Micromagnetism

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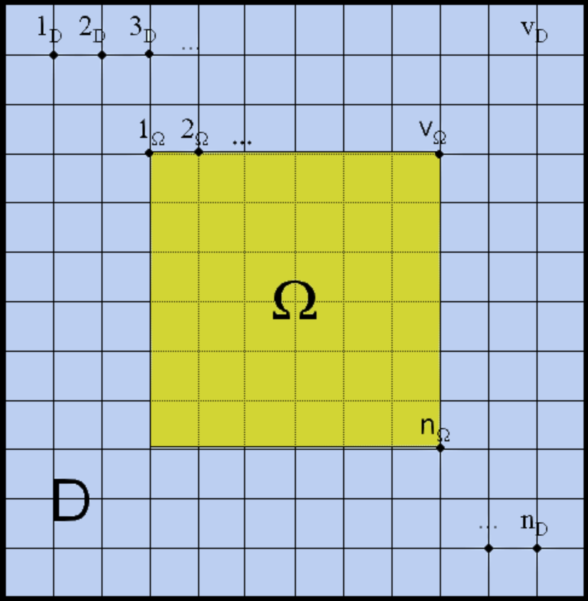
- Results



Equations

Landau-Livshits-Gilbert equation:

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{\mathrm{eff}} - \frac{\gamma\alpha}{M_s}\mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\mathrm{eff}}) \ .$$

Effective magnetic field:

$$\mathbf{H}_{\mathrm{eff}} = -\frac{\partial \mathcal{E}(\mathbf{m})}{\partial \mathbf{m}}$$

Equations

Gibbs free energy:

$$\mathcal{E}(\mathbf{M}) \quad = \quad \int_{\Omega} \frac{\bar{A}}{M_s^2} |\,\mathbf{grad}\,\mathbf{M}|^2 + \Phi(\mathbf{M}/M_s) - 2\mu_0 \mathbf{H}_e \cdot \mathbf{M}\, d\mathbf{x} + \frac{1}{2}\mu_0 \int_{\mathbb{R}^3} |\,\mathbf{grad}\,\psi|^2\, d\mathbf{x}\ .$$

Potential equation:

$$\mu_0 \Delta \psi = \operatorname{div} \mathbf{M} \quad \text{in} \mathbb{R}^3$$

$\mathbf{M}$ satisfies Neumann boundary conditions

Discretization: Time

Timestepping:

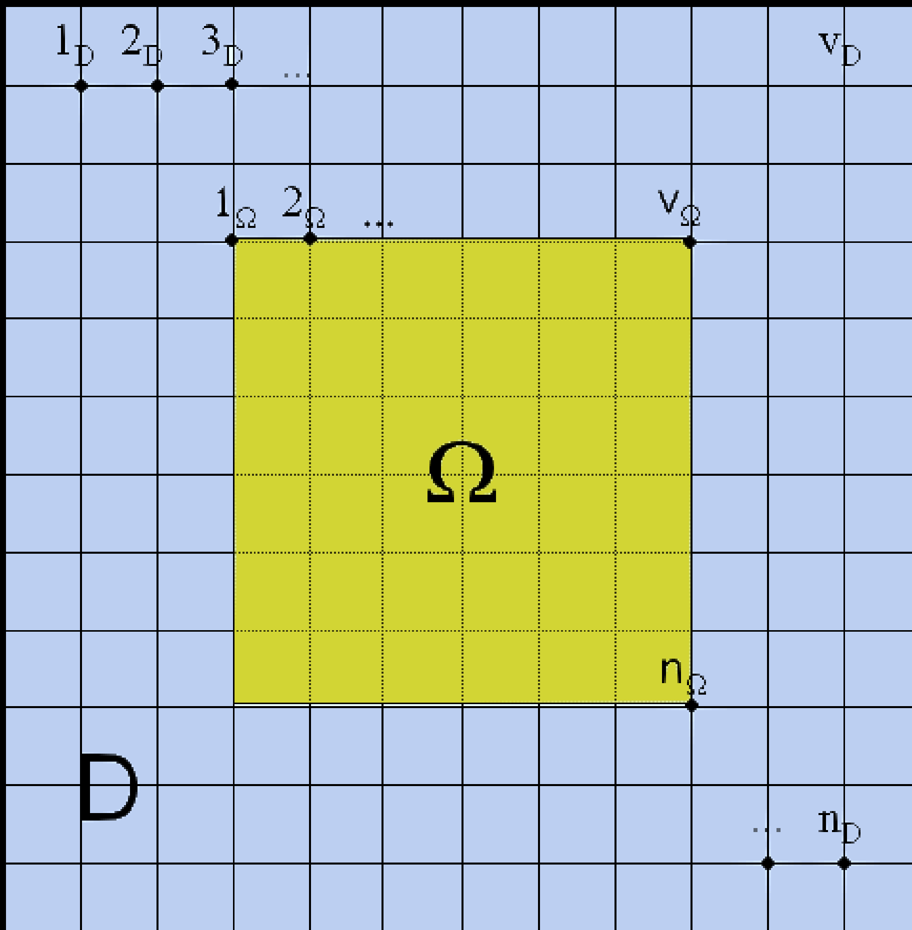
$$\delta \mathbf{m}^{n+\frac{1}{2}} = -\overline{\mathbf{m}}^{n+\frac{1}{2}} \times \mathbf{h}_{\text{eff}}^{n+\frac{1}{2}} - \alpha\, \overline{\mathbf{m}}^{n+\frac{1}{2}} \times (\overline{\mathbf{m}}^{n+\frac{1}{2}} \times \mathbf{h}_{\text{eff}}^{n+\frac{1}{2}})$$

