Transient Simulation of Eddy Currents in Ferromagnets

R. Hiptmair*

October 29, 2003

1 Description of the Problem

The eddy current model comprises the equations

$$\mathbf{curl}\,\mathbf{A} = \mathbf{B} \quad , \quad \mathbf{curl}\,\mathbf{H} = -\sigma\frac{d}{dt}\mathbf{A} + \mathbf{j}_0 \ ,$$

where $\mathbf{A}$ is the magnetic vector potential, $\mathbf{B}$ the magnetic induction, $\mathbf{H}$ the magnetic field, and $\mathbf{j}_0$ some (formal) exciting current. Temporal gauging is assumed for $\mathbf{A}$.

These equations are posed on a computational domain $\Omega \subset \mathbb{R}^3$ and, for the sake of simplicity, Ω is assumed to be bounded, and the scalar *conductivity* σ is to be uniformly positive almost everywhere. It can even be set to a positive constant σ_0.

Generically, the boundary value problem is posed on the whole space $\mathbb{R}^3$ with decay conditions for $\mathbf{A}$ and $\mathbf{H}$. However, since we restrict ourselves to bounded Ω, we have to impose boundary conditions. These can be of "PEC-type"

$$\mathbf{A} \times \mathbf{n} = 0 \quad \text{on } \partial\Omega \ ,$$

or of "PMC-type"

$$\mathbf{H} \times \mathbf{n} = 0 \quad \text{on } \partial\Omega \ .$$

The magnetic material properties of a soft ferromagnet are characterized by the relationship between $\mathbf{B}$ and $\mathbf{H}$, which are linked by a material law. A good approximation is

$$\mathbf{H}(\mathbf{B}) = \beta(|\mathbf{B}|)\frac{\mathbf{H}}{|\mathbf{H}|} \quad , \quad \beta(\xi) = \begin{cases} 0 & \text{if } \xi < B_0 \ , \\ \mu_0^{-1}(\xi - B_0) & \text{if } \xi \geq B_0 \ . \end{cases} \tag{1}$$

We can carry out different kinds of eliminations: getting rid of $\mathbf{H}$ leads to

$$\mathbf{curl}\,\mathbf{H}(\mathbf{curl}\,\mathbf{A}) = -\sigma\frac{d}{dt}\mathbf{A} + \mathbf{j}_0 \ , \tag{2}$$

*SAM, ETH Zürich, CH-8092 Zürich, hiptmair@sam.math.ethz.ch

whereas a formulation in terms of $\mathbf{H}$ reads

$$\mathbf{curl}\,\sigma^{-1}\,\mathbf{curl}\,\mathbf{H} = -\frac{d}{dt}\mathbf{B}(\mathbf{H}) + \mathbf{curl}(\sigma^{-1}\mathbf{j}) \ . \tag{3}$$

In (3) the dependence $\mathbf{B} = \mathbf{B}(\mathbf{H})$ is a formal one, because the material law (1) does not establish a bijective relationship between $\mathbf{B}$ and $\mathbf{H}$.

2 Connection with Phase Change

The classical Stefan problem describes the melting of ice. Denote by T the temperature and H the enthalpy and assume Fourier's law

$$\mathbf{j} = -\kappa\,\mathbf{grad}\,T \ ,$$

where $\mathbf{j}$ is the heat flux. At $T = 0$ ice melts and for a while the enthalpy will increase without further change in temperature, which means

$$T(H) = \begin{cases} \rho(H + H_0) & \text{for } H < -H_0 \ , \\ 0 & \text{for } -H_0 < H < H_0 \ , \\ \rho(H - H_0) & \text{for } H > H_0 \ , \end{cases}$$

with a constant heat capacity $\rho > 0$.

Fourier's law has to be supplemented by the conservation of energy

$$\operatorname{div}\mathbf{j} = -\frac{d}{dt}H + f \ ,$$

where f stands for a heat source. Elimination of $\mathbf{j}$ leads to the equation

$$\frac{d}{dt}H - \operatorname{div}(\kappa\,\mathbf{grad}\,T) = f \ , \tag{4}$$

which resembles (3).

3 Timestepping Schemes

References

[1] W. Jäger and J. Kacur, *Solution of porous medium type systems by linear approximation schemes*, Numer. Math., 60 (1991), pp. 407–427.

[2] ——, *Solution of doubly nonlinear and degenerate parabolic problems by relaxation schemes*, RAIRO, Modlisation Math. Anal. Numér., 29 (1995), pp. 605–627.

[3] J. Kacur, *Solution of some free boundary problems by relaxation schemes*, SIAM J. Numer. Anal., 36 (1998), pp. 290–316.

[4] ——, *Solution to strongly nonlinear parabolic problems by a linear approximation scheme*, IMA J. Numer. Anal., 19 (1999), pp. 119–145.