Diploma Project/Diplomarbeit

(Mathematics/Computational Science & Engineering)

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Impedance Boundary Conditions in Time Domain

Field. Coputational electromagnetism, numerical methods for parabolic PDE, numerical convolution, absorbing boundary conditions, *software development*

Problem. Alternating electromagnetic fields decay exponentially when penetrating a good conductor (*skin effect*). Therefore, a reasonable approximation of the electromagnetic Dirichlet-to-Neumann map of the interior of a good conductor is provided by the impedance boundary conditions

$$\mathbf{E_t} = \tfrac{1}{2}\sqrt{2}(1-i)\sqrt{\frac{\mu}{\sigma\omega}}\,(\mathbf{H}\times\mathbf{n})\ , \tag{1}$$

where $\omega > 0$ is a fixed angular frequency, characterizing the temporal variation of all electromagnetic fields. The conductivity σ and permeability μ are known material parameters.

Issues. The relationship (1) is valid in the frequency domain only. However, often sinusoidal temporal variation of the fields cannot be assumed, which forces us to resort to time domain methods. However, when formulation impedance boundary conditions in the time domain, we encounter temporal convolutions of the form

$$\mathbf{E_t}(\mathbf{x},t) = \int\limits_{t_0}^{t} k(\mu,\sigma,\tau-t)(\mathbf{H}\times\mathbf{n})(\mathbf{x},\tau)\,\mathrm{d}\tau\ . \tag{2}$$

In words, the boundary conditions become *non-local* in time. This renders a straightforware discretization of (2) prohibitively expensive, if many timesteps are to be carried out.

Simple model problem. Instead of the eddy current problem one can consider the second-order parabolic problem

$$\frac{\partial}{\partial t}(\sigma u) - \Delta u = f(\mathbf{x},t) \quad \text{in } \Omega\times]0;T[\ ,$$
$$u = 0 \quad \text{on } \partial\Omega\ ,$$

where

$$c(\mathbf{x}) = \begin{cases} c_0 \gg 1 & \text{in } \Omega_0 \subset\subset \Omega\ , \\ 0 & \text{in } \Omega\setminus\Omega_0\ . \end{cases}$$

The subdomain is assumed to have a smooth boundary. If σ_0 is very large, impedance boundary conditions on $\partial\Omega_0$ can provide a good approximation. Again, they involve a convolution in time.

Approaches. For the efficient approximation numerical evaluation of convolutions, for which the Laplace-transform of the convolution kernel is known, Lubich [1, 2] has developed a scheme for *fast convolution quadrature*. It involves the use of the trapezoidal rule for the approximate inversion of the Laplace-transform and thus reduces (2) to the parallel solution of a number of ODEs.

Task. A convolution formulation for the impedance boundary conditions for electromagnetic fields should be devised. Then, the fast convolution quadrature should be applied to it and the stability of the resulting scheme in the context of implicit timestepping should be investigated. The method should be implemented in the framework of an existing boundary element code developed by J. Ostrowski [3].

References

[1] C. Lubich, *Convolution quadrature and discretized operational calculus. I*, Numer. Math., 52 (1988), pp. 129–145.

[2] ——, *Convolution quadrature and discretized operational calculus. II*, Numer. Math., 52 (1988), pp. 413–425.

[3] J. Ostrowski, *Boundary Element Methods for Inductive Hardening*, PhD thesis, Fakultät für Mathematik und Physik, Tübingen, November 2002.

[4] A. Schädle, *Ein schneller Faltungsalgorithmus für nichtreflektierende Randbedingungen*, dissertation, Mathematisches Institut, Universität Tübingen, Tübingen, Germany, June 2002.

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