$$\mathbf{h}_{\text{eff}}^{n+\frac{1}{2}} := \eta \Delta \overline{\mathbf{m}}^{n+\frac{1}{2}} + Q\mathbf{d}\,(\mathbf{d} \cdot \overline{\mathbf{m}}^{n+\frac{1}{2}}) - \mathbf{grad}\,\overline{\psi}^{n+\frac{1}{2}} + \mathbf{h}_e(t_{n+\frac{1}{2}})$$

with

$$\frac{d\mathbf{m}}{dt}(t_{n+\frac{1}{2}}) \approx \delta \mathbf{m}^{n+\frac{1}{2}} := \frac{\mathbf{m}^{n+1} - \mathbf{m}^n}{\tau} \quad , \quad \mathbf{m}(t_{n+\frac{1}{2}}) \approx \overline{\mathbf{m}}^{n+\frac{1}{2}} := \frac{\mathbf{m}^{n+1} + \mathbf{m}^n}{2}$$

Discretization: Spatial



Discretization: Weak form

Weak form of LLG and potential equation:

$$\frac{1}{1+\alpha^2}\int_\Omega (\delta\mathbf{m}^{n+\frac{1}{2}} - \alpha\,\overline{\mathbf{m}}^{n+\frac{1}{2}} \times \delta\mathbf{m}^{n+\frac{1}{2}}) \cdot \mathbf{v}\,\mathrm{d}\mathbf{x} =$$

$$= \int_\Omega \eta\,\mathbf{grad}(\overline{\mathbf{m}}^{n+\frac{1}{2}} \times \mathbf{v}) \cdot \mathbf{grad}\,\overline{\mathbf{m}}^{n+\frac{1}{2}} - Q(\overline{\mathbf{m}}^{n+\frac{1}{2}} \times \mathbf{d})(\mathbf{d} \cdot \overline{\mathbf{m}}^{n+\frac{1}{2}})\mathbf{v} +$$

$$+ (\overline{\mathbf{m}}^{n+\frac{1}{2}} \times \mathbf{grad}\,\psi) \cdot \mathbf{v} - (\overline{\mathbf{m}}^{n+\frac{1}{2}} \times \mathbf{h}_e) \cdot \mathbf{v}\,\mathrm{d}\mathbf{x}$$

$$\int_D \mathbf{grad}\,\psi \cdot \mathbf{grad}\,\varphi\,\mathrm{d}\mathbf{x} = \int_D \overline{\mathbf{m}}^{n+\frac{1}{2}} \cdot \mathbf{grad}\,\varphi\,\mathrm{d}\mathbf{x}\;.$$

for all $\mathbf{v} \in (H^1(\Omega))^3$, and $\varphi \in H^1_0(D)$

Discretization

LLG $\rightarrow |\mathbf{m}|$ is invariant in time

Not the case for our discrete equations.

