Bachelor Thesis/Term Project

(Mathematics, Computational Science & Engineering)

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Non-equidistant approximate DFT based on Z-splines

Prerequisites. Basic calculus and numerical methods, in particular FFT [1].

Problem description. We are interested in the fast approximate evaluation of 1-periodic trigonometric polynomials [2]

$$p(t) = \sum_{j=-m}^{m} \gamma_j e^{-2\pi ijt} , \quad \gamma_j \in \mathbb{C} , \tag{1}$$

at arbtrary nodes $t_k \in [0, 1[$, $k = 0, \ldots, 2m$. In the case of equidistant nodes $t_k = e^{2\pi ik/(2m+1)}$ the computation of $p(t_k)$ can be done with $O(m \log m)$ operations using the famous FFT algorithm.

An approach to arbitrarily distributed t_k is outlined in [4]: the trigonometric polynomial is approximated by a function

$$q(t) = \sum_{j=-N}^{N} \kappa_j \varphi(t - \frac{j}{2N+1}) , \quad \kappa_j \in \mathbb{C} , \tag{2}$$

where φ is a compactly supported function that is concentrated around zero in frequency domain. Then FFT can be employed for the fast computation of the values $q(t_k)$ [4, Sect. 2].

Such functions are provided by the so-called Z-splines [3], which are compactly supported piecewise polynomials that come closer and closer to a sinc-function, when the polynomial degree is raised.

Issues. Accuracy of the approximations of the values $p(t_k)$ depending on the choice of N and polynomial degree of Z-splines.

Task. MATLAB implementation of the evaluation algorithm sketched above and numerical studies of its accuracy and performance.

References

[1] P. Duhamel and M. Vetterli, *Fast fourier transforms: a tutorial review and a state of the art*, Signal Processing, 19 (1990), pp. 259–299.

[2] R. Hiptmair, *Numerische mathematik für studiengang rechnergestützte wissenschaften.* Lecture Slides, 2005. http://www.sam.math.ethz.ch/˜hiptmair/tmp/NCSE.pdf.

[3] J. Sagredo, *Z-splines: Moment conserving cardinal spline interpolation of compact support for arbitrary interval data*, report, SAM, ETH Zürich, Zürich Switzerland, 2003.

[4] G. Steidl, *A note on fast Fourier transforms for nonequispaced grids*, Advances in Comp. Math., 9 (1998), pp. 337–352.