’729
- job size: ~ 2.2 GBytes
- Total CPU Time Usage: ~8 h
  ➜ 98 bias points
  ➜ run time without self-consistent optics: ~5 h

All benchmarks performed on Compaq AlphaServer ES45 1250 MHz (1 CPU)

Summary

- **Solve Helmholtz Equation**
  - ➜ Open Resonator Requires Absorbing Boundaries
  - ➜ Non-Hermitian Problem, Complex Eigenvalue
  - ➜ Discretization: Linear vs. Quadratic
- **Spectral Portrait**
  - ➜ Difficult to Find Lasing Modes Automatically
  - ➜ Track Modes for Perturbed Problem
- **Applications**
  - ➜ Tunable Air-Gap Filter
  - ➜ Coupled Opto-Electro-Thermal VCSEL Simulation

Conclusions / Outlook

- Eigenmode Analysis of Microcavity Lasers
  - ➜ Perfectly Matched Layer (PML) Boundary Condition is a Good Choice for Absorbing Boundary
  - ➜ Need Full-Wave Solution to Account for Diffraction Losses and Calculate Photon Life Time Accurately
  - ➜ Trade-Off Between Accuracy and Problem Size
- Improve Preconditioner
  - ➜ Possibility: First Order Elements for Preconditioner and Second Order Elements for Jacobi Correction Equation (Requires Hierarchical Finite Elements)

Acknowledgements

- **Oscar Chinellato, Peter Arbenz**
  Institute of Scientific Computing, ETH Zurich

  University of Kassel
- **Avalon Photonics**
  Hardware Partner
- **ISE Integrated Systems Engineering**
  Software Partner
- **Swiss Funding Agency**
  ➜ **Commission for Technology and Innovation (CTI)**
  ➜ **TOP NANO21 Programme.**

AVALON PHOTONICS

ISE TCAD

KTI CTI