Master Thesis Project

(Mathematics/Computational Science & Engineering)

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Resolution of Skin Layers in EM Simulation

Project to be conducted at ABB Corporate Research Center, Baden-Dätwil

Introduction. The numerical computation of trelectromagnetic fields in the presence of highly conducting materials like copper encounters the *skin effect*. This involves the emergence of exponential boundary layers in the field solution at metal surfaces. Standard finite element discretization entails resolving these boundary layers leading to a prohibitive number of elements. A remedy may be offered by generalized finite element approaches incorporating information about the behavior of the solution into the trial space.

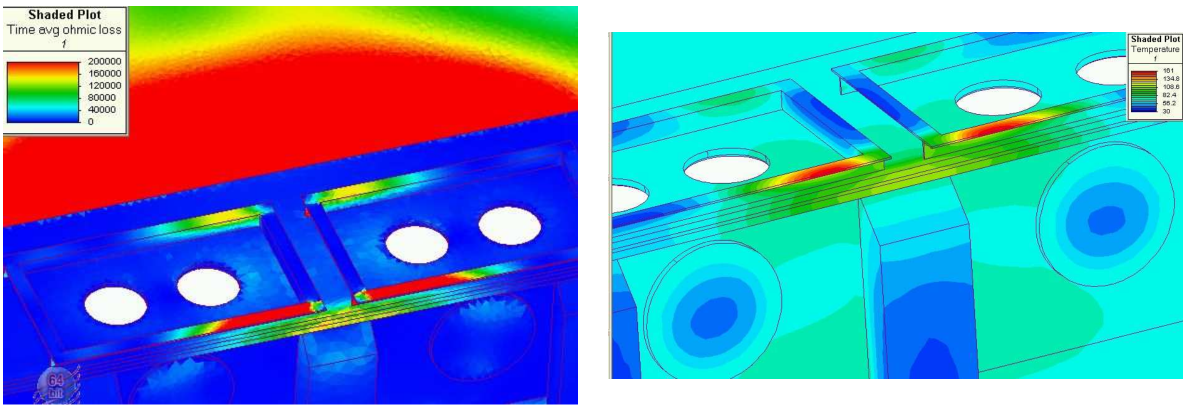

Figure 1: Ohmic losses (left) and temperature distribution (right) from an eddy current simulation displaying strong skin effect. Computations done at ABB corporate research Baden.

Model Problem. A simple scalar model problem arises from dimensional reduction based on translational symmetry. It yields the boundary value problem

$$-\Delta u + \sigma(\mathbf{x})u = 0 \quad \text{in } \Omega \quad , \quad \frac{\partial u}{\partial \mathbf{n}} - u = g \quad \text{on } \partial\Omega \; ,$$

posed on a domain $\Omega \subset \mathbb{R}^2$ with $\sigma(\mathbf{x}) > \sigma_0 > 0$ a.e. in Ω and $g \in H^{-\frac{1}{2}}(\partial\Omega)$. If $\sigma \gg 1$, a boundary layer will emerge, that is, the solution will exponentially decay away from $\partial\Omega$.

Boundary layers are hard to capture on finite element meshes produced by conventional mesh generators. A potential remedy is to use anisotropic meshes alined with the boundary, but often these are not available.

Approach. The general policy is to incorporate information about the exponential decay of the solution into the trial space on which a Galerkin discretization is built. This can be done by using *Trefftz-type basis functions*, that is, basis functions that provide solutions of the homogeneous equations

$$-\Delta u + \sigma_K u = 0 \; , \tag{1}$$

on each cell K of a mesh. Possible choices of such functions are

1. solutions of (1) with variation in one direction only ("plane wave solutions")
2. the images of harmonic polynomials under a Vekua transform associated with the differential operator of (1) ("generalized harmonic polynomials").

Unfortunately, blending such basis functions into a space of globally continuous functions is all but impossible. Therefore, the framework of *discontinuous Galerkin methods* (DG) seems attractive, because it dispenses with any continuity requirements for the local trial spaces.

Tasks.

- Derivation of suitable sets of local basis functions
- Study of the concept and implementation of DG methods
- Implementation of the Trefftz DG schemes for the 2D model problem (MATLAB or C++ based on a suitable finite element code)
- Numerical experiments to explore convergence and robustness.

Continuation. The project can be seminal research for a PhD thesis.