Master Thesis Project

(Applied Mathematics)

Supervisor: Prof. Dr. R. Hiptmair (SAM, D-MATH)

Interface Approximation and Finite Element Error Estimates

Prerequisites. Knowledge about the finite element discretization of 2nd-order elliptic boundary value problems. The techniques taught in the course about the numerical treatment of elliptic and parabolic PDEs should be mastered.

Problem description. On a polygonal domain $\Omega \subset \mathbb{R}^2$ we consider the 2nd-order elliptic boundary value problem

$$-\operatorname{div}(\sigma(\boldsymbol{x})\,\mathbf{grad}\,u) = f \quad \text{in } \Omega\,, \quad u = 0 \quad \text{on } \Omega\,. \tag{1}$$

The conductivity σ is assumed to be piecewise constant with respect to a partition $\overline{\Omega} = \overline{\Omega}_1 \cup \overline{\Omega}_2$ of the computational domain, see Fig. 1. The interface between the two subdomains is supposed to be *smooth*.

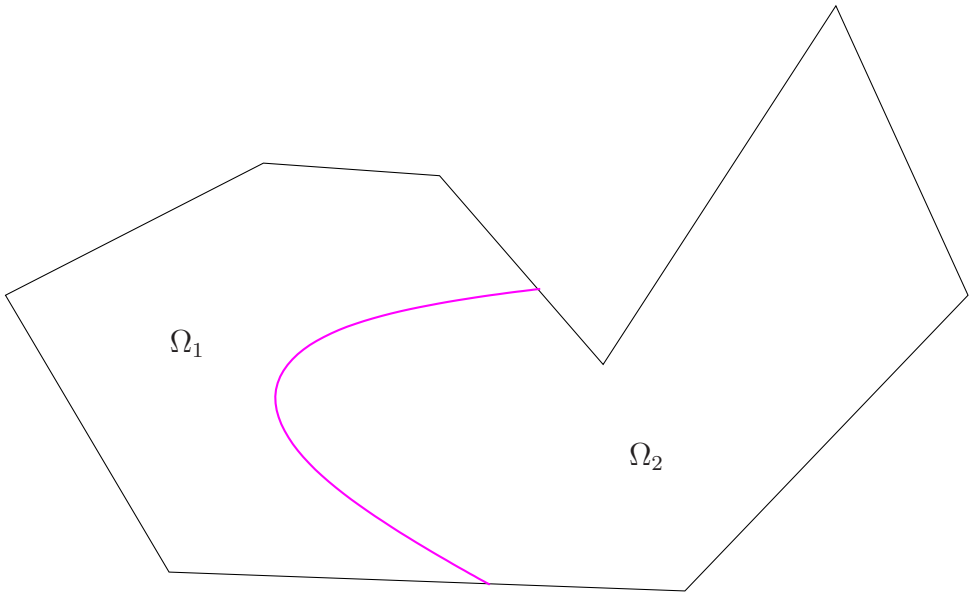

Figure 1: Partitioning of the domain Ω

However, in the context of a finite element approximation of u, one usually employs some approximation of the interface. The concrete setting is that of triangular piecewise linear Lagrangian finite elements with piecewise linear approximation of the interface. The value of σ is assumed to be constant in each triangle.

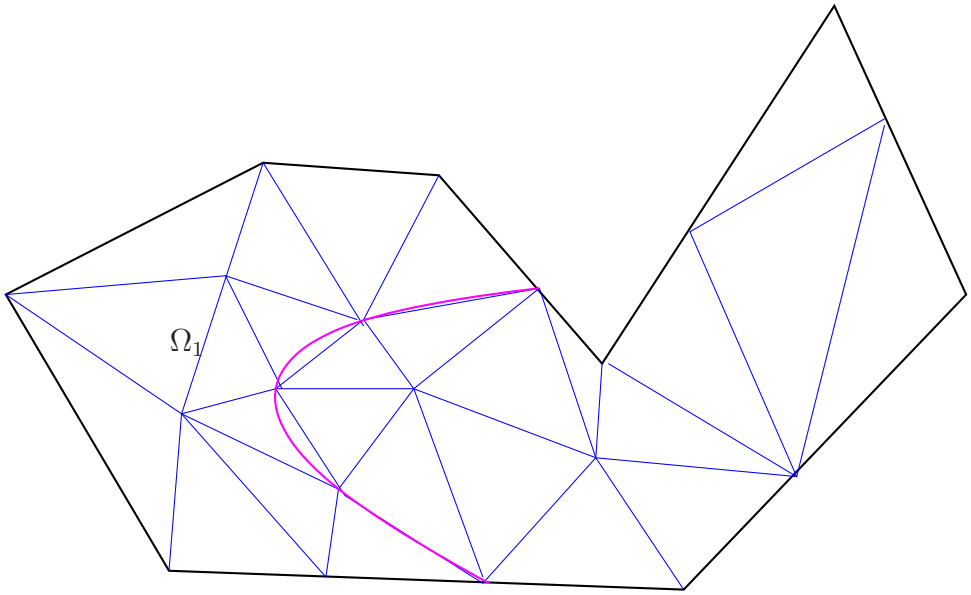

Figure 2: Finite element mesh

Issues. Proof of convergence in the presence of interface approximation.

Tasks. Use mapping techniques to derive a priori error estimates for the the linear finite element solution in L^2-norm and the energy norm. By mapping techniques the problem can be cast into a setting with exact interface approximation and perturbed conductivity. The 3D situation should also be considered.

Contact: Prof. Dr. Ralf Hiptmair
Seminar for Applied Mathematics, D-MATH
Room : HG G 58.2
: 01 632 3404
: hiptmair@sam.math.ethz.ch
: http://www.sam.math.ethz.ch/~hiptmair

.

References

[1] S. Brenner and R. Scott, *Mathematical theory of finite element methods*, Texts in Applied Mathematics, Springer–Verlag, New York, 2nd ed., 2002.

[2] Z.-M. Chen and J. Zou, *Finite element methods and their convergence for elliptic and parabolic interface problems*, Numer. Math., 79 (1998), pp. 175–202.