Need to introduce a *reduced integration* for coupling term:

$$\int_{\Omega} \eta \, \mathbf{grad}(\overline{\mathbf{m}}^{n+\frac{1}{2}} \times \mathbf{v}) \cdot \mathbf{grad}\, \overline{\mathbf{m}}^{n+\frac{1}{2}} \mathrm{d}\mathbf{x}$$

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$$\downarrow\downarrow\downarrow$$

$$\int_{\Omega} \eta \, \mathbf{grad}(I(\overline{\mathbf{m}}^{n+\frac{1}{2}} \times \mathbf{v})) \cdot \mathbf{grad}\, \overline{\mathbf{m}}^{n+\frac{1}{2}} \mathrm{d}\mathbf{x}$$

Newton-Algorithm

$$f_l(m) = kA^1(m_l - \widetilde{m}_l + \alpha(\widetilde{m}_{l+2}m_{l+1} - \widetilde{m}_{l+1}m_{l+2}))$$
$$+ \eta(A^3_{l+2}\overline{m}_{l+1} - A^3_{l+1}\overline{m}_{l+2}) - A_{l+1}\overline{m}_{l+2} + A_{l+2}\overline{m}_{l+1} = 0$$

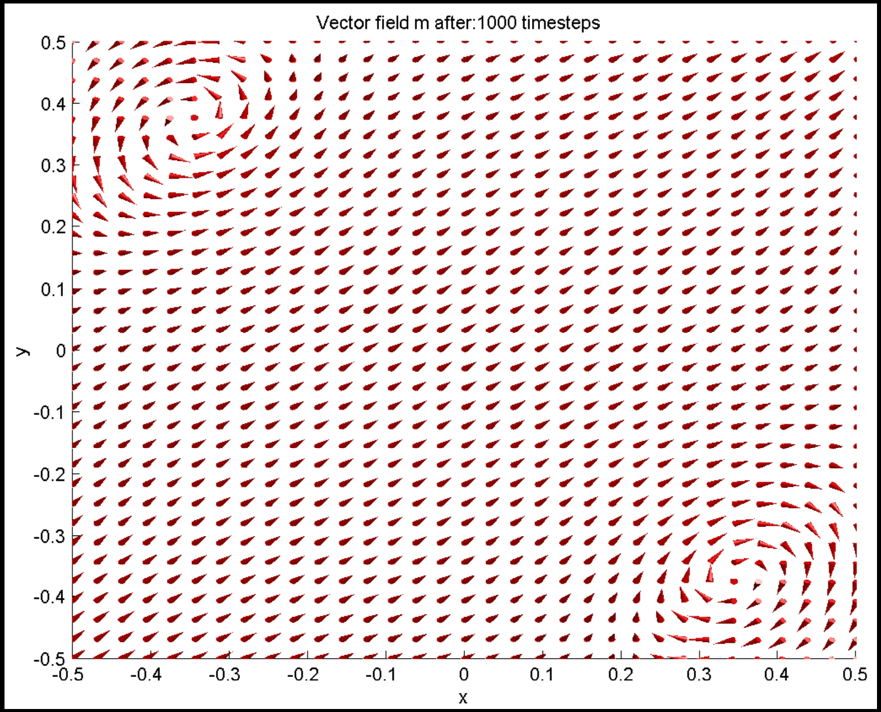
Start with $m = \widetilde{m}$

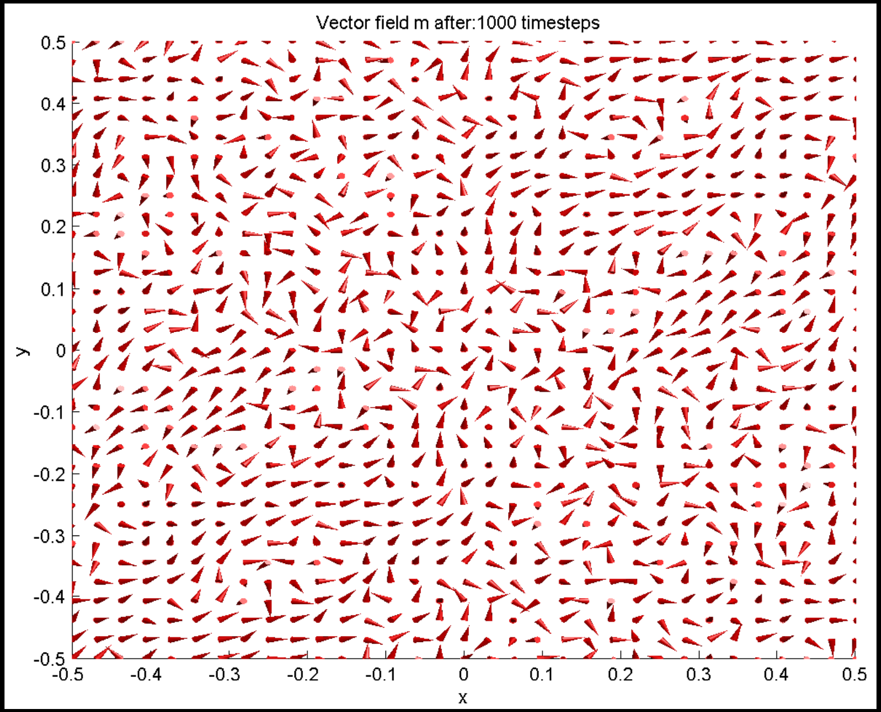
Do while norm(f(m)) > tol

- $m^{it+1} = m^{it} - (Df)^{-1} f(m^{it})$
- if it $>$ itmax, break and error

