Bachelor Thesis Project/ Term Project

(Mathematics, Computational Science & Engineering)

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Space-time Discontinuous Galerkin Methods

1 Space-time discontinuous Galerkin

Many variants of so-called discontinuous Galerkin methods (DG) have been developed for 2nd-order elliptic boundary value problems, for instance for

$$-\Delta u = f \quad \text{in } \Omega \subset \mathbb{R}^2 \quad , \quad u = 0 \quad \text{on } \partial\Omega \, . \tag{1}$$

The discontinuous Galerkin methods can be derived by consistent penalization of the associated Lagrangian functional [3], or by applying so-called numerical fluxes to a first-order system equivalent to (1), see [1]. It assumes that Ω is equipped with a mesh.

Now, we consider a space-time cylinder $\Omega \subset \mathbb{R}^2$, for instance $\Omega =]0,1[\times]0,T[$. By flipping the sign of one partial derivative in Δ we obtain the wave operator and the corresponding boundary value problem

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2}{\partial x^2 u} = f(x,t) \quad \text{in } \Omega \, , \tag{2}$$

$$u(0,t) = u(1,t) = 0 \quad \forall 0 \leq t \leq 1 \, , \quad u(x,0) = u_0(x), \; \frac{\partial u}{\partial t}(x,0) = v_0(x) \, , \quad 0 \leq x \leq 1 \, . \tag{3}$$

Formally at least, we can apply the manipulation leading to a DG scheme for (1) also to (2). To do so, it is natural to use a space-time tensor product mesh.

This will lead to discrete evolutions linking space-time moments of u on space-time mesh cells. These relationships will be local and yield "generalized 5-point stencils", see Fig. 1.

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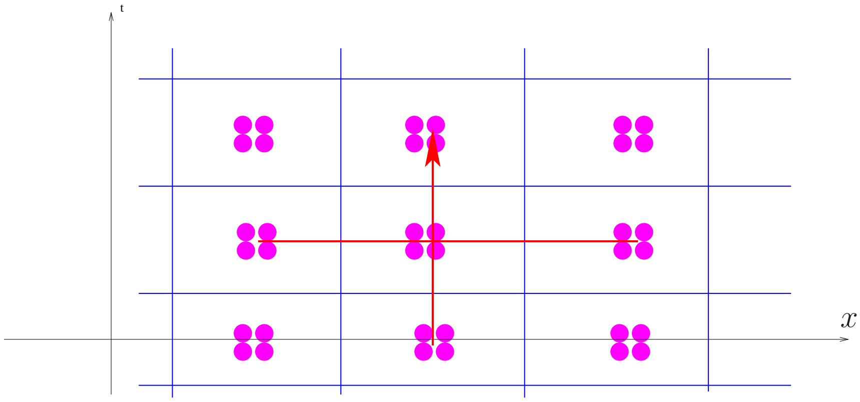

Figure 1: Space-time 5-point stencil, p.w. bilinear trial/test space

2 Tasks

1. Derivation of interior penalty DG equations for (2)
2. Investigation of stability using linear stability analysis [2, Sect. 1.7.2] for different values of stabilization parameter.
3. Numerical experiments to study the convergence of the method, if conditional stability can be achieved.

References

[1] D. Arnold, F. Brezzi, B. Cockburn, and L. Marini, *Unified analysis of discontinuous Galerkin methods for elliptic problems*, SIAM J. Numer. Anal., 39 (2002), pp. 1749–1779.

[2] R. Hiptmair, *Numerics of hyperbolic partial differential equations*. Online lecture notes, 2007. http://www.sam.math.ethz.ch/ hiptmair/tmp/NUMHYP_07.pdf.

[3] R. Hiptmair and C. Schwab, *Numerics of elliptic and parabolic boundary value problems.* Lecture slides. Available at http://www.sam.math.ethz.ch/˜hiptmair/NAPDE_06.pdf, March 2006.