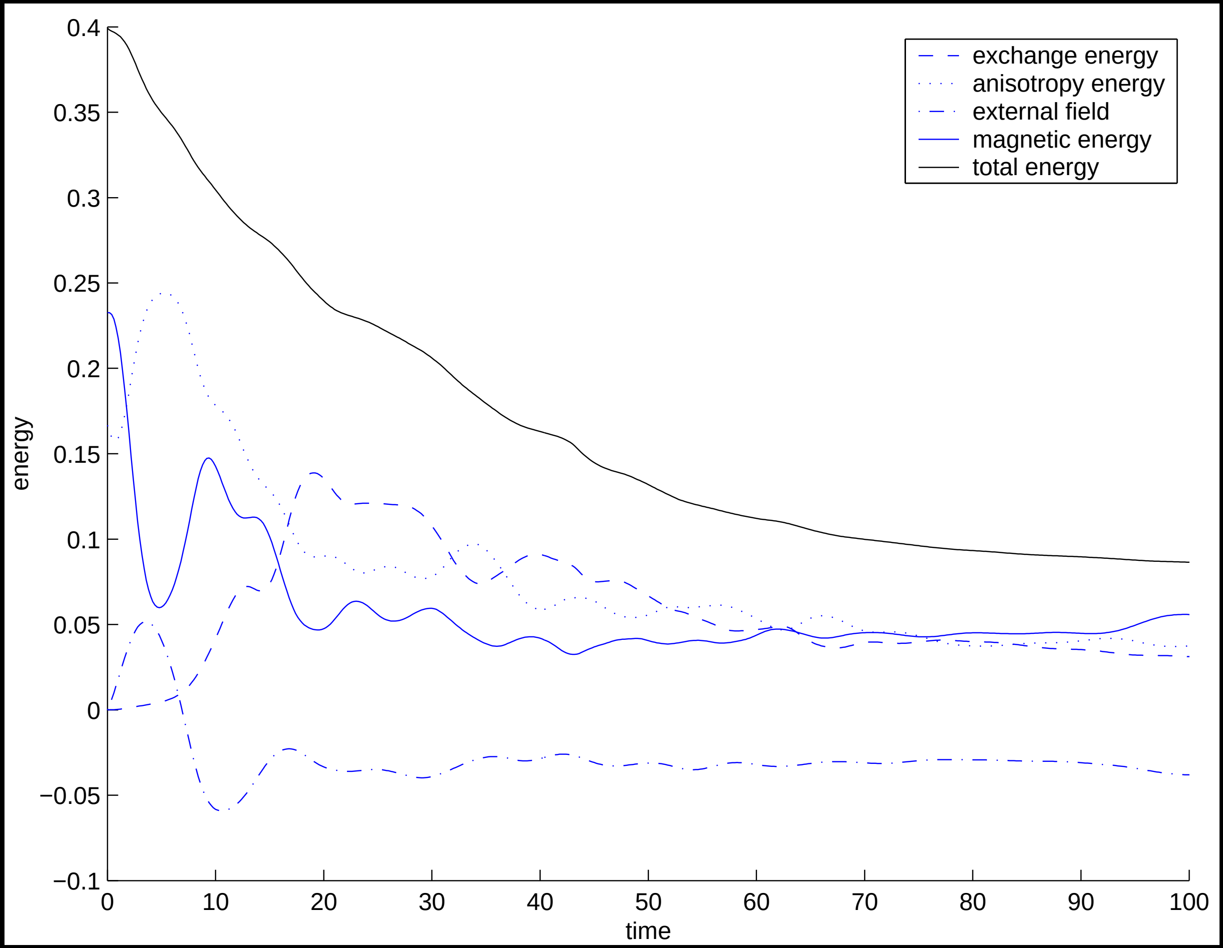
Results: Examples

Strong coupling vs weak coupling

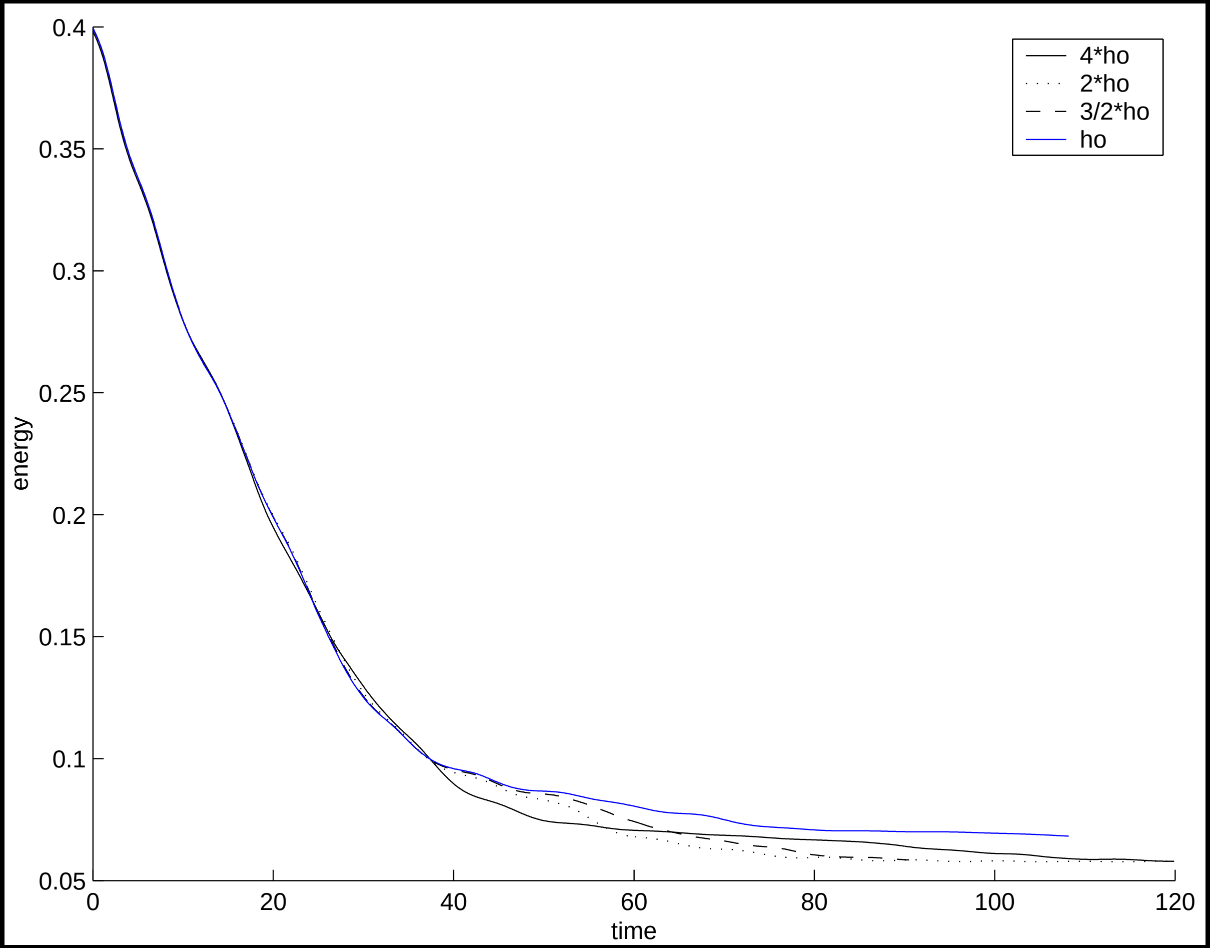




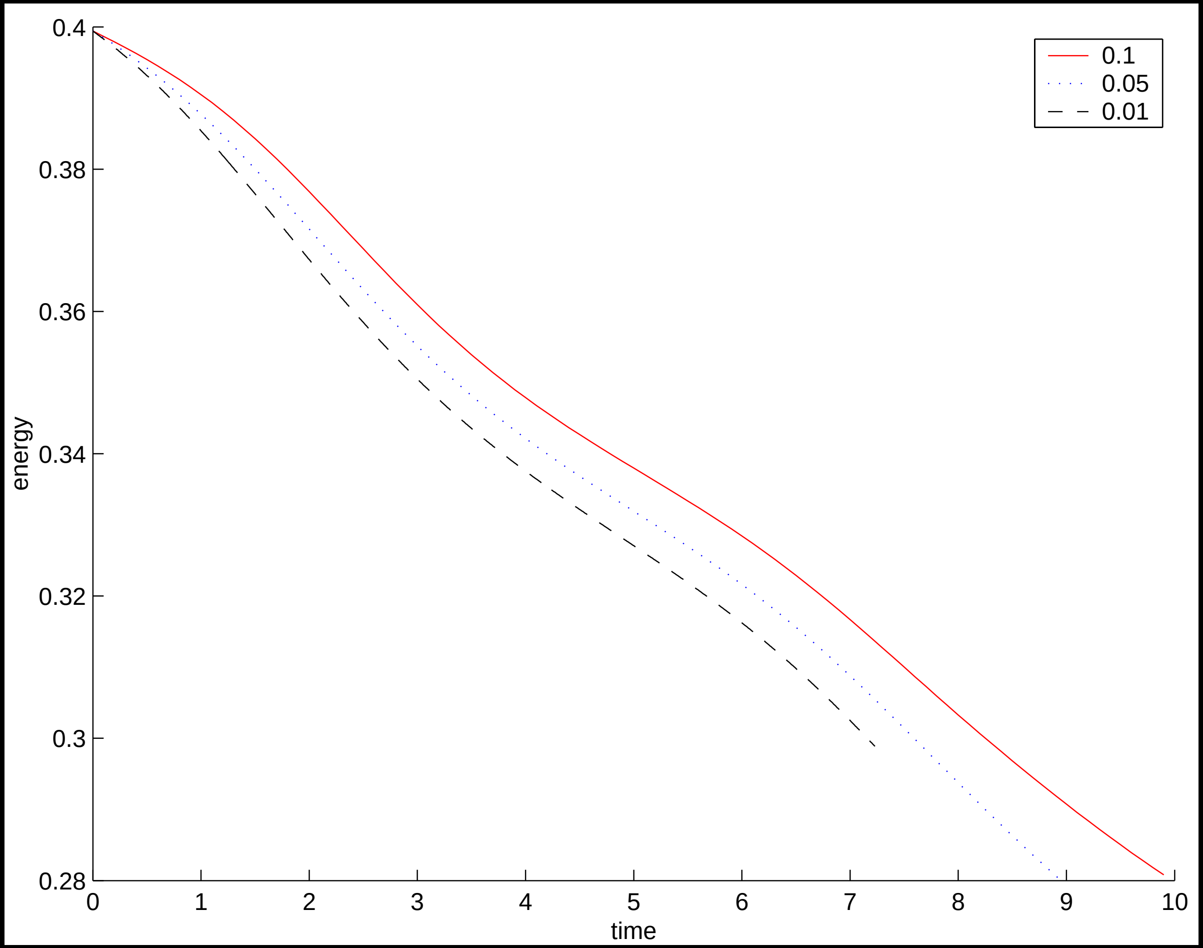
Results: Energy contributions



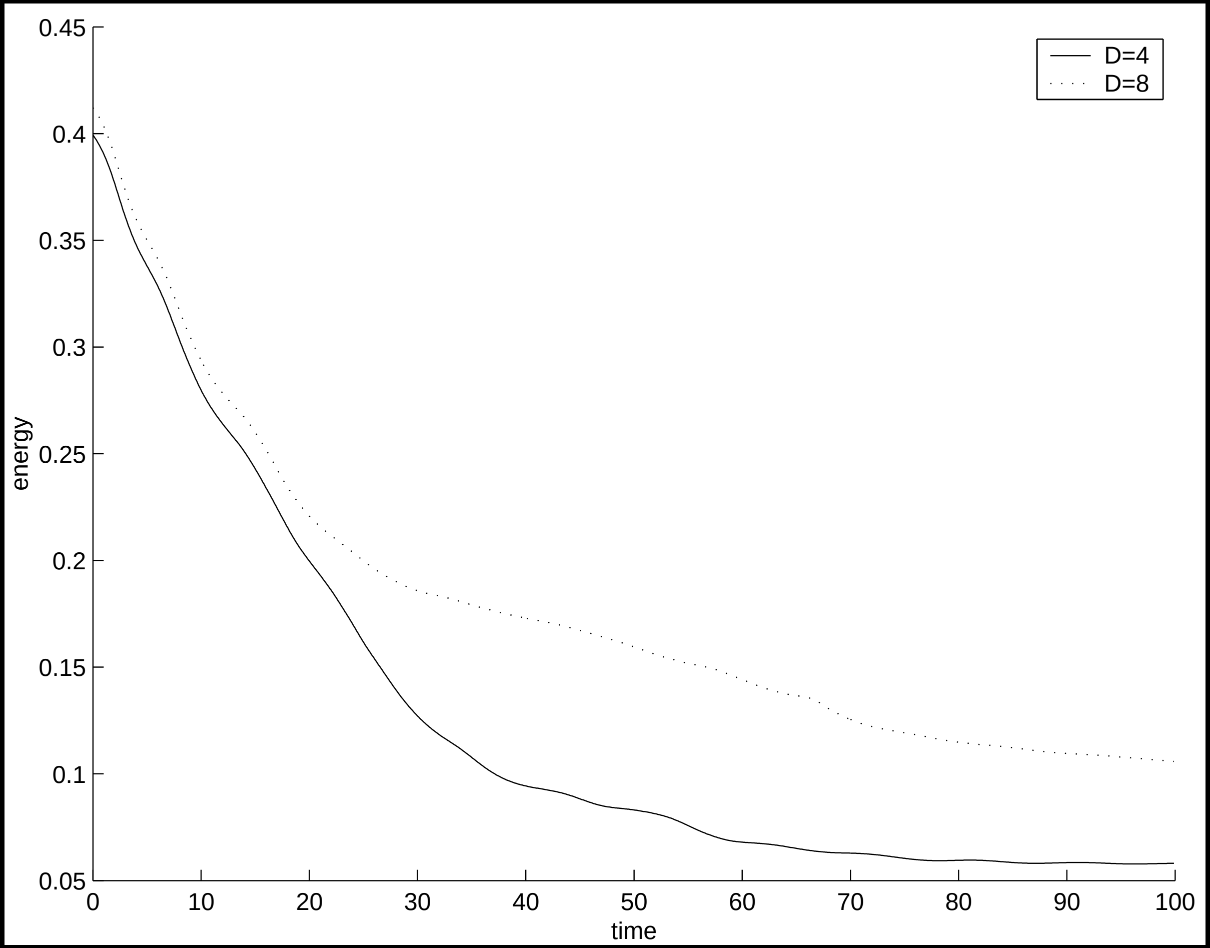
Results: Different mesh widths



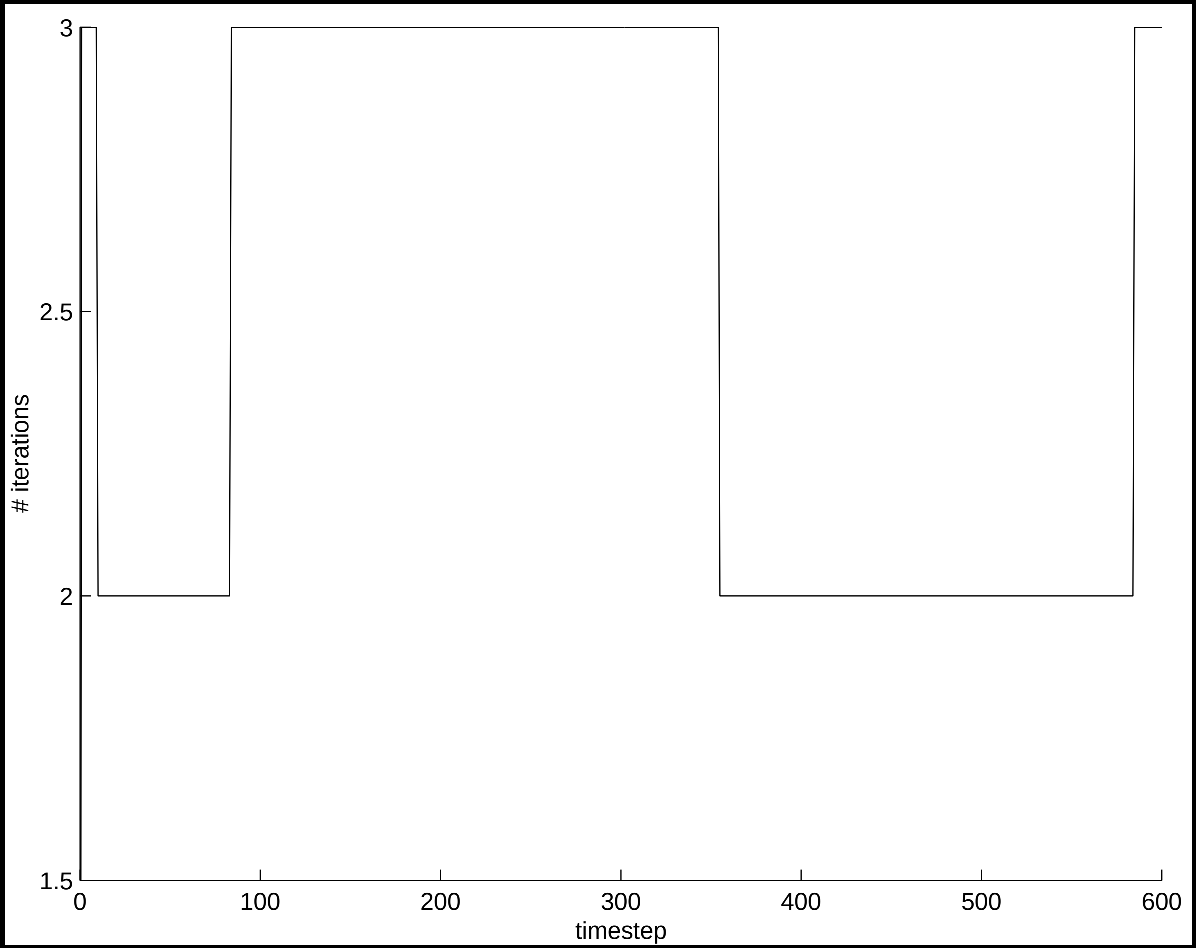
Results: Different timesteps



Results: Varying Size of D



Results: Convergence



Results: Movie example

Dynamical evolution of the magnetization